International Center for Mathematics

#36

Bulletin

March 2016

Editorial

The new direction board of CIM took office on the 5th of February of 2016. Accompanying this change, the CIM bulletin has a new editorial board that I am privileged to direct.

The main goal of the bulletin is to promote Mathematics and specially mathematical research. We hope to achieve this by supporting and advertising the activities of CIM and of its associates, which will include summaries and reports of the conferences, workshops and other scientific events, as well as, interviews to renowned mathematicians such as the invitees for the forthcoming *Pedro Nunes lectures*.

The bulletin will also publish review, feature, outreach and research articles, both invited and submitted. The idea is not to compete with other more oriented type of publications but rather make an effort to stimulate and present recent trends of research in mathematics.

We aim at publishing two issues of the bulletin per year.

Starting in the second issue of 2016, the bulletin will feature a series of historical articles about the work and lives of distinguished Portuguese mathematicians, opening with Pedro Nunes to celebrate the revival of the lectures named after him.

We note that most of the material for this issue of the bulletin was compiled by the previous editorial board, to whom we thank for the work and collaboration.

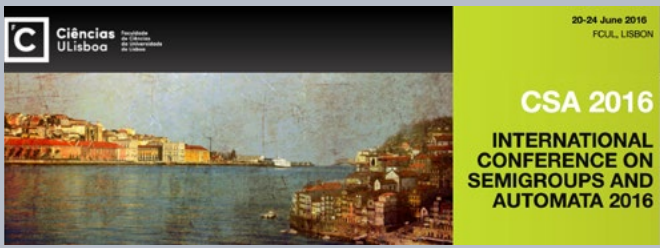
Jorge Milhazes Freitas
Centro de Matemática & Faculdade de Ciências
da Universidade do Porto
https://www.fc.up.pt/pessoas/jmfreita/

CIM's new Direction Board (from left to right): Henrique Oliveira (Vice-President–Secretary), Jorge Rocha (Vice-President), José Francisco Rodrigues (President), Isabel Figueiredo (Vice-President — Treasure), Jorge Milhazes de Freitas (Vice-President)

Contents

01 Editorial
02 Coming Events
04 Portugaliae Mathematica and its Exchange Library
11 Geometric Aspects of Modern Dynamics
13 An Interview with Cédric Villani
17 Representations, Higgs Bundles and Components: an Overview
24 The 109th European Study Group with Industry
26 Applications of Mathematics in Fluid Dynmics
32 An Interview with Julio Rebelo
35 Joint International Meeting of the American, European and Portuguese Matematical Societies, Porto, June 2015
37 An Interview with Samuel Lopes

Coming Events



CSA 2016—INTERNATIONAL CONFERENCE ON SEMIGROUPS AND AUTOMATA 2016 [June 20-24, 2016]

Celebrating the 60th birthday of Jorge Almeida and Gracinda Gomes

Faculty of Sciences (building C6), University of Lisbon, Campo Grande, Lisbon

Jorge Almeida (University of Porto) and Gracinda Gomes (University of Lisbon) have played a major role in the development of semigroups in Portugal. They will both be 60 years old in 2016, and a conference is being organized to celebrate their birthday.

csa2016@fc.ul.pt
http://ciencias.ulisboa.pt/en/conferencia/csa-2016

Workshop — 2nd Porto Meeting in Mathematics and Biology [June 15–17, 2016]

i3S — Instituto de Investigação e Inovação em Saúde
Auditório A (Corino de Andrade)
Rua Alfredo Allen, 208
4200-135 Porto, Portugal

The purpose of this meeting is to focus the attention on the many and varied opportunities to promote interactions between mathematics and biology. The meeting will be devoted to mathematical and computational modelling, analysis and simulation of problems arising in the context of biological applications. One main purpose of this meeting is to encourage collaboration between research centers in mathematics, computer sciences, physics and biology, located at the University of Porto. We hope this is a good reason to begin joint research on Mathematical and Computational Models in Biology.

There will be four short courses, of three one-hour lectures each. These will be given by invited distinguished researchers and should be specially designed to attract undergraduate students to distinctive and relevant formation profiles, motivate them during their study, and advance their personal training in Mathematics and Biology.The courses will be supplemented by a hands-on workshop on COPASI (see here), contributed short talks by other participants and posters of case studies.

Contacts

João Nuno Tavares [jntavar@fc.up.pt]
Dep. de Mat. da Fac. de Ciências da Universidade do Porto
Rua do Campo Alegre, 687
4169-007 Porto
Telm: 93 319 27 63

WEB: http://cmup.fc.up.pt/cmup/biomath/2016

MEDDS 2016 — Modelling and experiments in drug delivery systems [June 20–22, 2016]

Department of Mathematics of FCTUC
Apartado 3008EC Santa Cruz
3001-501 Coimbra, Portugal
Tel: +351-239791150; Fax: +351-239793069

Mathematical modelling is playing an increasingly important role in the field of medicine through the use of models and numerical simulations. These models represent an extremely useful tool to complement theoretical and experimental work and provide personalised approaches for patient treatment. Controlled drug delivery systems like drug-eluting stents, transdermal patches, and intravitreal implants have become a common tool in clinical practice. These drug delivery systems combine a platform with a drug in a such way that the active agent is efficiently released to a target organ, while maintaining the drug concentration within a therapeutic window during an extended period of time. Devices and drugs can be tuned to meet the individual patient needs.The Modelling and experiments in drug delivery systems 2016 (MEDDS2016) workshop aims to present state-of-the-art research on modelling and experimental approaches for drug delivery systems. The goal is to bring mathematicians, biologists, physicians and engineers together in an open discussion environment.The workshop will be held at the Department of Mathematics of the University of Coimbra (DMUC) on June 20–22, 2016 (how to reach DMUC). This event is financially supported by the University of Coimbra and the Ecole Polytechnique AXA Chair in Cardiovascular Cellular Engineering (sponsors).

Important deadlines

Abstract submission: April 17th • Early registration: May 15th

For more information, please contact
medds2016@mat.uc.pt.

WEB: http://www.uc.pt/en/congressos/medds2016

Recent Trends in Differential Equations [June 27–29, 2016]

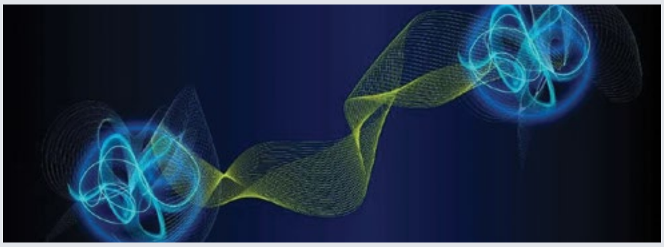
University of Aveiro
Aveiro, Portugal

The conference Recent trends in Differential Equations will celebrate the 75th birthday of Arrigo Cellina and the 60th birthday of Alberto Bressan and aims to bring together mathematicians engaged in research on differential equations, differential inclusions and set valued maps, calculus of variations, control theory, and applications.

