BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

December 2002 13

Contents

Coming Events

Thematic Term on Mathematics and Engineering

Coordinator

Isabel Maria Narra de Figueiredo (University of Coimbra)

Dates

June-September 2003

The **Thematic Term** for 2003 will be dedicated to Mathematics and Engineering. The application of mathematics to engineering is crucial to knowledge and the development of science. The main objective of the thematic term for 2003 is to improve and emphasize the interdependence between the most recent and important research fields in mathematics and the most important fields of contemporary engineering: informatics engineering, chemical engineering, mechanical engineering, civil engineering and electronics engineering.

The thematic term 2003 consists of four events. The first event is devoted to mathematics and informatics engineering and focuses on soft computing and complex systems. The second event deals with modelling and simulation in chemical engineering. The third event is related to modelling and numerical simulation in continuum mechanics. The fourth event is concerned with mathematics and telecommunications.

Each one of these events is an Advanced School and Workshop, where short courses, lectures and invited talks will be given by well-known invited scientists. So it is expected that the thematic term 2003 will attract a large number of postgraduate students, mathematicians and engineers, interested in contributing to the development of mathematics and its applications to engineering.

The programme of events is the following:

23-27 June: Workshop on Soft Computing and Complex Systems

Organizers

António Dourado Correia (Univ. Coimbra), Ernesto Jorge Costa (Univ. Coimbra), José Félix Costa (I. Superior Técnico - Lisbon), Pedro Quaresma (Univ. Coimbra).

Aims

The main scientific goal of the workshop is to introduce recent developments in mathematical techniques applied to complex engineering problems. In particular, the workshop will focus on different aspects of the area called soft computing, including fuzzy and conexionist systems, evolutionary computation, artificial life and complex systems.

Harnessing complexity is an important aspect of today problem solving. Complexity may be due to the presence of uncertain information or because the regularities of a system, we are trying to understand, cannot be briefly described. We will discuss recent developments in dealing with complexity, by means of introducing the methods and their sound mathematical foundations, as well as through the work of some difficult problems.

The workshop will be held at the Mathematics Department - University of Coimbra.

Lectures

Multi-criteria Genetic Optimisation

Carlos Fonseca, University of Algarve, Portugal

Neural Computation and Applications in Time Series and Signal Processing

Georg Dorffner, Department of Medical Cybernetics and Artificial Intelligence, University of Vienna, Austria

Analogic Computation

José Félix Costa, Department of Mathematics, Technical University of Lisbon, Portugal

State-of-the-art recurrent neural networks, with applications

Juergen Schmidhuber, IDSIA- Instituto Dalle Molle di Studi sull'Intelligenza Artificiale, Switzerland

Neuro-Fuzzy Modelling

Intelligent Control

Robert Babuska, Delft University of Technology, Holland.

For more information on this event, please visit the site

http://hilbert.mat.uc.pt/∼softcomplex/

30 June - 4 July: Workshop on Modelling and Simulation in Chemical Engineering

Organizers

Alírio Egídio Rodrigues (Univ. Porto), Paula Oliveira (Univ. Coimbra), José Almiro Meneses e Castro† (Univ. Coimbra), José Augusto Mendes Ferreira (Univ. Coimbra), Maria do Carmo Coimbra (Univ. Porto).

Aims

The main objective is to bring together mathematicians and chemical engineers to improve the understanding of the problems encountered in process engineering and tools available to solve them. To reach that objective the Workhop is designed:

- To provide the basis for mathematical modeling of chemical engineering systems
- To present some numerical methods to solve model equations in particular in cases of steep moving fronts
- To stress the use of dynamic simulators
- To introduce optimization techniques

The workshop will be held at the CIM headquarters: Complexo do Observatório Astronómico - Universidade de Coimbra.

Short Courses

Modeling in Chemical Engineering

S. Sotirchos and A. Rodrigues, University Rochester, USA and LSRE-FEUP, University of Porto, Portugal

Numerical Simulations with Advection-Diffusion-Reaction Systems

W. Hundsdorfer, Center for Mathematics and Computer Science, The Netherlands

Optimization and Control of Chemical Processes

N. Oliveira, University of Coimbra, Portugal

Invited talks

Adaptive finite element solutions of dependent partial differential equations using moving grid algorithms

J. M. Baines, Department of Mathematics, University of Reading, United Kingdom

Inorganic chemistry and mathematics

R.Mattheij, Department of Mathematics and Computer Science, Tech. University of Eindhoven, The Netherlands

Sensitivity Analysis for Differential-Algebraic Equations and More

Linda Petzold, UC Santa Barbara, USA

Splitting Methods for Advection-Diffusion-Reactions Problems

J. G. Verwer, Center for Mathematics and Computer Science, CWI, Amsterdam, The Neterlands

Numerical and Computational Challenges in Environmental Modeling

Z. Zlatev, National Environmental Research Institute, Denmark

For more information on this event, please visit the site

http://www.fe.up.pt/lsre/cim2msce/workshop.html

14-18 July: Advanced School and Workshop on Modelling and Numerical Simulation in Continuum Mechanics

Organizers

Luís Filipe Menezes (Univ. Coimbra), Isabel Maria Narra de Figueiredo (Univ. Coimbra), Juha Videman (I. Superior Técnico - Lisbon).

Aims

The scientific goals of this event are the following:

- to present some of the most important recent fields of research in mathematics and its applications to civil and mechanical engineering
- to promote the interdisciplinary aspects of the field by establishing contacts between mathematicians and engineers
- to provide an opportunity for Portuguese scientists to present and discuss their research work.

This event will take place at the Department of Mechanical Engineering - University of Coimbra.

Short Courses

Numerical analysis of discrete schemes approximating grade-two fluid models. Recent results and open problems

Vivette Girault (Université Pierre et Marie Curie, France)

Shape optimization

Patrick Le Tallec (École Polytechnique, France)

Advances in the finite point method for meshless analysis of problems in solid and fluid mechanics

Eugenio Oñate (CIMNE, Universitat Politècnica de Catalunya, Spain)

Mathematics and numerics of shell problems

Juhani Pitkäranta (Helsinki University of Technology, Finland)

Computational mechanics of solid materials at large strains

Cristian Teodosiu (Université de Paris Nord, France)

For more information on this event, please visit the site

http://www.math.ist.utl.pt/wmnscm/

8-12 September: Mathematical Techniques and Problems in Telecommunications

Organizers

Carlos Salema (I. Superior Técnico - Lisbon), Joaquim Júdice (Univ. Coimbra), Carlos Fernandes (I. Superior Técnico - Lisbon), Mário Figueiredo (I. Superior Técnico - Lisbon), Luís Merca Fernandes (I. P. Tomar).

Aims

The goals are three fold. Firstly we will try to identify and possibly provide solutions for a number of mathematical problems in the field of Telecommunications. Secondly we intend to disseminate among telecommunications engineers some mathematical techniques which are not widely known in this community even if they are being applied in modern communication techniques. Finally we would like to improve mutual understanding and recognition between mathematicians and telecommunication engineers, one of the heaviest users of mathematical techniques in the field of engineering.

This event comes in the follow-up of rather successful, even if less ambitious event, "Matemática em Telecomunicações: Que Problemas?" with similar objectives organized by IT in 1997.

This event will take place at the Instituto Politécnico de Tomar.

Invited Lectures

Combinatorial Optimization in Telecommunications

Mauricio Resende, ATT, USA

Transforms, Algorithms and Applications

Joana Soares U. Minho, Portugal

Controllability of PDE's and its Discrete Approximations

Enrique Zuazua U. A. Madrid, Spain

Evolutionary Computing

Eckart Zitzler SFIT, Switzerland

Stochastic Processes in Telecommunications Traffic

Ivete Gomes CEAUL, Portugal

For more information on this event, please visit the site

http://www.lx.it.pt/mtpt/

Third Debate on Mathematical Research in Portugal

Porto, 25 October 2003

Organizers: José Ferreira Alves (Univ. Porto), José Miguel Urbano (Univ. Coimbra).

This event will take place at the Pure Mathematics Department, University of Porto.

Themes

- Evaluation and Funding
- The Challenge of Excellence
- Mathematical Research in Industry

For more information on this event, please visit the site

http://www.fc.up.pt/cmup/jfalves/debate/

CIM News

ERCOM

CIM has been for some years a member of ERCOM, a network of European Research Centres on Mathematics.

ERCOM is an European Mathematical Society committee consisting of Scientific Directors of the member Centres, or their chosen representatives. Only centres for which the number of visiting staff substantially exceeds the number of permanent and long-term staff and that cover Mathematical Sciences broadly are eligible for representation in ERCOM. The eligibility of centres is decided by the EMS Executive Committee.

ERCOM aims to contribute to the unity of Mathematics, from fundamentals to applications.