Important dates

Registration and Abstract: May 15, 2016
Confirmation: May 25, 2016

WEB: http://rtde2016.weebly.com



6th IST-IME Meeting [September 5-9, 2016]

Instituto Superior Técnico
Lisbon, Portugal

This meeting, whose scope comprises Ordinary and Partial Differential Equations and related topics, aims to promote integration of researchers of both IST and IME (and other institutions), with the presentation of research works and plenary lectures of the participants.

This edition will honor Waldyr Oliva, our extraordinary mathematical bridge across the South Atlantic.

WEB: https://istime.math.tecnico.ulisboa.pt/announcement

For generic email questions about the meeting please contact
istime@math.tecnico.ulisboa.pt

Portugaliae Mathematica and its Exchange Library

by José Francisco Rodrigues*

In Portugal the first published books on mathematical sciences are dated of the Discovery Era, namely the remarkable Zacutus' astronomical tables (1496) and an influential book on Arithmetic's for the commerce by Gaspar Nicolás (1519), preceding the several books by Pedro Nunes (1502–1578) written in the Renaissance tradition. However, only in the XIX century Francisco Gomes Teixeira (1851–1933) founded the first mathematical journal in the Iberian world independent of any academic institution, the *Jornal de sciencias mathematicas e astronomicas* (1877–1902) [Ro]. This *Jornal* had the specific aim of ending Portugal's mathematical isolation and enhancing the direct contact with mathematicians from other countries and it reflected the significant increase of mathematical activity by Portuguese mathematicians, as observed in [S]. Although the majority of the authors of papers were Portuguese, it also included contributions from other European mathematicians like Ch. Hermite, M. D'Ocagne, E. Cesaro, G. Loria, or Ch. De la Vallée Poussin.

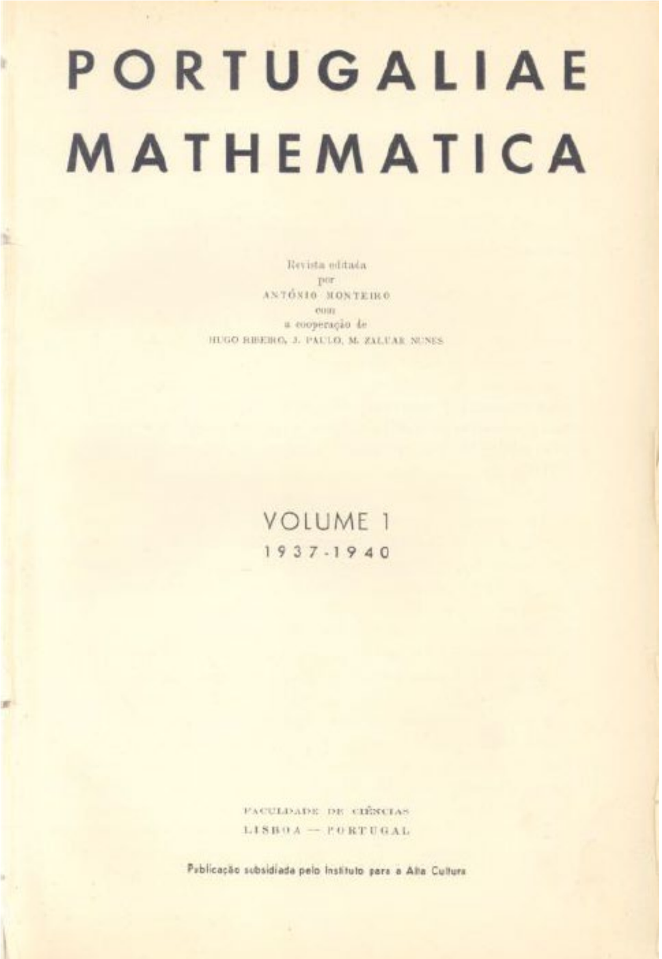
Gomes Teixeira, who became professor of Mathematics in 1883 at the *Academia Politécnica do Porto* and was the first *Reitor* of the University of Porto, from 1911 to 1929, contributed to create the basis of the rich periodicals component of the Library of the Faculty of Sciences of his University through the exchange of his *Jornal*, later called *Anais da Faculdade de Sciencias do Porto*, with other similar scientific journals. In a national survey of the 1930's [M1], the mathematics library in Porto was considered the most complete one in books and journals in the whole country.

A few years later, in 1937, António Aniceto Monteiro (1907–1980), the young forerunner of the mathematical modernism in Portugal, founded in Lisbon the journal *Portugaliae Mathematica*, just one year after having completed a thesis with Maurice Fréchet in Paris. This new research journal was created with the explicit purpose to contribute to the development of mathematical studies in Portugal and to archive all the Portuguese works unpublished or included in national and international journals [Ri1]. This modern version of the Teixeira's *Jornal*, created sixty years earlier, reflected a new mathematical movement in the country and had also the intention to contribute towards the international cooperation in the field, in particular, by initiating a successful exchange of mathematical periodicals.

Due to the poor conditions of the Portuguese mathematical libraries, this was one of the main contributions of the new journal. Indeed, during the initial period of its organization in 1937–1942, with the assistance of Hugo Ribeiro, José da Silva Paulo and M. Zaluar Nunes, Monteiro created in Lisbon the *Portugaliae Mathematica's Library* with the support of the *Instituto para a Alta Cultura* (IAC) [M1]. In 1940 the number of titles obtained by exchange was already of 60 and two years later it almost doubled to 116, being 10 from the USA and USSR each, 9 from Italy, Japan, Poland and Germany, 8 from UK and Romania, 7 from India, 6 from Belgium, etc. Half a century later, in 1990, after the reorganization of the *Portugaliae Mathematica's Library* done in the eighties by the *Centro de Matemática e Aplicações Fundamentais* (CMAF) at the University of Lisbon, that total number of titles in exchange was 143 [PG].