The purposes of ERCOM are:

- to constitute a forum for communications and exchange of information between the centres themselves and EMS
- to foster collaboration and coordination between the centres themselves and EMS
- to foster advanced research training on a European level
- to advise the Executive Committee of the EMS on matters relating to activities of the centres
- to contribute to the visibility of the EMS
- to cultivate contacts with similar research centres within and outside Europe

The current members of ERCOM are:

- **Abdus Salam International Centre for Theoretical Physics**, Trieste, Italy.
  www.ictp.trieste.it
- **Centre International de Rencontres Mathématiques**, Luminy, France.
  www.cirm.univ-mrs.fr
- **Centre de Recerca Matemàtica**, Barcelona, Spain.
  www.crm.es/
- **Centre for Mathematical Physics and Stochastics**, Aarhus, Denmark.
  www.maphysto.dk

- **Centro Internacional de Matemática**, Coimbra, Portugal.
www.cim.pt

- **Centrum voor Wiskunde en Informatica**, Amsterdam, The Netherlands.
www.cwi.nl

- **Emmy Noether Research Institute for Mathematics**, Ramat-Gan, Israel.
www.cs.biu.ac.il/∼eni

- **Erwin Schrödinger International Institute for Mathematical Physics**, Vienna, Austria.
www.esi.ac.at

- **Euler International Mathematical Institute**, St Petersburg, Russia.
www.pdmi.ras.ru/EIMI

- **European Institute for Statistics, Probability and Operations Research**, Eindhoven, The Netherlands.
www.eurandom.nl

- **Institut des Hautes Études Scientifiques**, Bures-Sur-Yvette, France.
www.ihes.fr

- **Institut Henri Poincaré, Centre Emile Borel**, Paris, France.
www.ihp.jussieu.fr

- **Institut Mittag-Leffler**, Djursholm, Sweden.
www.ml.kva.se

- **International Centre for Mathematical Sciences**, Edinburgh, UK.
www.ma.hw.ac.uk/icms/

- **Isaac Newton Institute for Mathematical Sciences**, Cambridge, UK.
www.newton.cam.ac.uk

- **Istituto Nazionale di Alta Matematica**, Rome, Italy.
indam.mat.uniroma1.it

- **Lorentz Center**, Leiden, The Netherlands.
www.lc.leidenuniv.nl

- **Mathematical Research Institute**, Nijmegen, The Netherlands.
www.sci.kun.nl/mri/

- **Mathematisches Forschungsinstitut Oberwolfach**, Oberwolfach, Germany.
www.mfo.de

- **Max-Planck-Institut for Mathematics**, Bonn, Germany.
www.mpim-bonn.mpg.de

- **Max Planck Institute for Mathematics in the Sciences**, Leipzig, Germany.
www.mis.mpg.de

- **Stefan Banach International Mathematical Center**, Warsaw, Poland.
www.impan.gov.pl/BC/

- **Thomas Stieltjes Institute for Mathematics**, Leiden, The Netherlands.
www.stieltjes.org

CIM Publications

CIM has just added two more items to its series of monographs and volumes of proceedings. The complete list is now as follows:

1. Pedro V. Silva, *Introdução à Teoria Combinatória de Semigrupos Inversos*, 1996.
2. João Tiago Mexia, *Introdução à Teoria Estatística do Risco*, 1996.
3. S. A. Robertson, *Three Talks on Convex Bodies*, 1997.
4. J. A. Green, *One Hundred Years of Group Representations*, 1997.
5. Paul A. Fuhrmann, *Linear Algebra and Control - Lecture Notes*, 1998.
6. Isabel N. Figueiredo (ed.), *Escola de Elementos Finitos e Aplicações*, 1998.
7. A. Ornelas, A. C. Barroso, J. Palhoto de Matos, J. Matias and P. Pedregal (ed.), *Mathematical Methods in Materials Science and Engineering - International Summer School*, 1999.
8. Grant Walker, *Some Aspects of the Action of Matrices over F_p on Polynomials*, 1998.
9. J. F. Queiró (ed.), *A Investigação Matemática em Portugal: Tendências, Organização e Perspectivas*, 1999.
10. Nazaré M. Lopes and E. Gonçalves (ed.), *On Nonparametric and Semiparametric Statistics*, 1999.
11. A. Sequeira (ed.), *International Summer School on Industrial Mathematics*, 1999.

12. A. Sequeira (ed.), *International Summer School on Computational Fluid Dynamics*, 1999.

13. A. Sequeira (ed.), *Navier-Stokes Equations and Related Topics (International Summer School)*, 1999.

14. L. Trabucho and J. F. Queiró (ed.), *O ensino da Matemática na universidade em Portugal e assuntos relacionados*, 2000.

15. M. Field, *Complex Dynamics in Symmetric Systems*, 2000.

16. M. Golubitsky and I. Stewart, *The Symmetry Perspective: From Equilibria to Chaos in Phase Space and Physical Space*, 2000.

17. L. N. Vicente (ed.), *Segundo Debate sobre a Investigação Matemática em Portugal*, 2001.

18. J. M. Guedes and H. Rodrigues (ed.), *Bone Mechanics: Mathematical and Mechanical Models for Analysis and Synthesis*, 2002.

19. J. Martins and E. B. Pires (ed.), *Mathematical and Computational Modeling of Biological Systems*, 2002.

The complete text of volumes 9, 14 and 17 (the proceedings of the three CIM debates) can be found online at the CIM website.

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the University of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form

http://www.cim.pt/cim.www/cim_app/application.htm

CIM on the WWW

Complete information about CIM and its activities can be found at the site

http://www.cim.pt

This is mirrored at

http://at.yorku.ca/cim.www/

Warp Drive With Zero Expansion

by José Natário

Departamento de Matemática
Instituto Superior Técnico

Introduction

As everyone knows, Einstein's Relativity forbids all material objects (or even signals) to travel faster than light. What is sometimes ignored is that this is a *local* statement: speed with respect to an observer can only be defined in a neighborhood of this observer. For instance, it is well known that the universe is expanding, all galaxies (on average) speeding away from each other. An analogy which is particularly well suited is the surface of an expanding balloon, with the galaxies as points on this surface. Although the galaxies are not moving with respect to the balloon's surface, the distance between them is increasing; if they are sufficiently far apart (i.e., if the balloon is large enough), then the distance will increase faster than 300000 kilometers per second. So in a sense they will be moving faster than the speed of light with respect to each other. This is indeed what happens with galaxies at the edge of the visible universe. The "thou shall not travel faster than light" commandment in this analogy simply forbids objects to travel faster than light with respect to the balloon's surface. (Incidentally, this analogy also shows that there is no "center of the universe" where the Big Bang occurred; the Big Bang simply means the epoch where the balloon was very small and very hot - in a sense it happened in all points of space).

These ideas were used by Miguel Alcubierre ([Alc94]) to construct (in theory) a "warp drive", allowing a spaceship to travel faster than light, by deforming space in the following manner: take a ball containing your spaceship (the "warp bubble"), and keep it undeformed; contract space in front of the bubble, expand space behind it. Since there is no *a priori* constraint on the speed of contraction/expansion, it is possible to move the bubble from one point to another as quickly as one wishes.

In what follows we will explain exactly how this is done within the mathematical framework General Relativity, show how it can be generalized and see how this attempt at circumventing Einstein's prohibition is doomed to fail.

General Relativity

General Relativity is the physical theory of space, time and gravitation. It states that spacetime is a 4-dimensional Lorentzian manifold (i.e., a pseudo-Riemannian manifold (M, g) for which the metric g has signature $(-,+,+,+)$), satisfying the *Einstein equation*

$$G = 8\pi T,$$

where the *Einstein tensor* G is just the trace-reversed Ricci tensor,

$$G = R - \frac{\operatorname{tr} R}{2} g,$$

and the *energy-momentum tensor* T describes the matter content of the spacetime. Thus any 4-dimensional Lorentzian manifold can be thought of as a spacetime containing the matter described by

$$T = \frac{1}{8\pi} G.$$

However, an arbitrary choice is almost certain to generate an unphysical energy-momentum tensor.

A nonzero tangent vector $v \in TM$ is said to be *timelike*, *lightlike* or *spacelike* according to whether $g(v,v) < 0$, $g(v,v) = 0$ or $g(v,v) > 0$ (the zero vector is by definition spacelike). A curve $c : \mathbb{R} \to M$ whose tangent vector $\dot{c}$ remains in one of the above classes is given the same name. Timelike curves are interpreted as possible histories of test particles with nonvanishing rest mass (which must travel slower than light); the length

$$\tau = \int_{t_0}^{t_1} |g(\dot{c}(t), \dot{c}(t))|^{\frac{1}{2}} dt$$

is then the time measured by the particle between the events $c(t_0)$ and $c(t_1)$. Lightlike curves are interpreted as possible histories of test particles with vanishing rest mass (which must travel at the speed of light, e.g., photons or neutrinos).

If c is a geodesic, i.e.,

$$\nabla_{\dot{c}}\dot{c} = 0,$$

then

$$\frac{d}{dt}\left[g(\dot{c},\dot{c})\right] = 2g\left(\dot{c},\nabla_{\dot{c}}\dot{c}\right) = 0.$$

Thus geodesics are always curves of a given type. Timelike geodesics are interpreted as the histories of free-falling test particles with nonzero rest mass; the fact that they are geodesics means that free-falling particles measure *more* time between any two (sufficiently close) events than any other particle. Lightlike geodesics are interpreted as the histories of free-falling test particles with zero rest mass; the extremality property in this case is that no other massive or massless particle can travel between two (sufficiently close) events on the lightlike geodesic.

Unlike Newtonian mechanics, General Relativity provides no canonical way of splitting spacetime into space plus time. A possible choice is to take an arbitrary *spacelike hypersurface*, i.e., a hypersurface $\Sigma \subset M$ whose orthogonal vector field is timelike, and consider its evolution along the orthogonal geodesics. A local chart (x^1, x^2, x^3) on Σ can therefore be extended to a local chart (t, x^1, x^2, x^3) on M (from this point on called an *Eulerian chart*), where t is the arclength (time) measured along the orthogonal (timelike) geodesic. In these local coordinates, the metric is just

$$g = -dt \otimes dt + \gamma(t),$$

where

$$\gamma(t) = g_{ij}(t, x^1, x^2, x^3)dx^i \otimes dx^j$$

must be a Riemannian metric on Σ (we are using the summation convention on the indices $i, j = 1, 2, 3$). This allows us to interpret General Relativity as describing the evolution of a Riemmannian metric $\gamma(t)$ on the 3-dimensional manifold Σ. This metric yields the distances measured between nearby Eulerian observers.

The Einstein equation can then be formulated in terms of γ and the *extrinsic curvature*

$$K = \frac{1}{2}\frac{\partial\gamma}{\partial t}.$$

It implies

$$\frac{\partial}{\partial t}(\operatorname{tr} K) - \operatorname{tr}(K^2) = -8\pi\left(T_{00} - \frac{1}{2}\operatorname{tr} T\right);$$
$$\operatorname{tr} R + (\operatorname{tr} K)^2 - \operatorname{tr}(K^2) = 16\pi T_{00},$$

where $R = R(t)$ is now the Ricci tensor of (Σ, γ) (not (M, g)).

It is also possible to show that

$$\operatorname{tr} K = \frac{1}{(\det\gamma)^{\frac{1}{2}}}\frac{\partial}{\partial t}\left[(\det\gamma)^{\frac{1}{2}}\right].$$

In other words, $\operatorname{tr} K$ measures the fractional variation of the volume element for Eulerian observers: $\operatorname{tr} K < 0$ in some region means that the volume of that region is decreasing.