António Monteiro was also the driving force of a modernized professional and autonomous mathematics activity that he describes in 1942 in [M2]: "It is unquestionable that we see today in our country a truly effervescence of activities in the field of mathematical sciences. This assertion is shown by the successive appearance, in the short period of five years, of: 1) *Portugaliae Mathematica*, founded in 1937; 2) *Seminário Matemático de Lisboa* (1938), that changes its name into Semi-

* Centro de Matemática, Aplicações Fundamentais e Investigação Operacional da Faculdade de Ciências da Universidade de Lisboa



PORTUGALIAE
MATHEMATICA

Revista editada
por
ANTÓNIO MONTEIRO
com
a cooperação de
HUGO RIBEIRO, J. PAULO, M. ZALUAR NUNES

VOLUME 1
1937-1940

FACULDADE DE CIÊNCIAS
LISBOA — PORTUGAL

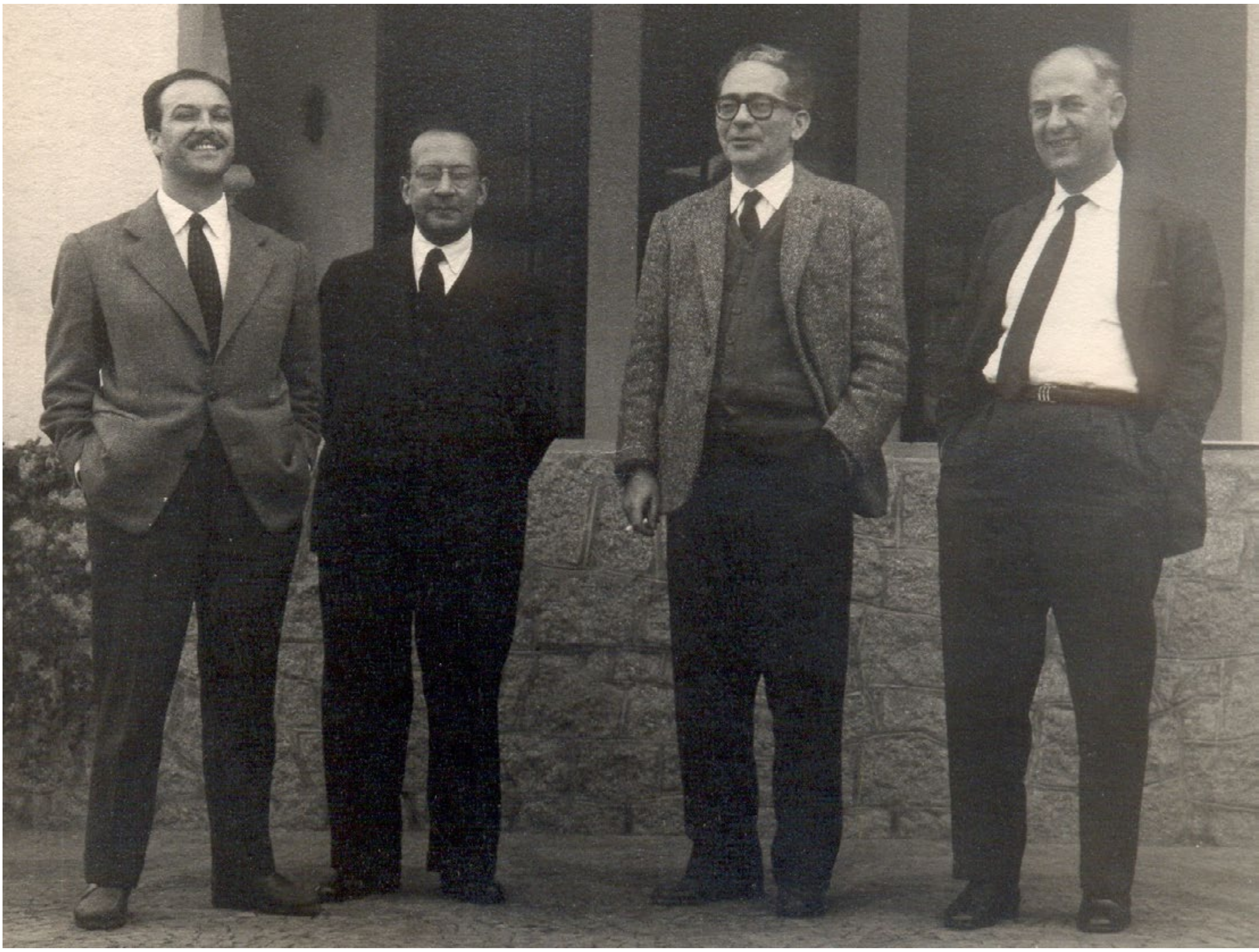
Publicação subsidiada pelo Instituto para a Alta Cultura

nário de Análise Geral in November of 1939; 3) *Centro de Estudos de Matemáticas Aplicadas à Economia*, founded by the 1st group of the Instituto Superior de Ciências Económicas e Financeiras (1938); 4) *Gazeta de Matemática*, January 1939; 5) *Centro de Estudos Matemáticos de Lisboa*, founded by the *Instituto para a Alta Cultura*, in February of 1940; 6) *Sociedade Portuguesa de Matemática*, 12th December 1940, 7) *Centro de Estudos Matemáticos do Porto*, founded also by the *Instituto para a Alta Cultura*, in February of 1942."

Portugaliae Mathematica started the first two volumes with Monteiro's thesis and an interesting blend of articles by young Portuguese researchers, such as Monteiro himself, Ruy Luis Gomes, Hugo Ribeiro, and José Sebastião e Silva, together with the inclusion of a few papers of the established mathematicians J. Vicente Gonçalves (1896–1985) and A. de Mira Fernandes (1884–1958), who supported the new journal by allowing the reprint of his papers from Italian journals. In those first volumes, the small group active around Monteiro in the *Centro de Estudos Matemáticos de Lisboa anexo à Faculdade de Ciências*, in the period 1940–1942 published several original contributions, in particular, solving and extending some questions proposed in the Fréchet's 1928 important book *Les espaces abstraits et leur théorie consideré comme introduction à l' Analyse générale*.

Fifty years later, Hugo Ribeiro wrote in [Ri2]: "I worked frequently with Monteiro, Zaluar Nunes and Silva Paulo in some old office room of the School of Science at the University of Lisboa. All of us did, of course, our best to help but I must emphasize, for it was and is my understanding, that Monteiro, alone, started the journal and took all crucial initiatives."

In 1942, the third and last volume of *Portugaliae* sponsored by the IAC shows an increasing international collaboration. It includes a long paper by John von Neumann, and, among others, also papers by M. Fréchet and R. Cacciopoli. In spite of the international recognition of the journal, the subsequence volumes of *Portugaliae Mathematica*, starting with the fourth volume corresponding to the years 1943/1945 and containing articles by G. Ascoli and H. Hopf, were published without any financial support from the state, a situation that continued until the reorganization in the end of the seventies already under the new democratic regime. The financial support came from the *Junta de Investigação Matemática* (JIM) and the Portuguese Mathematical Society (SPM). The JIM was a remarkable private association created

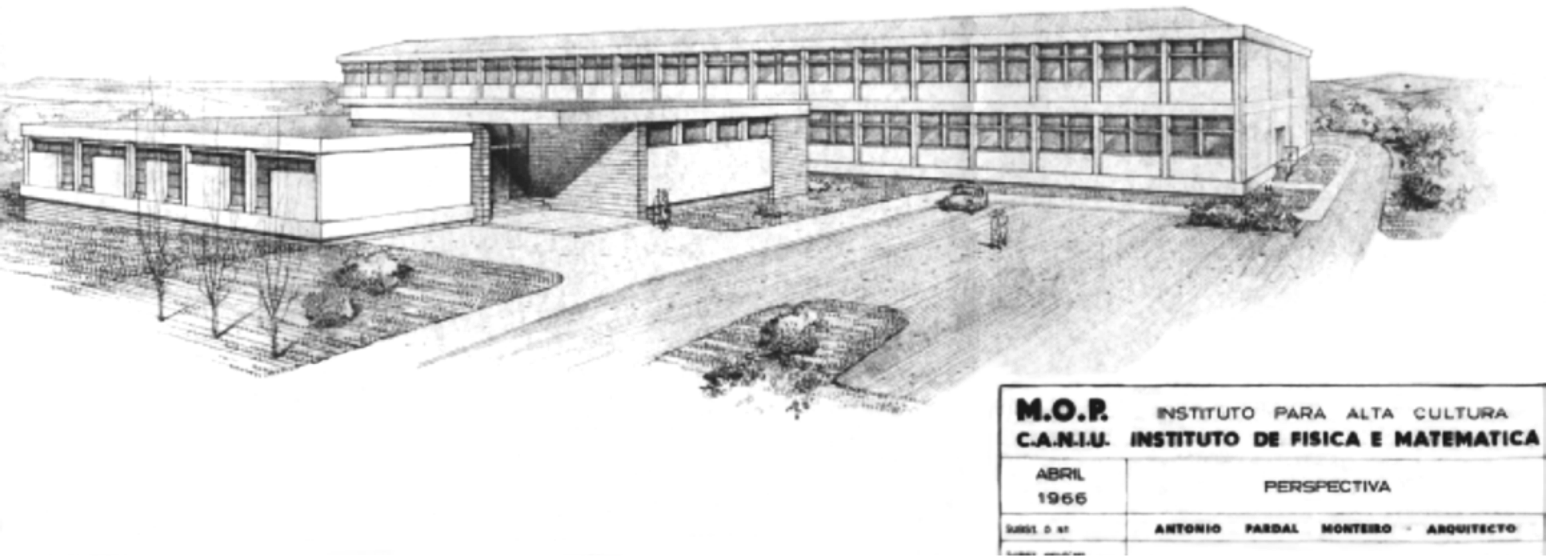


A. Pereira Gomes, Ruy Luis Gomes, António Monteiro and M. Zaluar Nunes, at the 2nd Colóquio Brasileiro de Matemática, in 1959, São Paulo, Brazil, were the representatives of the resistance of a generation.

in 1943 by the initiative of Ruy Luis Gomes, professor at the Faculty of Sciences of Porto, Mira Fernandes and António Monteiro with the purpose of promoting mathematical research and sponsoring publications, fellowships and international missions of Portuguese mathematicians [Ro].

In spite of the anti-intellectual offensive of the dictatorial regime in Portugal of 1947, that expelled from the University and the country several scientists and professors, creating an oppressive and retrograde cultural atmosphere and, in particular, would affect in a dramatic way the development of mathematical sciences in the country for the next three decades, *Portugaliae Mathematica* had survived. Under the direction of Zaluar Nunes, who became in fact the journal director from 1945 until 1967, replacing António Monteiro after his departure to Rio de Janeiro due to political and economical reasons, the list of contributors to the twenty five volumes of *Portugaliae* contains the names of all the relevant Portuguese mathematicians active in those two decades, some with several articles, like José Sebastião e Silva (1914-1972), who published in volume 9 (1950) his thesis presented at the Faculty of Sciences of Lisbon in 1948 [SS]. Within the long list of international collaborations, we may find not only the names of distinguished mathematicians, such as the already mentioned M. Fréchet, J. von Neumann, R. Caccioppoli, G. Ascoli and H. Hopf, but also W. Sierpinsky, L. Nachbin, L. de Broglie, P. Erdös, I. Kaplansky, M. Peixoto, J. Dieudonné, G. Köthe, C. Foias and J.-L. Lions. The majority of the editorial committee, that included also Ruy Luis Gomes, was in that period in exile, but shared Monteiro's vision of transforming *Portugaliae Mathematica* into a "truly organ of Portuguese mathematical culture". In fact, "History has seen this journal also as a remarkable example of intellectual resistance during the oppressive environment of indifference and repression of the dictatorship" [Ro].

From 1967 until the reconstruction of the the *Sociedade Portuguesa de Matemática* (SPM) in 1977, *Portugaliae Math-*



The architect's vision of the Instituto de Física e Matemática (1971-1975) in Lisbon.

ematica continued in Lisboa, published by the *Tipografia de Matemática*, and kept its role of an internationally accepted mathematical journal with a relevant exchange with several other mathematical journals. It was necessary the moral donation by António Monteiro of the title of *Portugaliae Mathematica* to the SPM and its official registration by the Society in December 1978 to establish the normal conditions for the recover of its international credibility [PG], under the responsibility of a new Editorial Committee directed by Alfredo Pereira Gomes.

The regularization of the periodicity of its publication, from volume 36 onwards took some years. That volume, corresponding to the year 1977, with the exception of the first issue, was published under the new Editorial Committee only in October 1980, with the financial support of the Calouste Gulbenkian Foundation. The *Portugaliae Mathematica's Library* was then re-established with a renewed list of exchanges with similar journals, following a protocol of 1983 with the *Centro de Matemática e Aplicações Fundamentais* (CMAF), the research mathematical centre originated in 1975 with the support of the INIC, the National Institute for Scientific Research that had replaced the IAC, resuming the old tradition of the *Centro de Estudos Matemáticos de Lisboa*, which was active from 1952 until 1972 under the direction of J. Sebastião e Silva. This agreement of mutual support of mathematical libraries was reconfirmed in 1991 with the CMAF, and, after the extinction of INIC at the end of 1992, continued to be supported by the *Complexo Interdisciplinar* at the University of Lisboa, in the new Library of the renewed building of the former *Instituto de Física e Matemática* of the IAC, where the journals were available to the Portuguese researchers during a period of more than 22 years.