Most models of matter are described by energy-momentum tensors satisfying both the *strong energy condition*, which implies

$$T_{00} - \frac{1}{2}\operatorname{tr} T \geq 0,$$

and the *weak energy condition*, which implies

$$T_{00} \geq 0$$

(confusingly the strong energy condition does not imply the weak energy condition). If T satisfies the strong energy condition and $\operatorname{tr} K$ does not vanish at some point then our Eulerian chart must break down at some value of t: indeed, in this case the Einstein equation implies that

$$\frac{\partial}{\partial t}(\operatorname{tr} K) - \operatorname{tr}(K^2) \leq 0.$$

Using the inequality

$$(\operatorname{tr} A)^2 \leq n\operatorname{tr}(A^2)$$

(which holds for any real $n \times n$ symmetric matrix A) one can easily prove that starting from a nonzero value $\operatorname{tr} K$ must blow up in finite time. This breaking down of the Eulerian chart can either be a coordinate singularity or a genuine geometric singularity (meaning that M is geodesically incomplete); indeed this can be thought of as a primitive version of the famous Penrose-Hawking singularity theorems.

Warp Drive Spacetimes

We will now describe a class of spacetimes which can be understood simply by studying a (time-dependent) vector field in Euclidean 3-space. These will then be used to construct our warp drives. If you find what follows a bit too technical you can turn to the short summary in the beginning of section 4.

Definition 1. *A* warp drive spacetime (M, g) *is defined by taking* $M = \mathbb{R}^4$ *with the usual Cartesian coordinates* $(t, x, y, z) \equiv (t, x^i)$ *and*

$$g = -dt \otimes dt + \sum_{i=1}^{3}(dx^i - X^i dt) \otimes (dx^i - X^i dt)$$

for three unspecified bounded smooth functions $(X^i) \equiv (X, Y, Z)$.

The Riemannian metric γ induced in the spacelike hypersurfaces $\{dt = 0\}$ is just the ordinary Euclidean flat metric. A warp drive spacetime is completely defined by the vector field

$$\mathbf{X} = X^i \frac{\partial}{\partial x^i} = X \frac{\partial}{\partial x} + Y \frac{\partial}{\partial y} + Z \frac{\partial}{\partial z},$$

which we can think of as a (time-dependent) vector field defined in Euclidean 3-space.

Notice carefully that the chart (t, x^i) is *not* an Eulerian chart; Eulerian observers' histories are integral curves of the unit normal vector to the $\{dt = 0\}$ hypersurfaces,

$$n = \frac{\partial}{\partial t} + X^i \frac{\partial}{\partial x^i} = \frac{\partial}{\partial t} + \mathbf{X}.$$

Proposition 2. *The extrinsic curvature tensor is*

$$K = \frac{1}{2}\left(\partial_i X^j + \partial_j X^i\right) dx^i \otimes dx^j.$$

Proof. The extrinsic curvature tensor is given by

$$K = \frac{1}{2}\pounds_n \gamma = \frac{1}{2}\pounds_{\left(\frac{\partial}{\partial t} + \mathbf{X}\right)}\gamma.$$

Now

$$\pounds_{\frac{\partial}{\partial t}}\gamma = \pounds_{\frac{\partial}{\partial t}}\delta_{ij} dx^i \otimes dx^j = \frac{\partial \delta_{ij}}{\partial t} dx^i \otimes dx^j = 0$$

(where δ_{ij} is the Kronecker delta). On the other hand, since $\mathbf{X}$ is tangent to the spacelike hypersurfaces, we can use the usual formula for the Lie derivative of the metric,

$$\pounds_{\mathbf{X}}\gamma = \left(\delta_{kj} D_i X^k + \delta_{ik} D_j X^k\right) dx^i \otimes dx^j =$$

$$\left(D_i X^j + D_j X^i\right) dx^i \otimes dx^j,$$

where D stands for the Levi-Civita connection determined by γ. Since γ is just the flat Euclidean metric, $D = \partial$ and we get the formula above. □

Corollary 3. *The expansion of the volume element associated with the Eulerian observers is given by* $\nabla \cdot \mathbf{X}$.

Proof. We just have to notice that

$$\operatorname{tr} K = K^i{}_i = \partial_i X^i.$$

□

Corollary 4. *A warp drive spacetime is flat wherever* $\mathbf{X}$ *is a Killing vector field for the Euclidean metric (irrespective of time dependence). In particular, a warp drive spacetime is flat wherever* $\mathbf{X}$ *is spatially constant.*

Proof. Since the spacelike surfaces are flat, all curvature comes from the extrinsic curvature. Thus the spacetime will be flat wherever the extrinsic curvature is zero, *i.e.*, wherever $\pounds_{\mathbf{X}}\gamma = 0$. □

In particular, the Einstein equation implies that there is no matter in these regions. Also there is no geodesic deviation, and hence no tidal forces.

Theorem 5. *Non flat warp drive spacetimes violate the weak or the strong energy condition.*

Proof. We already know that if the strong energy condition holds and $\operatorname{tr} K \neq 0$ at some event, then $\operatorname{tr} K$ blows up in finite time. Since $\nabla \cdot \mathbf{X}$ is finite, the strong energy condition can only hold if $\operatorname{tr} K \equiv 0$. However, it follows from the Einstein equation that

$$T_{00} = \frac{1}{16\pi}\left(\operatorname{tr} R + (\operatorname{tr} K)^2 - \operatorname{tr}(K^2)\right)$$

where $R = 0$ is the Ricci tensor of the flat Cauchy surfaces $dt = 0$. Thus if $\operatorname{tr} K = 0$ we have $T_{00} \leq 0$, and $T_{00} = 0$ *iff* $K \equiv 0$. Consequently if the spacetime does not violate neither the strong nor the weak energy conditions it must be flat. □

Warp Drive With Zero Expansion

We have seen in the previous section that given a time-dependent smooth bounded vector field $\mathbf{X}$ in Euclidean 3-space we can construct a Lorentzian manifold with a global chart $\{t, x^i\}$ having the following properties:

1. The space sections $\{dt = 0\}$ are just Euclidean 3-space;
2. The free-fall Eulerian observers move in this Euclidean 3-space with velocity $\mathbf{X}$;
3. The fractional volume variation of these observers is (unsurprisingly) $\nabla \cdot \mathbf{X}$;
4. There exists no matter nor tidal forces wherever $\mathbf{X}$ is spatially constant;
5. Unfortunately, the strong or weak energy conditions are always violated.

If

$$\dot{c} = \dot{t}\frac{\partial}{\partial t} + \dot{x}^i \frac{\partial}{\partial x^i}$$

is the tangent vector to a timelike geodesic, we must have

$$g(\dot{c}, \dot{c}) < 0 \Leftrightarrow -\dot{t}^2 + \sum_{i=1}^{3}(\dot{x}^i - X^i \dot{t})^2 < 0 \Leftrightarrow \left\| \frac{d\mathbf{x}}{dt} - \mathbf{X} \right\| < 1,$$

where

$$\mathbf{x} = x^i \frac{\partial}{\partial x^i} = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}.$$

This can be readily interpreted as meaning that the speed of any test particle with nonvanishing rest mass with respect to the Eulerian observers must be smaller than the speed of light (which is normalized to 1). However, *there is no a priori limit for the speed* $\mathbf{X}$ *of the Eulerian observers themselves.* This was used by Miguel Alcubierre ([Alc94]) to construct the following example of a spacetime in which superluminal travel is possible:

Example 6. *Choose*

$$X = v_s f(r_s);$$
$$Y = Z = 0,$$

with

$$v_s(t) = \frac{dx_s(t)}{dt};$$
$$r_s = \left[(x - x_s)^2 + y^2 + z^2\right]^{\frac{1}{2}},$$

where $x_s(t)$ *is arbitrary and* $f : [0, +\infty) \to [0, 1]$ *is a smooth function such that* $f' \leq 0$, $f = 1$ *in a neighborhood of the origin and* $f = 0$ *in a neighborhood of infinity. Let us call the region* $f(r_s) = 1$ *the* interior of the warp bubble *and the region* $f(r_s) = 0$ *the* exterior of the warp bubble*. In both these regions* $\mathbf{X}$ *is spatially constant, and hence they contain no matter and generate no tidal forces; nevertheless, Eulerian observers inside the warp bubble move with arbitrary speed* v_s *with respect to Eulerian observers outside the warp bubble (there is no reason why* v_s *should be smaller than* 1*).*

The expansion of the volume element associated with the Eulerian observers in this example is

$$\operatorname{tr} K = \partial_x X = v_s f'(r_s) \frac{x - x_s}{r_s}.$$

Since $f' \leq 0$, *we see that volume is decreasing in front of the bubble and increasing behind it. This compression/expansion was thought to be a fundamental ingredient in the warp drive mechanism; we will presently see that it's not. Alcubierre also found that the energy conditions were violated at the bubble's wall (i.e., the region where* $f' \neq 0$*), as we now know to be unavoidable.*

It is convenient to replace the x *coordinate with*

$$\xi = x - x_s(t).$$

This effectively corresponds to replacing X *with* $X - v_s$*, so that the Eulerian observers inside the bubble stand still whereas the Eulerian observers outside the bubble move with speed* v_s *in the negative* ξ*-direction. Obviously* $\operatorname{tr} K$ *retains its value, but now*

$$r_s = \left(\xi^2 + y^2 + z^2\right)^{\frac{1}{2}}$$

does not depend on the coordinate t*.*

Definition 7. *The vector field* $\mathbf{X}$ *is said to* generate a warp bubble with velocity $\mathbf{v}_s(t)$ *if* $\mathbf{X} = \mathbf{0}$ *for small* $\|\mathbf{x}\|$ *(the* interior of the warp bubble*) and* $\mathbf{X} = -\mathbf{v}_s(t)$ *for large* $\|\mathbf{x}\|$ *(the m exterior of the warp bubble)*

To construct a warp drive with zero expansion all one has to do then is to find a divergenceless field generating a warp bubble with velocity $v_s(t)\frac{\partial}{\partial x}$ (see [Nat02] for details on how to do this).

The Alcubierre warp drive can be pictured as contracting space in front of the warp bubble and expanding it behind; a zero expansion warp drive can be thought of as sliding the warp bubble through normal space.

Lightlike geodesics and horizons

Besides violating energy conditions, warp drive spacetimes have much more serious problems, namely horizons. To see evidence of this, let us consider the case of a vector field $\mathbf{X}$ generating a warp bubble with velocity $v_s \frac{\partial}{\partial x}$ satisfying

$$\frac{\partial \mathbf{X}}{\partial t} = \mathbf{0} \ \left(\Rightarrow \frac{dv_s}{dt} = 0\right).$$

Since null geodesics must satisfy

$$g(\dot{c}, \dot{c}) = 0 \Leftrightarrow dt^2 = \sum_{i=1}^{3} (dx^i - X^i dt)^2 \Leftrightarrow \left\| \frac{d\mathbf{x}}{dt} - \mathbf{X} \right\| = 1,$$

we see that a flash of light outside the warp bubble can be pictured in the Euclidean 3-space as a spherical wavefront which is simultaneously expanding with speed 1 and moving in the direction of $\mathbf{X}$ with speed $\|\mathbf{X}\| = v_s$. Thus it is clear that if $v_s > 1$ then events inside the warp bubble cannot causally influence events outside the warp bubble at large positive values of x, as no particle emitted from inside the bubble can reach those points. Assuming cylindrical symmetry about the x-axis, there will be a point on the positive x-axis where $\|\mathbf{X}\| = 1$; the cylindrically symmetric surface through this point whose angle α with $\mathbf{X}$ is given by

$$\sin \alpha = \frac{1}{\|\mathbf{X}\|}$$

is a horizon, in the sense that events inside the warp bubble cannot causally influence events on the other side of this surface (see figure 1). Notice that away from the warp bubble we have

$$\sin \alpha = \frac{1}{v_s}$$

which is the familiar expression for the Mach cone angle. Also notice that the interior of the warp bubble is causally disconnected from part of the bubble's wall. This is the so-called you-need-one-to-make-one problem with the warp drive: the warp bubble wall, where your (unphysical) matter fields live, cannot be generated from inside the bubble. You'd need someone who was already traveling faster than light to generate it for you.

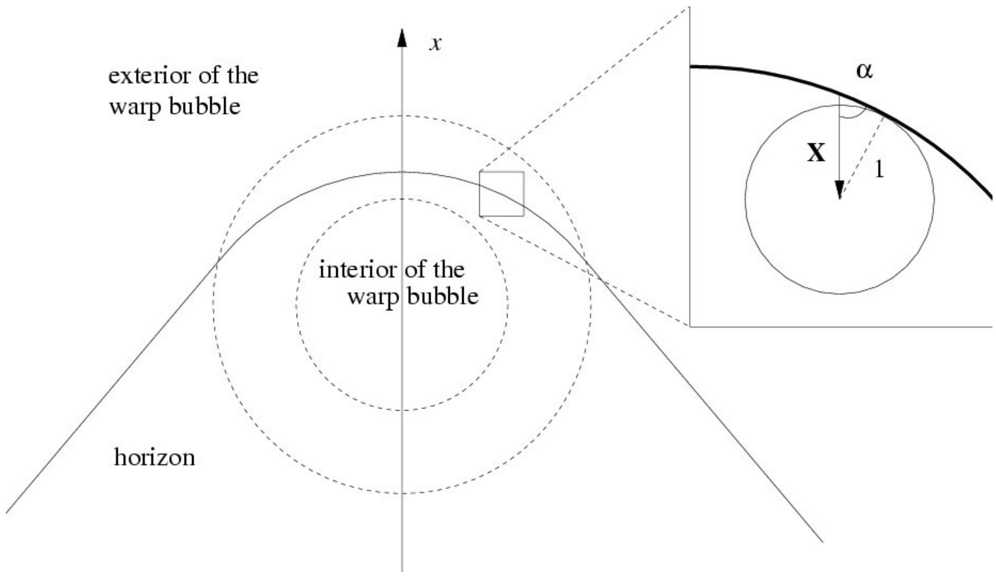

Figure 1: Computing the horizon.

Bibliography

[Alc94] M. Alcubierre, *The warp drive: hyper-fast travel within general relativity*, Class. Quantum Grav. **11** (1994), L73–L77.

[LL97] L. Landau and E. Lifshitz, *The classical theory of fields*, Butterworth-Heinemann, 1997.

[Nat02] J. Natario, *Warp drive with zero expansion*, Class. Quantum Grav. **19** (2002), 1157-1165.

[Wal84] R. Wald, *General relativity*, University of Chicago Press, 1984.

What's New in Mathematics

A new kind of Science?

"By relying on mathematical equations to describe the world, scientists for centuries have grossly limited their powers of explanation, asserts Stephen Wolfram" is the start of Richard Monastersky's piece *(Chronicle of Higher Education*, May 17, 2002) on the publication of Wolfram's long-awaited opus, "A New Kind of Science". The book is described by Jim Giles (*Nature*, May 16, 2002) as "a call for researchers to turn away from calculus and other conventional mathematical tools ... ." What is to replace calculus? Since John Conway's "Game of Life" (with roots in von Neumann's work in the 1940s, but first brought to wide attention by Martin Gardner in the October 1970 *Scientific American*) we have all known that a cellular automaton can start from a couple of simple rules and generate patterns of amazing complexity. Wolfram's fundamental innovation, as best reported by Edward Rothstein (*New York Times* "Arts and Ideas" section, May 11, 2002) is to posit that such automata are actually at work behind the complex systems (turbulence, consciousness, the local structure of space-time) that currently baffle scientific inquiry. "Not only can complex designs and processes arise from the simplest of rules, but ... simple rules actually lie behind the most sophisticated processes in the universe." And the corollary: some complex processes cannot be handled by scientific laws in the way we know them. "All we can do in such cases is discover the simple rules that give birth to the complexity. ... Everything else can be found only by 'experiment': the process must run its course."

New/old math probes the Big Bang.

"A reconstruction of the initial conditions of the universe by optimal mass transportation" is the title of an article in the May 16, 2002, *Nature* by an international team mostly based at the CNRS Observatoire de la Côte d'Azur in Nice. "We show that, with a suitable hypothesis, the knowledge of both the present non-uniform distribution of mass and of its primordial quasi-uniform distribution uniquely determines" a map from present positions to the respective initial ones. The mathematics they use, which they call the Monge-Ampère-Kantorovich (MAK) method, goes back in part to Monge's solution of how best to move a pile of dirt from one location to another: you construct a "cost" function and minimize it. They have tested the MAK reconstruction on "data obtained by a cosmological N-body simulation with 1283 particles," and exhibit the results. Caution: they note that "when working with the catalogues of several hundred thousand galaxies that are expected within a few years, a direct application of the assignment algorithm in its present state would require unreasonable computational resources."

The number line is real.

The number line is real. Psychologically speaking. That's the conclusion reached by a team of psychologists (Marco Zorzi, Konstantinos Priftis, Carlo Umiltà) at the University of Padua. In "Neglect disrupts the mental number line" (*Nature*, May 9, 2002) they examine right-brain-damaged patients with persistent left neglect: these patients "show a spatial deficit for left-side stimuli. ... When asked to mark the midpoint of a line, they miss the midpoint and place it to the right. The misplacement increases as a function of line length, with a crossover effect (leftward displacement) for very short lines". The team showed that exactly the same systematic errors occurred in mental operations when the patients were asked to *name* the midpoint of an integral segment $[a, b]$ given its endpoints a and b. The errors occur in the same direction whether the endpoints were given in increasing or in decreasing order, e.g. 1-9 or 9-1, leading them to observe that "the number line is canonically orientated in a left-to-right manner". They conclude: "Although most people focus on symbolic aspects of numbers, ... thinking of numbers in spatial terms (as has been reported by great mathematicians) may be more efficient because it is grounded in the actual neural representation of numbers". The reference is to Hadamard's "The Mathematician's Mind" (Princeton, 1996) which describes his own use of mental imagery but in coordinate-free terms: "a confused mass, ..., a point rather remote from the confused mass, ..., a second point a little beyond the first, ..." etc. (his visualization of Euclid's infinity-of-primes theorem). He also quotes Einstein: "The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought."

Stephen Wolfram and *The New York Times*.

George Johnson reviewed "A New Kind of Science" in the New York Times Book Review, June 9, 2002. Johnson begins with the book as a physical object: "1,263 pages ... and 583,313 words," intimidating perhaps but with marvelous pictures. "Certainly no one has worked so hard to produce such a beautiful book." He then contrasts Wolfram's publishing style (everything, all at once) with "the normal thing," i.e. regularly posted unreadable papers in "fashionable zines" like Physical Review Letters or Physica D. Johnson presents a cogent digest of Wolfram's main tenet: "the algorithm is the pure, elemental expression of nature; the equation is an artifice." And several examples. "One idea after another comes spewing from the automata in Wolfram's brain." The publication of Wolfram's treatise was also covered in the Science Times for June 11. In "Did This Man Just Rewrite Science?" Dennis Overbye relays opinions from several scientists who have worked the same turf. Here is Edward Fredkin, a BU physicist and longtime proponent of viewing nature as a computer: "For me this is a great event. Wolfram is the first significant person to believe in this stuff. I've been very lonely." Fredkin goes on: "An equation is just a thing you write down on a piece of paper. $E = mc2$ can't keep you warm." But programs are different. "Put them in the computer and they run." George Johnson is at bat again in "What's So New in a Newfangled Science?" (The Week in Review, June 16). "Interesting ideas rarely spring up in isolation" is the theme of this article, making up for Johnson's neglect of that topic in the Book Review. He surveys some of the current work on the algorithmic universe, including MIT's Seth Lloyd, the author of 'Lloyd's hypothesis' (Everything that's worth understanding about a complex system can be understood in terms of how it processes information), and BU's Fredkin. He concludes: "That is how an idea progresses. But sometimes it takes a bombshell to bring it to center stage." and in fact, as Johnson tells us at the start of the piece, " 'A New Kind of Science' was holding its own last week atop Amazon's best-seller chart, along with 'Divine Secrets of the Ya-Ya Sisterhood' and 'The Nanny Diaries.' "

A tough math problem in Internet routing.