Under the initiative of A. Pereira Gomes, that directed Portugaliae during 1978-1995, with the help of the new Editorial Committee, the volume 39, corresponding to the year 1980, appeared a few years later, again with support from the Calouste Gulbenkian Foundation, and was published in homage to the memory of its founder, just deceased at the age of 73 in Bahia Blanca, Argentina, where he lived for many years. This volume, in addition of the pages dedicated by his friends and colleagues to aspects of his life, work and personality, includes 24 original papers in homage to Monteiro and integrates also his long memoir [M3], which had been written during Monteiro's stay in Lisbon at the CMAF in 1977–1978, as visiting researcher, and was awarded with the 1978 Gulbenkian Science prize.

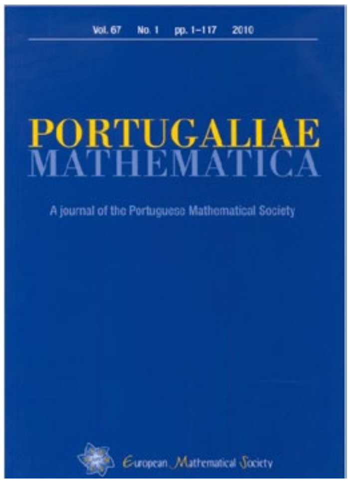
Also in the beginning of the eighties, during the normalization process of publication, Portugaliae dedicated a special volume, the 41st corresponding to 1982, to José Sebastião e Silva, the most significant Portuguese mathematician of the XX century who published nine papers in this journal from 1940 until 1960. This volume collected 42 papers presented at the international Symposium on Functional Analysis and Differential Equations held in Lisbon, organized by the *Sociedade Portuguesa de Matemática*, on the occasion of the tenth anniversary of the decease of the distinguished Portuguese analyst, and reflected the new atmosphere and internationalization of the renewed mathematical community at the dawn of the Portugal integration in the European Union.

With its fiftieth anniversary, the publication of volume 47 in 1987 corresponded to a stabilized editorial situation,

A view of the research Library of the Complexo Interdisciplinar of the University of Lisbon, where the Portugaliae Mathematica's Library was available from 1993 until 2015.

which then had recovered its international acceptance and had attracted again the collaboration of Portuguese mathematicians to the journal. With the financial support of INIC, that volume also initiated a new era of its publication, since for the first time it was electronically composed in TeX. From 1996 until 2006 it was partially supported by CMAF and *Centro de Álgebra*, as well as by the other Mathematics research units of the University of Lisbon, that provided the remaining financial funds for *Portugaliae Mathematica*, complementing its subscriptions and a support from the *Fundação para a Ciência e a Tecnologia* (FCT), the continuation of the INIC after 1993.

During the World Mathematical Year (WMY2000) the Editorial Committee of *Portugaliae Mathematica* was composed by six members from the Universities of Lisbon and Coimbra, under the direction of João Paulo Dias, a team in charge from 1996 to 2006. The Editorial Board of 31 members was composed by 15 Portuguese mathematicians, including two living abroad, and among the 16 other foreign members, being three of them North American, one Brazilian and the other 12 Europeans. Currently, the director is Luis Nunes Vicente, from the University of Coimbra.



The electronic edition of *Portugaliae Mathematica* of the first 50 volumes, from 1937 until 1993, were integrated in the digital *Biblioteca Nacional de Portugal* [W1]. The volumes 51 (1994) up to 63 (2006) are also in Open Access and are integrated in The Electronic Library of Mathematics (EMIS) [W2].

Starting with volume 64 (2007), with the support of four research units associated with the *Faculdade de Ciên-*

Fernando Pestana da Costa, President of the Portuguese Mathematical Society (SPM), with José Artur Martinho Simões, Director of the Faculty of Sciences of the University of Lisbon (FCUL), after the signature of Agreement on the Portugaliae Mathematica's Library the 11th November 2015

cias da Universidade de Lisboa (FCUL) and the FCT, *Portugaliae Mathematica* is being published by the EMS-Publishing House [W3], under an agreement of the SPM with the European Mathematical Society (EMS).

In the spring of 2015, with the transfer of the four research units of Mathematics affiliated with the FCUL from the *Complexo Interdisciplinar* of the University of Lisbon, together with their libraries, to the building of the Mathematics Department on the campus of the FCUL, the *Portugaliae Mathematica's Library*, that currently keeps an exchange of about one hundred titles with similar journals, was also transferred to the Library of the FCUL, where is maintained and made available to the scientific community under a renewed agreement with the SPM.

References

[M1] A. A. Monteiro, *Serviço de Inventariação da Bibliografia Científica Existente em Portugal — Relatório, Instituto para a Alta Cultura*, Lisboa, 1939.

[M2] A. A. Monteiro, *Movimento Matemático*, Gazeta de Matemática, nº10 (1942), 25–26.

[M3] A. A. Monteiro, *Sur les Algèbres de Heyting symétriques*, Portugaliae Math. 39 (1980), 1–237

[N] J. von Neumann, *Approximative properties of matrices of high finite order*, Portugaliae mathematica, 3 (1942), 1–62.

[PG] A. Pereira Gomes, *Portugaliae Mathematica, um marco histórico na investigação matemática portuguesa*, Publicações de História e Metodologia da Matemática, n.º5 Dep. Matemática, Univ. Coimbra, 1997.

[Ri1] H. Ribeiro, *Actuação de António Aniceto Ribeiro em Lisboa entre 1939 e 1942*, Portugaliae Math. 39 (1980), v–vii.

[Ri2] H. Ribeiro, *Just half a century ago . . .* , Portugaliae Math. 44 (1987), iii–iv.

[Ro] J.F. Rodrigues, *Portuguese Mathematical Journals, some aspects of (almost) periodic research publications*, in pp.601–627 of The Practice of Mathematics in Portugal, Edited by L. Saraiva & H. Leitão, Universidade de Coimbra, 2004 — (Portuguese translation in Boletim da SPM, n. 50, Maio 2004, 19-36).

[S] L. Saraiva, *A Survey of Portuguese Mathematics in the Nineteenth Century*, CENTAURUS, vol.42 (2000), pp.297–318.

[SS] J. Sebastião e Silva, *As funções analíticas e a análise functional*, Portugaliae Math. 9 (1950), 1–130.