A tough math problem in Internet routing is described in "Guessing secrets: applying mathematics to the efficient delivery of Internet content" by Ivars Peterson in the April 6, 2002, *Science News*. Internet route optimizers need to determine the geographical source of a webpage request in order to connect that "client" with the nearest server holding the webpage. The request comes via an intermediate computer called a nameserver, but only the nameserver's address is immediately available. The client's address must be ascertained by a kind of "20 questions" game with the nameserver. E.g. "is the first digit a '1'?" The problem becomes interesting when, as is often the case, the client has two or more addresses, because then the nameserver still gives a yes-or-no answer. Peterson presents an worst-case example with three addresses and an honest but inscrutable answering algorithm that makes it impossible to guess any digit of any of the addresses. In general, when the information is available, how should one ask the questions to obtain it most efficiently? The matter, which is related to "list decoding" of ambiguous messages, is treated by Tom Leighton, Ron Graham and Fan Chung in the Electronic Journal of Combinatorics.

Primes in the *Times*.

Sara Robinson again, in the August 8, 2002, *New York Times*: "New Method Said to Solve Key Problem in Math." The problem is "to tell quickly and definitively whether a number is prime," a problem that has "challenged many of the best minds in the field for decades." *Quickly* here means *in polynomial time*. The new method is an algorithm devised by Manindra Agrawal, Neeraj Kayal and Nitin Saxena of the Indian Institute of Technology in Kanpur. Robinson explains that the discovery has little immediate commercial significance, since the probabilistic algorithms currently in use are faster and accurate enough for practical purposes. But the theoreticians have loved it ever since it was announced (by e-mail) on Sunday, August 4th. It is simple and elegant enough so that Carl Pomerance of Bell Labs, who got the news Monday morning, was able to explain it to his colleagues in an "impromptu seminar" that very afternoon; he commented to Robinson: "This algorithm is beautiful." The "AKS" paper is available online (http://www.cse.iitk.ac.in/news/primality.pdf) in PDF format. It bears as epigraph a quotation from Gauss (1801): "The problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to one of the most important and useful in arithmetic. ... Further, the dignity of the science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated." The story was also reported by the Associated Press ("Prime Riddle Solved"); the Times story was picked up in *The Hindu* on August 9 ("New algorithm by three Indians").

Solving an Escher puzzle.

"The Print Shop" is one of Maurits Escher's more paradoxical creations. In the lower left-hand corner we see, through a window, a man looking at a print on the wall of a print shop. But the top of the print swells out of the shop and as we follow it clockwise through the picture it leads us back to the outside of the shop where we started, so the shop itself is in the print. This is a continuous version of the "picture within itself" that we see, in the US, on Land O'Lakes Butter boxes and in Holland on packages of Droste chocolate. The center of the print has "a large, circular patch that Escher left blank. His signature is scrawled across it." So Sara Robinson describes it in the July 30, 2002, New York Times, where she tells how Hendrik Lenstra, a mathematics professor at Berkeley and at Leiden, solved the riddle of what goes in the center. The key turned out to be the revelation, by a friend who had watched Escher at work, that the artist had kept the distortions conformal (i.e. angle-preserving, like the Mercator projection). Lenstra was able to exploit this feature to give a complete mathematical analysis of the print, and to fill in the patch. The solution has been beautifully presented on the web-page Escher and the Droste effect (http://escherdroste.math.leidenuniv.nl/) on the Leiden website. The page shows the original print and an amazing animation of the solution. Do not miss this.

143-Year-old Problem.

143-Year-old Problem Still Has Mathematicians Guessing – the headline stretches almost across the top of a page in the July 2, 2002, *Science Times*. And right in the middle is a picture of the man himself, with the caption "In 1859, Bernhard Riemann made a hypothesis on prime numbers that hasn't been proved or refuted." The occasion is a meeting at NYU earlier this year, where "more than a hundred of the world's leading mathematicians" gathered to "swap hunches, warn of dead ends and get new ideas that could ultimately lead to a solution" of the Riemann Hypothesis. Bruce Schechter wrote this article, a beautiful piece of mathematical reporting. It blends ancient history (Hardy, Gauss, Riemann) with modern history (Hugh Montgomery, Peter Sarnak, Andrew Wiles) and enough authentic background about prime numbers, complex numbers and the zeta function to keep the exposition honest. Of course after this wonderful buildup the news is disappointing, if not surprising: "Mathematicians at the conference agreed that there was no ... clear evidence of a trail head" from which to set off in pursuit of the still elusive hypothesis. Even more tantalizing, the Riemann Hypothesis now appears as the door to a universe of undiscovered mathematics. As Montgomery puts it: "It should be the first fundamental theorem."

Post-mortem on The Geometry Center.

The analysis is carried out by Jeffrey Mervis in Science for July 26, 2002: "The Geometry Center, 1991-1998. RIP." The Geometry Center was created at the University of Minnesota as one of the first NSF-funded Science and Technology Centers. "From the start, the Geometry Center faced long odds. Even its mission was controversial." The mission was "to attempt to introduce computer graphics and visualization into pure mathematics and geometry," Mervis was told by David Dobkin, who chaired the center's governing board. "It wanted to change the field, but people weren't ready for that." Another problem was the budget: $2 million a year from NSF funds otherwise typically doled out in $25,000 parcels to single investigators. "We were immediately a target for people who said we didn't deserve all that money," said Richard McGehee, who directed the Center during its final years. There is no lack of suspects, and Mervis glances at several others. But he gives the final word to Don Lewis, head of the NSF mathematics division at the time: "I didn't see any progress, so I pulled the plug." The Geometry Center which, as McGehee remarks, "had one of the first 100 Web sites", lives on virtually (http://www.geom.umn.edu/) at the U of M.

Perfect Graphs.

Perfect Graphs and the "Strong Perfect Graph Conjecture" are the topic of a News Focus piece by Dana Mackenzie in the July 5, 2002, *Science*. As Mackenzie explains it the definition involves two invariants of a graph. The first, ω, is the size of the biggest clique (set of nodes each of which is one step away from all the others). The second, χ, is the number of colors it takes to color the nodes so that no two adjacent nodes are the same color. So χ is always bigger than ω; if the numbers are equal, the graph is *perfect*. Mackenzie: "A perfect graph is like a perfect chocolate cake: It might be easy to describe, but it's hard to produce a recipe." A conjecture due to Claude Berge (CNRS, Paris) has been around since 1960: every imperfect graph contains either an "odd hole" or an "odd anti-hole." This is the Strong Perfect Graph Conjecture (SPGC). The odd hole is "a ring of an odd number (at least 5) of nodes, each linked to its two neighbors but not to any other node in the ring." The odd anti-hole is "the reverse: Each node is connected to every other node in the ring except its neighbors." The news is that a proof of the SPGC has been announced by Paul Seymour (Princeton), G. Neil Robertson (OSU) and Robin

Thomas (Georgia Tech). The proof is worth $10,000 (put up by fellow "perfect-graph aficionado" Gerard Cornuejols) and "the early betting is that they will collect the prize."

Neurons do Math.

Neurons do Math, in the brains of monkeys and frogs, at least. This is the message of Single brain cells count, a Nature Science Update for September 6, 2002. The update, by John Whitfield, describes two recent sets of experiments. Monkeys: A. Nieder, D.J. Freedman and E.K. Miller (*Science*, **297** 1708-1711 (2002)) "showed groups of dots to macaques, and recorded the output from individual neurons in the monkeys' prefrontal cortex. ... The neurons ignore the dots' size, shape and arrangement and hone in on their number. Each cell's response peaks at its preferred number and tails off on either side." Frogs: C.J. Edwards, T.B. Alder and G.J. Rose (*Nature Neuroscience* **5** 934-936, available online) sampled neurons in the brains of female frogs (*Hyla regilla*) to understand how they distinguished between the aggressive calls and the advertisement calls of males of their species. The only difference between the two calls is their speed. "Female frogs' male-detector neurons fire only after they hear five or more rapid pulses, Rose and his colleagues find. If the pulses are too close or too far apart, the counter resets to zero - as if the nerve cells measure the spaces between pulses, rather than the sounds themselves."

A mathematical phase transition.

Phase transitions occur in physical systems, often at a certain "critical temperature" (e.g. ice to and from water at zero degrees C). In "Analytic and Algorithmic Solution of Random Satisfiability Problems" (*Science*, August 2, 2002), Marc Mézard, Giorgio Parisi and Riccardo Zecchina (Orsay) bring methods from statistical mechanics to study a phase transition which occurs in a purely mathematical context: the probability that a randomly generated k-SAT problem has at least one satisfying ("SAT") assignment. The k means that each constraint involves exactly k variables, so $(A + B + c)(a + D + e)(b + E + C)(d + a + b) = 1$ is a 3-SAT problem with four constraints in the five Boolean variables A, B, C, D, E, with $a = \neg A$, etc. The $+$ is the logical "or": $x + y = 1$ unless $x = y = 0$, and multiplication is the logical "and": $xy = 0$ unless $x = y = 1$. In this example $A = 0$, $B = 1$, $C = 1$, $D = 0$, $E = 1$ is a "satisfying assignment." The role of temperature is played by the ratio α of the number of logical constraints to the number of variables. Clearly when there are many more variables than constraints the probability of a satisfying assignment is high, and vice-versa. David Mitchell, Bart Selman and Hector Levesque showed experimentally about 10 years ago that the transition from high to low occurs abruptly at a critical value α_c near 4.3 for $k = 3$ and in addition that the computing time necessary to settle the problem peaks dramatically near α_c. Mézard and his colleagues pin down α_c to 4.256 and locate another transition point $\alpha_d = 3.921$ such that between α_d and α_c "the space of configurations breaks up into many states, and there exists a nontrivial complexity" thus partly explaining the computation peak observed by Mitchell *et al*. They remark "From the strict mathematical point of view, the phase diagram we propose should be considered as a conjecture," an invitation for mathematicians to get involved in this aspect of mathematics.

The next big thing.

The *Chronicle of Higher Education* (September 30, 2002; Section B, page 4) invited experts in Geography, Math, Information Technologies and Criticism to tell us "What will be the next big thing?" in their fields. The mathematics respondent was John Ewing of the AMS. "The next big thing in mathematics? Biology. ... The mathematics involved in studying the genome and the folding of proteins is deep, elegant, and beautiful ... a spectacular new area of research that is certain to grow enormously in the next 10 years." Ewing goes on: "During the coming decades, scientists and mathematicians will come to see the false distinctions between pure and applied mathematics. ... More and more, mathematicians will see their subject as underlying all science and social science – not as a humble servant but as an essential companion."

Originally published by the American Mathematical Society in What's New in Mathematics, a section of e-MATH, in

http://www.ams.org/index/new-in-math/home.html

An Interview with with Thomas J. Laffey

Professor Laffey, please tell us about your formative years. How did you go into Mathematics, and what are your recollections of your university studies?

I was born and raised on a small farm in the West of Ireland. Incomes in the area were low and, while neither of my parents had second level education themselves, they strongly encouraged me to study, as success at school was seen as the gateway to greater status and income. In the prevailing society, the medical doctor and schoolteacher had greatest status, even though their incomes would not match that of the bigger farmers and shopkeepers. I succeeded in winning a State scholarship to study at University College, Galway (UCG, now known as National university of Ireland, Galway (NUIG)). While this was, until recently, the smallest of the Irish universities, it was the one at which all State scholars were required to study. The State scholarships were very prestigious, since only about 20 per annum were awarded and they were also seen as the most lucrative available for open competition. The two programmes generally considered the most prestigious among those offered in UCG at that time were Mathematical Science and Classics (Latin and Greek) and the majority of the State scholars studied one of those. As a result, UCG produced a large proportion of Irish students who went on to do doctorates in Mathematics . (For other reasons, University College, Cork also produced a large number). The head of the Mathematics Department at UCG was Professor Sean Tobin, a group theorist, and, as a result, there was a strong algebra content in the courses offered. I was especially attracted towards research in group theory – in particular, I found the textbook of Walter Ledermann "Introduction to the theory of finite groups" inspiring and the tricky exercises and examples discussed by Tobin fascinating and challenging. I still remember a day in 1964 when, during a lecture on solvable groups, Tobin told us that a paper had just appeared by Feit and Thompson proving that all finite groups of odd order are solvable – it was clear from his description of it that we were not expected to read it for the examination!

You went on to study with Ledermann for your doctorate.

At that time, one had to choose between doing a PhD in the U.S. or the U.K. There was little tradition of research in Mathematics in Irish universities; the number of staff was very small and lecturers tended to have very large teaching loads. Also, there was no funding in place to support graduate students – this situation has only changed in recent times. I wrote to Walter Ledermann seeking support to do a doctorate at the then new University of Sussex (to which he had moved from Manchester) . After interview there, I was offered and accepted a tutorial studentship (the British equivalent of a teaching assistantship). The contrast between UCG and Sussex at that time was striking. Facilities at Sussex, such as the library, offices, restaurant etc., demonstrated wealth and style. The 1960s were a period of great economic development in Ireland , but the starting point for this was very low, and, despite the ongoing improvement in the country, university expenditure was very limited and the system functioned at a barely adequate level.

Thomas Laffey

I already had a Masters degree from UCG, so when I went to Sussex in September 1966, I was not required to do any further coursework, and I immediately began research. In the book by Curtis and Reiner entitled "Representation theory of finite groups and associative algebras", I was greatly impressed by Jordan's Theorem which states that there is a function J defined on the natural numbers with the property that if G is a finite subgroup of the group $GL(n, \mathbb{C})$ of invertible $n \times n$ matrices with complex entries, then G has an abelian normal subgroup A with $|G/A| \leq J(n)$. Several explicit functions J were known, as a result of work of Jordan, Schur, Blichfeldt, Speiser , and I was interested in trying to get a better one. Ledermann (who, as an undergraduate in Berlin, was taught by Schur), encouraged me to work on this problem and, more generally, on the structure of finite subgroups of $GL(n, \mathbb{C})$ as a function of n. He was a most helpful and encouraging advisor and he was always available for discussion on group theory questions. He arranged for me to travel every week to Kings College, University of London, to participate in an advanced course on the representation theory of finite groups given by E.C. Dade, who was visiting the UK from Caltech in 1966-67 and I found this very stimulating. He also arranged for me to meet Walter Feit, for advice on problems concerning finite linear groups. There were a number of group theorists at Sussex at that time, but their interest was largely in theory of infinite groups, and I did not have much interaction with them. A paper which greatly influenced my work was "On a theorem of H.F. Blichfeldt" (Nagoya Math. J. 5 (1953) 75-77) by Noboru Itô, in which it is shown that if $p > n$ is a prime which divides the order of a finite solvable subgroup G of $GL(n, \mathbb{C})$, and G does not have a normal abelian Sylow p-subgroup, then n is a power of 2 and $p = n + 1$ is a Fermat prime. I learnt several useful techniques from this paper – the proof is a wonderful demonstration of the minimal counterexample approach - and I was quickly able to build on them to get a best possible Jordan function for finite solvable groups and this formed the backbone of my thesis. During my second year at Sussex, I was more relaxed and started to read more widely in algebra and number theory and I attended all the colloquia and workshops taking place in the department. I shared an office with some graduate students of John Kingman in probability theory and David Edmunds in analysis and had many interesting and informative discussions with them on real and complex analysis. I submitted my thesis in June 1968. Roger Carter, whose name I was familiar with because of his discovery of Carter subgroups, was the external examiner and he arranged an invitation for me to attend the finite group theory meeting at the Mathematisches Forschungsinstitut in Oberwolfach that August and I first presented my thesis results there. That meeting was one of the high points of my research career; there I met several of the leading figures in the subject whose names were known to me through their papers and reputation. Among them were John Thompson, John Conway, Sandy Green, Helmut Wielandt, Bertram Huppert, Reinhold Baer, Michio Suzuki, Wolfgang Gaschütz. I still remember Thompson's lecture on Conway's group and its connections with unimodular lattices, modular forms and Ramanujan congruences There was a tremendous air of excitement and expectation about the future of the theory of finite simple groups.

What did you do after you obtained your PhD?

I applied for and obtained an advertised position as assistant lecturer in Mathematics at University College, Dublin (UCD) and took up duty there in September 1968. Though this was, and still is, by far the largest of the Irish universities, I did not know much about it before taking up employment there. Because of the very high teaching loads that had to be carried out by each lecturer in previous times, there was not a strong research tradition in Mathematics. However, as a result of the improving economy, more staff were then being appointed and teaching loads were being reduced to about ten lectures per week, so research was becoming more feasible. There was only one other algebraist in the department when I was appointed, the late Fergus Gaines. Fergus had been an undergraduate at UCD and then did his PhD at Caltech under the supervision of Olga Taussky Todd. In order to be able to discuss research with someone, I decided to read the literature related to Fergus's work. I learnt the number theory and ring theory required to read the papers of Olga Taussky Todd. I also, for the first time, became aware of the difference between the American style PhD and the one I did. I was impressed by the fact that Fergus had attended advanced courses and sat examinations on a wide variety of topics and had an altogether wider knowledge of Mathematics, before embarking on his research. Much later, when we came to set up our own PhD programme here, I was one of these who successfully argued for basing it on the American model.

You lived in the US for a while. Was it there that you came to know Paul Erdös?

I got the opportunity to spend a sabbatical year at Northern Illinois University (NIU) in DeKalb during the session 1972-1973. This place was the home of Henry Leonard, Harvey Blau and John Lindsey II, whose names were well-known to me in the context of finite linear groups. Suddenly, I was in an environment with several researchers in algebra. There I made my first contact with semigroup theorists – Bob McFadden and Don McAlister, both incidentally originally from Northern Ireland, headed a most active research team in the subject and I attended their twice-weekly seminars. There was a very active research group in Ring Theory headed by Bill Blair and John Beachy (though John was away in 1972-73) and I also participated in their seminars. The group theorists were very active also and I fully participated in their seminars. There was also a constant stream of colloquium speakers. I also found the analysts asking interesting question on matrix theory, some of which I could answer because of my earlier reading. This caused me to think seriously about doing research in matrix theory for the first time. Northern Illinois has a great reputation in the area of analytic number theory because of the presence of John

Selfridge (who was chairman of the department when I visited) and his research group. This group had a constant stream of research visitors – the first semester I was there, Paul Erdös, Derrick Lehmer and John Brillhart were there and I attended their seminars. I got to know Erdös and I often used to give him a ride to the department in my car on frosty mornings as he used to live in the same part of DeKalb as me, though I did not interact with him mathematically. Had I foreseen the concept of Erdös number, I might have tried to think of some question on which we might have collaborated. While some recent biographies of him suggest that he had no interests outside Mathematics, I found that he had considerable interest in current affairs. Many years later, in 1986, I met him at the ICM in Berkeley and he immediately started to give me a detailed account of a news report earlier that day on a fairly obscure political incident in Northern Ireland.

How did your research interests evolve over the years?

Having returned to Dublin in Autumn 1973, I worked on ring theory questions and gradually moved to ring theoretic questions about matrices such as simultaneous triangularizability, Research activity was increasing in Dublin and I also interacted with the functional analysis group at Trinity College, Dublin led by Trevor West. Olga Taussky Todd wrote to me in connection with some questions arising in a paper I wrote on simultaneous triangularization and thus began what was for me a long and very fruitful correspondence. Through Olga's influence, I worked on the L-property, simultaneous reducibility of matrices and on integral similarity and factorization results for integer matrices. Fergus Gaines was not so research oriented and I began to address questions which arose in work in his thesis or in his joint work with Bob Thompson or in the thesis (also supervised by Olga Taussky Todd) of Helene Shapiro. I also got to know two other former students of Olga – Charlie Johnson and Frank Uhlig – and also Hans Schneider and Richard Brualdi at Madison. Through Trevor West, I got to know Jaroslav Zemánek and the operator theorists connected to the Banach Center. As a result, by the late 1970s, I had moved entirely into linear algebra and functional analysis.

You have been heavily involved in mathematical competition activities. What are your thoughts and experience concerning Mathematical Olympiads for the young?