[W1] http://purl.pt/index/pmath/PT/index.html

[W2] http://www.emis.de/journals/PM/

[W3] http://www.ems-ph.org/journals/journal.php?jrn=pm

Geometric Aspects of Modern Dynamics

by Alberto Pinto, Helena Reis, and Renato Soeiro

The conference *Geometric Aspects of Modern Dynamics* was held at the Department of Mathematics of the Faculty of Sciences of the University of Porto from 11 through 15 January 2016. The event was partially supported by the following institutions: Centro de Matemática da Universidade do Porto (CMUP), Centro Internacional de Matemática (CIM), Fundação Luso-Americana para o Desenvolvimento (FLAD), Fundação para a Ciência e a Tecnologia (FCT), Institut de Mathématiques de Toulouse (IMT) and Reitoria da Universidade do Porto (UP).

The conference brought together more than 70 experts in dynamical systems coming from various countries and including several field leaders for a program consisting of 24 talks. The scientific and organizing committees for the conference consisted of M. Abate (*University of Pisa*, Italy), A. Glutsyuk (*ENS-Lyon*, France and *HSE-Moscow*, Russia), M. Lyubich (*Stony Brook*, US), J. Raissy (*University of Toulouse*, France), J. Rebelo (*University of Toulouse*, France) and H. Reis (*University of Porto*, Portugal).

Broadly speaking, dynamical systems has to do with determining the asymptotic behavior of systems that *evolve with time*. The beginning of the theory is generally ascribed to Poincaré's investigations of the qualitative behavior of solutions of differential equations. The point of view of dynamics, however, was gradually enlarged to encompass the iterations of a diffeomorphism/endomorphism, more general finitely generated (semi) group actions, foliations and so on. It is the fact that the nature of these maps, differential equations and so on can be extremely varied that accounts for the existence of some many different trends in dynamical systems of which hyperbolic dynamics, conservative dynamics, group actions, and complex dynamics are examples.

Complex dynamics

Complex dynamics can roughly be described as the part of dynamics where the system under study has a holomorphic nature, regardless of it is a differential equation, a diffeomorphism or an endomorphism. It then just natural that the techniques and specific phenomena lying in the scope of complex analysis become an all important tool in the area. Basic examples of dynamics belonging to the universe covered by *complex dynamics* include most classical differential equations such as Gauss hypergeometric equation, Halphen systems, Riccati equations, Painlevé equations. It also includes the iteration of rational fractions on the Riemann sphere and questions about convergence of root-finding algorithms for polynomial equations.

Dynamical systems is an area strongly represented in the Portuguese mathematical community and the country counts on recognized groups working on (partially) hyperbolic dynamics, strange attractors, and conservative dynamics to name only a few. In contrast, there are very few Portuguese mathematicians working on *Complex dynamics* although some of them have acquired international recognition. Time seems then ripe to make the area of Complex dynamics better known not only to our students but also to our colleagues working in other branches of dynamics.

A conference in dynamical systems emphasizing complex dynamics

The conference *Geometric Aspects of Modern Dynamics* featured a wide range of classical topics in dynamics such as those stemming from classical hyperbolic dynamics (SRB measures, topologically/smoothly equivalent systems and so on), real analytic geometry and Hilbert's problem, counting problems and lattices in spaces of negative curvature. A definite emphasis was, however, put on complex dynamics as around 2/3 of the program was devoted to topics connected with complex dynamics in a large sense. As a matter of fact, the approach to complex dynamics taken by the organizers was precisely to try to work out a *bigger picture* for the field and, ultimately, this attempt at considering complex dynamics in a broader sense constituted a distinctive trait of our conference with respect to most conferences devoted to complex dynamics. This point of view deserves to be further elaborated here.

The origin of complex dynamics goes back to the nineteenth century and to the study of classical differential equations. Landmarks in this direction are Riemann's theory on Gauss hypergeometric equation and Poincare's work on Riccati equation which led Poincare to study basic properties of Kleinian groups. Around the same time E. Schröder studied the convergence of Newton's method and obtained some fundamental results concerning the iteration of rational fractions on the Riemann sphere. The study of complex differential equations was then continued by Painlevé while Julia and Fatou have laid the foundations of the iterative theory of rational fractions. After the works of Painlevé and of Fatou-Julia, the subject of *complex dynamics* has developed into two main strands, namely *iterative dynamics*, where the main object of study is the dynamics arising from one single holomorphic automorphism/endomorphism, and *holomorphic foliations* whose aim is to understand the dynamics of the holomorphic foliation associated with a (possibly meromorphic) differential equation. Although each of these strands has evolved into a large body of results and methods, it is perhaps a bit disappointing to realize that communication between experts in *iterative theory* and *holomorphic foliations* has not been very effective.

Over the past twenty five years or so, the community formed by mathematicians working in complex dynamics has hoped for a more unified theory where each of the above mentioned main strands in the field would be able to benefit from progresses made in the other. Establishing connections between two topics in Mathematics is always useful and often sparks some accelerate progress. For example, in recent years, the *iterative side* of complex dynamics has benefitted immensely from its connections with Kleinian groups while techniques of *holomorphic foliations* have found new applications in complex algebraic geometry. Yet, and despite of the efforts of some outstanding mathematicians such as Dennis Sullivan and Etienne Ghys, examples of situations where relevant interactions between these two *sides* of complex dynamics have occurred are still not numerous.

In the conference of Porto some concerted effort was done in terms of stimulating collaboration between *iterative theory* and *holomorphic foliations*. Common features between the problems studied in *iterative theory* and in *holomorphic foliations* have long been identified but, somehow, the differences between them have out-weighted the similarities and provided major obstacles to a unified framework. Probably the simplest and greatest obstacle to bring the themes together can be summarized as follows: in *iterative theory* we study one single and globally defined map whereas the dynamics of a *holomorphic foliation* is encoded by a collection of maps that are only locally defined. Very recently, however, some very subtle but promising directions to develop deeper collaborations between the two sides have emerged and some of them were highlighted in the talks.

Interactions between different branches of dynamics

As mentioned above, a significant part of the conference program was devoted to promising directions for developing a more unified theory in complex dynamics. In more general terms, it is also true that the idea of breeding new connections between the several themes in dynamics represented in the conference was ubiquitous in the designing of the program.

It is well-known that the abstract setting of ergodic theory often provides a useful language in which problems from different areas of dynamics can be formulated in a unified way. In fact, when it comes to complex dynamics, the developing of a suitable ergodic theory for foliations is widely viewed as a fundamental step to build effective bridges with the *iterative theory*. Ergodic theory for holomorphic foliations was the subject of talks by J. Rebelo and by N. Sibony. The organizers were pleased to see one expert from *iterative theory* and one expert in *holomorphic foliations* giving two very closely related talks about the same topic.

Another interesting aspect was J.-P. Ramis talk which focused on producing new relevant examples of *iterative dynamics* out of Painlevé equations. In turn, R. Roeder discussed examples of endomorphisms of the complex projective plane without invariant foliations. Similarly M. Abate talked about the existence of parabolic curves for germs of diffeomorphisms tangent to the identity: a topic in which the local theory of foliations provides important tools.