Every mathematician knows that one of the appealing aspects of Mathematics is the exhilaration and sense of achievement one gets by solving a challenging problem. People of my generation often experienced this first in the context of tricky exercises in Euclidean geometry, but this material is no longer taught at second level. Mainly through his initiative, Finbarr Holland of University College, Cork and I decided to try and run the American High School Mathematics Examination (AHSME) in Irish schools from 1978 onwards. The aim was to further interest in Mathematics among the more talented second level students. The competition organisers generously gave us permission to copy and distribute the papers to interested schools and the competition has been held annually here since 1978. The participating teachers manage the examination in their schools and the scores are coordinated by Finbarr and me and (small) prizes are awarded to the top performers. The reaction to this initiative was very positive, particularly from the more prestigious schools. The questions on the AHSME (and in the follow-up Invitational Competition) are elegant and tricky and a good score is indicative of mathematical potential. It served the purpose of increasing interest in the subject among mathematically gifted students well. However, Finbarr and I had the greater goal of trying to get funding in order to have an Irish team participate in the annual International Mathematical Olympiad (IMO). Unemployment was very high in Ireland in the 1980s and the tax take was low, so the State had difficulty paying its bills and there was little optimism that it would fund IMO participation. We planned to send a team to the IMO as soon as the host country was near, preferably the U.K,, as travel costs would be manageable within the small resources gained from limited sponsorship of the mathematics competition. Completely out of the blue, we got funding to send a team to the 1988 IMO in Canberra. Due to Irish being deported to the penal colony in Van Diemens Land (Tasmania) or voluntarily going to Australia to escape the Irish famine in the 19th century, over 50% of Australians claim some form of Irish heritage and, as the IMO there was nominated an official event of the 1988 Bicentennial Celebrations, the Irish Government wished to be represented at it. (It also helped that the chief organiser of the event was Peter O'Halloran, an Australian of Irish heritage, and that he lobbied the right people!). The Irish Department of Education has supported Irish participation in the IMO since 1988. Each year, they supply those involved in preparing students for the IMO with a list of the top performers in Mathematics in the Junior Certificate Examination (an examination taken by all students about age sixteen) and students on this list (through their school principals) are then invited to attend problem solving sessions and compete for a place on the Irish IMO team each year while they are still in secondary school. The training programme is offered in five universities, UCD, UCC, NUI Galway, NUI Maynooth and the University of Limerick, and usually takes place on Saturday mornings. While Irish performance at the IMO has not been impressive (like Portugal, we are definitely in the "amateurs" rather than "professionals" section), there have been considerable benefits in increasing interest in Mathematics among the very bright students. Several of our top postgraduate students in recent years have said that their first contact with UCD was the AHSME and the Saturday morning training sessions. One professor at Trinity College pointed out to me that the process also has an interesting sociological consequence – namely, most of the top students in the university system in all subjects now know one another, since these students tend to score very highly in Junior Certificate Mathematics and meet one another at the Saturday morning training sessions. I worry at times that the sheer difficulty of the IMO questions discourage some talented students from pursuing a career in the subject at university, but we explain very carefully the exceptional nature of the IMO, comparing it with running a 100 metre race against Marion Jones or using some other similar analogy, and the feedback from students and their teachers has been very positive.

Can you give us a short overview of the Irish university system?

Our system is modelled on the British one. Generally, courses to students were offered at two levels "general" and "honours", the honours courses being more challenging. Traditionally, the majority of students took the general courses, while the more talented took the honours ones. Entry to postgraduate programmes was open only to honours graduates and required a high level of performance in the honours courses. In recent times, both in the U.K. and Ireland, there has been a movement towards "honours only" programmes. Several factors have contributed to this: 1. Increased state funding has enabled students from wider socio-economic groups to go to university, while, at the same time, the number of places available did not increase proportionately, so the greater competition for places has led to an improvement in the academic calibre of the students gaining admission. 2. The status of the general degree also diminished in the view of employers. 3. New universities and technological institutes were created which, while getting weaker students than the established universities, produced large numbers of honours graduates. Here, we now offer traditional Honours Mathematics, with an emphasis on rigour and preparing students for graduate study later, and also an Honours programme called Mathematical Studies, more geared towards students going into employment in second level teaching or financial services immediately after obtaining a primary degree. The majority of Mathematics graduates come through the Mathematical Studies stream. We recently established a new programme called Mathematical Science in which a small select intake of students take a selection of Honours courses in Mathematics, Mathematical Physics and Statistics (with Computer Science also in their first year) and this has proved very popular with top students.

One concept that still survives here and in the U.K and some former members of the British Empire, is that of having an external examiner (or extern). In each subject, the university appoints an academic from another university to act in this position, usually on a three or four year contract. The duties of the extern are to vet the examinations, approve their standard and content and, through sampling, approve the marking and adjudicate on issues such as borderline cases of pass/fail or the grade of honours to be awarded. In particular, in most cases, the award of first class honours to candidates is approved on an individual basis by the extern. The extern has "total power" in that his recommendations are normally treated as sacrosanct by the university. While here, the extern visits the department about a week before the board meetings at which the results are approved and his recommendations are conveyed in written form to the meetings, in some universities the extern attends the final board meeting in person. During the mid-1990s, I acted as extern examiner in Honours Mathematics at the University of St. Andrews. There, the extern attends the final board meeting and signs the official record.

The system was originally established to ensure that the standard required for degrees in the various universities in Great Britain and its Empire were approximately the same (unlike, for example, the situation in the U.S., where the standard varies widely from university to university). However, as universities have increased in number, size and diversity of programmes, the influence of the system in uniformizing standards has diminished, but it is still valuable in guaranteeing certain minimal levels of knowledge and achievement, particularly in the case of first class honours graduates.

Ireland is sometimes mentioned as a kind of 'miracle economy'. How has this influenced the university system?

The past seven years has been a period of unprecedented economic growth in Ireland. Suddenly, the country changed from one with high unemployment to one in which anyone seeking employment could get it. Skill shortages in the computer industry arose and, as well as putting plans in place to increase the number of computer science graduates, the government started to actively encourage qualified people from inside the EU and also from the U.S., Australia, India and countries from the former Eastern Block to come to Ireland to satisfy the demand. Jobs were available immediately on graduation, so fewer students stayed on to do postgraduate studies. Incentives were put in place to encourage more students to go into graduate study and more foreign students availed of these and came to do their graduate studies here. Greater emphasis was put on the universities' mission of fostering research, vis a vis its teaching role. A policy of setting up a number of specialised research institutes in university campuses was put in place and also funding was made available, partly through the EU, to enable the appointment of postdoctoral researchers. As a result, universities now have some academic researchers with no teaching duties. While the number is small, this represents a major change in the structure. On the other hand, the increased wealth in society, combined with the increase in population due to the arrival of migrant workers, led to enormous increases in house prices and this made it more difficult for the university to attract foreign academics to fill senior positions.

At lower level, the government successfully sought to encourage a much higher percentage of secondary school-leavers to go on to third level through the remission of tuition fees at universities and institutes of technology, and other incentives. Companies such as Apple, Intel and Microsoft, which employ thousands of people here, hire mainly graduates, probably reflecting the fact that they have their roots in the American system where third level education is the norm. Another consequence of the economic changes has been that supermarkets, restaurants and other businesses in the services industry where salary levels are traditionally low, have difficulty hiring sufficient people on a full-time basis, and, as a result, senior second-level and all university students have unlimited opportunities to get part-time employment. Currently, the vast majority of students work about 20 hours per week (usually in the evenings or at weekends) during term-time in paid employment. This has led to a reduction in the time available for study and consequent downward pressure on the standards achievable in their courses. It also means that students are less interested in reforming the world and only a tiny minority take part in the traditional student activities of left-wing politics and anti-capitalist demonstration. The aftermath of September 11 and the general downturn in the computer industry is just starting to effect life seriously here and many of the recent developments, whose affordability was predicated on continuing high economic growth, are now being reviewed and it is likely that a period of financial stringency is on its way.

How is the job market for mathematics graduates in Ireland nowadays?

Over recent years, the job opportunities for Mathematics graduates in Ireland have become much more diverse. Up till then, the principal employment area for graduates with general degrees was second level teaching, while the smaller number of honours graduates either stayed in academic life or obtained positions in the central statistics office, the meteorological service or more generally, in the technical areas of the civil service. A smaller number obtained positions in operations research, principally in the international companies KPMG and Andersen Consulting, which have major facilities here. Nowadays, the majority of Mathematics graduates are employed in financial services – Dublin has a very successful international financial services centre (its success is enhanced through a low taxation policy) – and in the computer industry – this is a major component of Irish industry at present. While demographic changes, consequent of much lower average family size, are occurring here as elsewhere in Europe, lower class size has meant that there continue to be vacancies for second level teachers and, in fact, many schools are not able to find sufficient Mathematics graduates to teach their Mathematics courses and it is not uncommon for even the more advanced second level courses to be taught by graduates in commerce or agricultural or biological science who have had only one year of Mathematics at university.

As a subject to study at university, Mathematics has declined in popularity, and, while the total intake of university students has greatly increased each year, the gross total taking Mathematics has actually reduced. Two factors which contribute to this are (i) a continually reinforced philosophy in the media that technical knowledge is boring and that fuzzy thinking is to be encouraged (ii) a general perception that, as a subject, Mathematics is "hard". The same factors have led to decreased interest in physical science, computer science and engineering, despite the needs of the economy. Another noticeable change in students' outlook now is the emphasis they place on obtaining a qualification that should guarantee a large income for them in the future. Thus Medicine, Law, Actuarial and Financial Studies attract many of the brightest students. University entrance in Ireland is based on a points score compiled from a student's mark in six subjects at the State-run Leaving Certificate Examination, taken by all students at the completion of their second-level studies. There is a very good correlation between the total points scores and the Mathematics marks achieved by students – this correlation appears to be lower in the case of other subjects – and there is also an excellent correlation between the total points score and university performance in the traditionally challenging disciplines.

What are your views about future trends in mathematics?