On a different direction, A. Guillot and H. Reis have talked about complex differential equations with uniform solutions along with some applications to problems of complex geometry. Concerning (pseudo) differential geometry, M. Pollicott and N. Tholozan discussed lattices in negative curvature and some related problems with ergodic theoretic nature.

Apart from *holomorphic foliations*, there was also a number of talks on Henon maps a topic where *iterative theory* in complex dynamics comes close to *real* dynamics problems related to SRB measures and strange attractors, subjects well represented in the University of Porto, and J.-F. Alves, E. Bedford, M. Martens, A. Pinto, and M. Yampolsky have all given talks connected with this circle of ideas.

Conclusion and sources for students and newcomers

The literature on dynamical systems is vast and continuously increasing as the area remains very active. The collection of *Handbook of Dynamical Systems* offers a good overview of many aspects of what might be called *real* dynamics, as opposed to complex dynamics in the sense described above. Yet the literature on complex dynamics is huge as well. People interested in holomorphic foliations may benefit largely from classical material including Painlevé's *Leçons sur la théorie analytiques des Équations Différentielles* or P. Ince's classic *Ordinary Differential Equations*. Modern expositions of the theory are given in [3], [4], [6]. For the *iterative side* of complex dynamics, two good introductions are provided by the books of Carleson-Gamelin and of Milnor [2], [5]. For higher dimensional theory, we refer the reader to [1]. Additional useful information, including announcements for several past and upcoming conferences in the field, can be obtained from the Dynamical Systems web-page of Stony Brook: http://www.math.stonybrook.edu/dynamical-systems.

References

[1] M. Abate et al, *Holomorphic Dynamical Systems*, Lecture Notes in Mathematics 1998, Springer-Verlag, Berlin-Heidelberg. 2010

[2] L. Carleson and T. Gamelin, *Complex dynamics, Universetext: Tracts in Mathematics*, Springer-Verlag, New York. 1993

[3] Y. Ilyanshenko and S. Yakovenko, Lectures on Analytic Differential Equations, Graduate Studies in Mathematics, Volume 86, AMS Providence Rhode-Island (2008).

[4] K. Iwasaki, H. Kimura, S. Shimomura, and M. Yoshida, From Gauss to Painlevé, a modern theory of special functions, Braunschweig-Vieweg, (1991).

[5] J. Milnor, Dynamics in One Complex Variables, Annals of Mathematics Studies, Number 160, Princeton University Press (2006).

[6] J. Rebelo and H. Reis, Local Theory of Holomorphic Foliations and Vector Fields, available from arXiv:1101.4309.

An Interview

with Cédric Villani

by **Jorge Buescu**

Fields Medalist Cédric Villani visited Portugal in November 2015. True to his fame as a polymath, he was a member of the jury for the international film festival LEFFEST, gave a lecture on maths for the general public at the University of Coimbra, launched the Portuguese edition of his book Théorème Vivant and held several other speaking engagements. In the midst of all this he still found the time to discuss with us Maths, the Universe and Everything.

Pour faire les Maths au plus haut niveau, il faut poursuivre une idée obsessivement. Peux-tu décrire ça?

C'est très simple: on commence par de la curiosité, on se pose une petite question . . . puis on y réfléchit fort . . . puis on y pense jour et nuit, cela devient une obsession, on y met toutes ses forces vives, et chaque nouvelle idée vient renforcer le projet; on écrit, on réécrit, on recommence . . . On finit par y porter un intérêt vital . . . Il est très important de pouvoir se mettre dans un tel « état obsessif », même si ce n'est que temporaire. Dans Théorème Vivant, cette montée de l'obsession est rendue en partie par l'invasion progressive du texte par des formules.

Il est clair dans ton livre Théorème Vivant que, avant avoir déclenché la Médaille Fields, les maths occupaient tout ton temps. Comment la Fields a-t-elle changé ta vie?

Dans les années de l'avant médaille Fields, c'était effectivement très plein de mathématique . . . mais cela n'a pas toujours été ainsi. Le début de ma thèse était marqué par ma propension a faire toutes sortes de choses : j'allais au cinéma, au concert, je me suis même fait élire président de l'association des élèves a l'École normale supérieure. Mes journées étaient alors occupées de tout sauf des maths, et ma carrière a bien failli se terminer avant d'avoir commencé !! La médaille Fields m'a fait revenir a cet état en me confiant toutes sortes de missions publiques — représentation, discours, administration, coopération, montage de projets . . . je suis devenu président d'association, auteur, personnage public, etc. C'est une nouvelle vie, ou même plusieurs nouvelles vies, qui se sont ouvertes a moi.

Te manque-t-il l'engagement journalier avec les maths? Tu crois que tu retourneras à ton ancienne vie ou les nouveaux défis te séduisent en trop?

Oui, l'engagement journalier me manque . . . mais on ne peut pas tout faire dans la vie ! Le piano me manque, les sorties régulières au cinéma me manquent, on pourrait continuer la liste longuement et c'est ainsi. Le plus grave n'est pas quand le temps vous manque, c'est quand l' envie vous manque. Par ailleurs je continue a donner des cours et a perfectionner mes notes de cours, cela est très important pour moi aussi. Et, pour ce qui est de l'avenir, je ne m'engage en rien — je n'ai jamais envisage mon avenir a longue durée, et ne compte pas le faire maintenant ! Pour l'instant j'ai des défis a faire aboutir d'ici a la fin de mon mandat de directeur a l'Institut Henri Poincaré, c'est une priorité.

Comment expliques-tu l'excellence de l' école mathématique française?

C'est en premier lieu une question d'histoire : une tradition qui remonte aux Lumières, des institutions fécondes, un héritage de la Révolution française. Mais aussi une question d'esprit : les français sont si friands d'universalisme, d'abstraction, d'absolu . . . A mes collègues étrangers je dis souvent en plaisantant « la mathématique est l'art de découvrir les lois absolues du monde et de les expliquer a la Terre entiere, n'est-ce pas précisément ce que les Français font sans cesse et sur tous les sujets ? »

En 2014, après plus de 70 ans, pour la première fois une femme [Mariam Mirzhakani] déclenche la Médaille Fields. Pourquoi a-t-il fallu autant de temps? Les femmes ne font pas de bonnes maths?