Having being a graduate student in the late 1960s, I was influenced by the Bourbaki philosophy of formal definition based non-intuitive pure Mathematics "for its own sake." It is interesting to see the changes in this view over the years. Nowadays, one delights in pointing out applications of one's work outside the subject; the influence of physics in suggesting concepts to study, particularly in geometry, or that of computer science or electrical engineering in doing the same in relation to algebra, is stressed by leaders in the field. The subject has broadened and become more diverse because of these applications, while the status of Mathematics based on a new concept defined by a set of axioms "for its own sake" has reduced in my view. At the same time, progress on classical problems, particularly in number theory, has been achieved through the use of enormous technical machinery, such as arithmetic geometry in which number theory, algebraic and differential geometry, topology and analysis are combined to work effectively . This might be called "total Mathematics" by analogy with "total theatre". While one may regard the Bourbaki philosophy as passé, Grothendieck's ideas continue to inspire. It is hard to predict what areas will be seen as the most prestigious in, say, thirty years time. In the 1960s and 1970s, finite group theory, especially the classification of all finite simple groups, attracted universal acclaim, while nowadays, algebraic geometry and number theory and their connections, as in the Langlands programme, have a similar status. The fact that research in the area requires great background knowledge makes it very a demanding area for a student working towards a doctorate. The theory of finite simple groups reached that point in the mid 1970s and the subject became unpopular as an area in which to do a PhD. While Mathematics is often likened to a knowledge pyramid, it is important that a researcher need only know a relatively small part of the pyramid in order to make progress.

In Algebra, emphasis on commutative algebra has increased, and since this area has strong links with theoretical and algorithmic questions in computation, I expect this trend to continue.

Finally, we would be grateful for some words on your connection to Portugal, as well as your views of mathematics in Portugal in light of your participation in the 1999 assessment committee.

Since my involvement with linear algebra dates from the mid-1970s, it was only at that time that I became aware of the work of Graciano de Oliveira and his group at Coimbra. I think that I first learnt about it from Bryan Cain of Iowa State University at a conference in Santa Barbara. I was very honoured to be invited to speak at the first international linear algebra conference in Coimbra and this was also the occasion of my first visit to Portugal. Since then, I have maintained strong links with the algebraists in Coimbra and Lisbon. The research groups in linear and multilinear algebra have strong international visibility and contain a number of leaders in research in these subjects. I quickly also became familiar with the research in semigroups in Lisbon and the great strength of the subject there as well as in Coimbra and Porto. However, it was only as a member of the FCT triennial evaluation panel in 1999 that I got a global picture of mathematical research in Portugal and the structure of its research programmes. The traditional system whereby graduates are appointed to essentially permanent teaching positions at a young age and then do their PhDs while carrying out heavy teaching duties is quite unlike the systems I had previously encountered. The legal separation of the teaching functions of academics from their research functions was also a surprise. I was quite impressed by the level of research activity throughout Portugal and the number of conferences, workshops etc. organized there. The negative impact of large teaching loads and, especially, the necessity of offering examinations in the same course several times to cater for individual students' whims, made the high level of activity even more creditable. Given that my primary interest is in algebra, I found it interesting that, in Portugal, research in this subject is mainly concentrated in linear and multilinear algebra , semigroup theory, category theory and algebraic logic. The level of achievement in these areas is excellent and gains the country international recognition and status. However, and allowing for the fact that Portugal is a small country, I was surprised that there was not a greater variety of areas being investigated and, in particular, that there was very little research interest in number theory. I can understand the reason for the lack of diversity and believe that, without doubt, the situation will change as time goes on. The strong international links between leading research groups here and cognate centres abroad is to be lauded. The research centres produce a very impressive amount of material, such as textbooks in Portuguese, strategies for second-level Mathematics teaching etc., which, while not published internationally, makes a valuable contribution to the health of Mathematics in the country. Generally, the system of funding centres appears to work well and researchers are much more encouraged in their work than the corresponding people in Ireland.

Interview by J. F. Queiró

After completing a doctorate in Sussex University under the supervision of Walter Ledermann, Thomas Laffey joined the Mathematics Department of University College Dublin in 1968 and has remained there since. He served two terms as head of department (1986-90 and 1996-99). His principal research interests are in algebra, particularly in finite group theory and algebraic linear algebra.

He was the founding editor of the Newsletter (now Bulletin) of the Irish Mathematical Society and is currently one of the two editors of the Mathematical Proceedings of the Royal Irish Academy and a member of the editorial board of three other journals.

Gallery

José Joaquim Dionísio

Professor J. J. Dionísio was for years almost a legend to me. If my memory serves me right, I started hearing about him around 1957 or 58, and later I studied carefully some of his articles in the field of Linear Algebra. It was a time when mathematical research was almost unknown in Portugal, so to read papers by one of the few Portuguese active researchers was a stimulus to my imagination. I believe it was only in 1970 that I met him for the first time, at the School of Sciences, Lisbon University.

Prof. Dionísio was an assistant at Coimbra University, where he received his doctorate in 1954, and he moved to the University of Lisbon in 1956, where he stayed for the rest of his career.

Prof. Dionísio was one of the people who had a large influence on my own scientific career. Another one was Prof. Luís de Albuquerque, with whom I was in close contact since my student days. It was through him that I received the influence of J. Dionísio, then unknown to me. Luís de Albuquerque had been his friend for a long time, and he often talked to me about Dionísio's work, which I studied long before I met him for the first time.

As a professor, Dionísio taught courses in several areas. In research, he worked mainly in Linear Algebra, although he published articles in other fields in Portuguese journals. He was a cultivated man. He gave particular attention to the History of Mathematics, teaching a course on it and contributing to the Biographic Dictionary of Authors.

I had the opportunity to be in close contact with him in the periods 1972-75 and 1984-89, when I was his colleague at the School of Sciences, Lisbon University.

Perhaps the most remarkable aspect to which I can bear witness is his influence, unknown to many, on the Portuguese Linear Algebra group. Although indirect, this influence was important.

When I graduated, I knew two people interested in Linear Algebra: Dionísio and Albuquerque.

José Joaquim Dionísio

I never knew which of them was the first to have this interest, nor their mutual influence nor even why they chose the field. I asked Prof. Albuquerque, who, as far as I can remember, did not give a complete answer, and I believe his interest in the subject came about more or less by chance. He may have become curious about matrices during his undergraduate studies. A few years after his doctorate, Albuquerque spent a year in Germany, where he studied stochastic processes. I remember him talking a lot about stochastic matrices.

Again I recall that, at the time, the situation in Portugal concerning research was backwards, very different from today. Research was seldom mentioned, or it was described as something mysterious. It is impossible to understand what was done then without first recalling the university atmosphere.

Since then, everything changed a lot, and today Portugal has an important Linear Algebra School, with groups working in several universities. Some people think I started this, but the initiators were actually Profs. Dionísio and Albuquerque. I played a role because I was clever to make the right choice for the year of my birth. Thanks to that, my role was to serve as a bridge between Dionísio and Albuquerque and the generation after me (I do not mention any names, so as not to hurt anyone: if this happened it would be solely my fault, as I am no longer following their large research output).

As I said before, it is impossible to understand what is done without understanding the time in which one lives. By today's criteria, what counts to be recognized as a mathematician is the number of publications and the quality attributed to the journals in which they appear. Journal quality is measured by the quality of published papers and vice versa. This is a vicious circle that mathematicians, not usually fond of vicious reasoning, strangely accept very well! By these criteria, or even by the international standards of the time in which they lived, the work of the two mathematicians we are referring to will, no doubt, be far from the first places in any kind of ranking. In truth, it is my judgement that neither of them discovered any important theorems. Nevertheless, they had a very important influence that lasted longer than their lives.

In those times, the Portuguese university did not have doctoral programmes (even today, by European standards, the situation is not bright) and the very notion of a supervisor did not exist. If it weren't like that, one could say that Prof. Luís de Albuquerque was my supervisor and Prof. Dionísio was a kind of indirect supervisor, and unaware of it. I corresponded with the former for many years, especially before my doctorate and during absences fom Coimbra. But not so with the latter, since Prof. Dionísio was not in the habit of answering letters. I wrote to him twice before meeting him. The first time was in 1966, asking him for advice on my doctoral studies and on a place to pursue them. My second letter to him was sent around three years later, with an invitation for him to give a talk at a small meeting I organized in Coimbra in September 1969. He didn't answer either of them. I don't know why. Maybe he found writing difficult, much as I do writing the present article.

Speaking about indirect influences, and the assessing of CV's by counting research papers, I mention another significant and little known episode. I must mention a third person who had great influence on Linear Algebra research in Portugal, probably without ever becoming aware of that. That person was Leon Mirsky. I never talked with him but we corresponded a lot. Once I went to look for him at his university, in Sheffield, but unfortunately he was away. In another occasion I saw him but did not talk to him, and I still regret this. He was giving a talk in Oxford and I was in the audience. I was very young and was too shy to address him. Well, L. Mirsky published a paper (Inequalities and Existence Theorems in the Theory of Matrices) which was expository, precisely the kind of paper that nowadays is not highly valued, not for lack of quality but because it contains no new results, even if the journal had a good ranking. That paper, which I studied carefully, discussed a range of open problems. It was a great source of inspiration, with many consequences. In fact, many papers came to be published, by me but especially by others after me, which had their roots in that work of Mirsky. That explains why the word 'prescribed' appears so often in those papers. I even believe that, without the inspiration of Mirsky's paper, the Sá-Thompson interlacing inequalities (one of the most remarkable results in Linear Algebra, whose full consequences are still to be discovered) would not have been found.

When talking about the Portuguese Linear Algebra school, the names of L. Albuquerque and J. Dionísio should always be remembered. And the name of L. Mirsky as well. I think that, of the three, only L. Albuquerque was aware of his influence.

Graciano de Oliveira

Editors: Jorge Buescu (jbuescu@math.ist.utl.pt)
F. Miguel Dionísio (fmd@math.ist.utl.pt)
João Filipe Queiró (jfqueiro@mat.uc.pt).

Address: Departamento de Matemática, Universidade de Coimbra, 3000 Coimbra, Portugal.

The CIM Bulletin is published twice a year. Material intended for publication should be sent to one of the editors. The bulletin is available at http://www.cim.pt.

The CIM acknowledges the support of: Fundação para a Ciência e a Tecnologia, Apoio do Programa Operacional Ciência, Tecnologia, Inovação do Quadro Comunitário de Apoio III; Departamento de Matemática da Universidade de Coimbra; Departamento de Matemática do Instituto Superior Técnico da UTL; Centro de Matemática da Universidade de Coimbra; Banco Comercial Português; Império Bonança, Companhia de Seguros, S.A.