Certaines raisons sont historiques : pendant longtemps les femmes n'étaient pas admises a étudier (pensez a Hypatie d'Alexandrie . . . ou a Sophie Germain faisant semblant d'être un homme pour communiquer avec Gauss). L'absence de modèles a contribué a entretenir l'idée d'un manque de dispositions des filles pour les sciences fondamentales; c'est pour cela que la médaille de Maryam Mirzakhani, en brisant cette fatalité, constitue un événement très important. Mais il y a aussi d'autres raisons qui sont plus subtiles a analyser et vont au-delà des stéréotypes les plus simples : des questions de comportement face a la compétition, de capacité a s'immerger dans un environnement sélectif et compétitif; dans mon expérience cela rebute souvent les jeunes filles; cela va aussi avec le manque de confiance en soi de beaucoup d'entre elles. Pourtant les carrières de sciences fondamentales, par la relative liberté qu'elles procurent, sont assez favorables aux jeunes femmes, et je recommande vivement a celles qui ont du gout pour ces disciplines de l'envisager pour leur carrière. Notons enfin que dans certaines cultures (Proche-Orient, Moyen-Orient), ce sont les femmes qui sont statistiquement les plus motivées dans ces sujets, et d'ailleurs le fait que c'est une iranienne qui a été la première a décrocher la médaille Fields le rappelle. Cela démontre bien aussi que la question est éminemment culturelle.

Aurais-tu quelque conseil à donner aux mathématiciens portugais?

Il est très important de voyager et prendre des idées a droite et a gauche, même si l'on souhaite rester

basé dans son pays ! Pour le Portugal, le cas de mon collègue Jean-Claude Zambrini, professeur a Lisbonne, me vient en tete : c'est a l'étranger et avec le jeu des rencontres qu'il a pu trouver ses sujets de prédilection.

Quel conseil donnerais-tu à jeunes apprentices de mathématiciens?

D'abord, être fier et confiant de travailler dans un sujet qui est en train de se développer plus que jamais. Ensuite, ne pas hésiter a se spécialiser dans un premier temps, tout en restant curieux de tout et prêt a approfondir dans le futur de nouveaux sujets. Et puis bien laisser une place au jeu de la chance et du hasard pour façonner la carrière.

Auxquels projets professionnels te consacres-tu aujourd'hui?

Beaucoup — trop, certainement! L'agrandissement de l'Institut Henri Poincaré en est un; cela passe par de grands travaux de rénovation (pour lequel j'ai obtenu de fortes subventions publiques), la création d'un fonds de dotation, des partenariats privés, la mise en place d'un musée des sciences orienté sur la mathématique, pour tous publics . . . C'est le gros projet de mon second mandat de direction. Je fais également des actions de coopération avec l'Afrique : chaque année j'y enseigne dans des formations de niveau master, et je participe a une multitude de conseils scientifiques. Enfin je suis toujours éditeur de revues mathématiques et membre de nombreux conseils scientifiques.

Pourquoi as-tu senti l'appel de la vulgarisation scientifique après la Médaille Fields?

C'était naturel dans l'environnement scientifique de Lyon — en mathématique, tout particulièrement l'influence d'Etienne Ghys qui avait donné l'exemple. J'avais même suivi une formation CNRS sur la communication avec les médias. Mais surtout, c'est tout le monde — toute la société — qui m'a sollicité. Les écoles m'ont contacté, les chaines de télévision, les radios, les journaux . . . quand on en fait, et que cela marche bien, alors on est invité a en refaire, encore et encore. Chaque année je reçois des centaines et des centaines d'invitations a donner des conférences publiques ou a participer a des émissions, débats, dossiers, etc. Rien qu'en répondant a une petite proportion des demandes, j'ai un programme de conférences complet.

Et que retiens-tu de ces cinq années de très fort engagement public?

Première leçon: nous — chercheurs, scientifiques — sommes très populaires, des que nous pouvons affirmer notre individualité et notre prise de parole.

Deuxième leçon : le contact avec le grand public est très enrichissant, si on le travaille sérieusement. Troisième leçon : le contact avec le public, cela se travaille et s'améliore; il y a des difficultés, des pieges, et on apprend a les contourner. Quatrième leçon : cela prend des années de tisser les liens et les expériences jusqu'a avoir un impact conséquent. Cinquième leçon : c'est a coup d'émotions (rires, émerveillement, sens du tragique, art) que l'on établit les liens forts.

Tu as une expérience unique. Veux-tu nous en raconter un ou deux épisodes particulièrement remarquables?

Il y a eu des épisodes forts. La publication de mes livres en a été un : avant tout, Théorème vivant (plus de 100 000 exemplaires, traduit en 12 langues . . .); mais aussi « Les Rêveurs lunaires », un roman graphique (bande dessinée) original qui m'a donné 18 mois de collaboration extraordinaire avec un très grand dessinateur, devenu un ami proche. Mon expérience associative a été très marquante : j'ai pris la présidence de l'association Musiques, fondée par le musicien et ingénieur Patrice Moullet; il s'agit de concevoir des instruments de musique nouveaux pour faire jouer a la fois des artistes professionnels et des jeunes handicapés. La visite du centre de polyhandicap avec lequel nous travaillons a été un des moments les plus émouvants de ma vie. Les visites organisées dans mes tournées ont été souvent très fortes, depuis le désert en Arabie jusqu'au muséum d'histoire naturelle du Minnesota (ou j'ai pu tenir dans mes mains des ossements fossiles très rares de dinosaures), en passant par la bibliothèque de Coimbra, les sites archéologiques du Liban, etc. etc. Et les passages médiatiques ont aussi été l'occasion de moments forts. Un jour, sur un plateau télévision, dans une émission très populaire, je savais que j'aurais exactement une minute pour évoquer ma discipline. J'ai préparé trois objets illustratifs — une édition des Eléments d'Euclide, un « Gomboc » (une forme mathématique fascinante découverte récemment par des chercheurs hongrois) et un téléphone portable — pour illustrer toute la gamme de ce que représente la mathématique. Grand succès !

Tu viens d'être choisi, avec Elvira Fortunato, pour intégrer le plus importante Conseil Scientifique de la CE. Penses-tu te dédier aussi aux politiques?

Il m'est arrivé de m'engager dans quelques élections en France, en particulier j'ai été président du comité de soutien d'Anne Hidalgo pour les élections a la Ville de Paris. Mais mon engagement politique durable, c'est en tant qu'européen fédéraliste : je reprends ainsi un flambeau que portait Henri Cartan en son temps. Ma nomination au sein du Conseil Scientifique s'inscrit bien dans cet engagement. Je crois qu'il est important que nous scientifiques soyons impliqués dans la vie politique; nous avons notre mot a dire.

Quels sont tes intérêts au-delà des Maths?

La musique, la lecture, le cinéma, les bandes dessinées, les voyages . . . un peu tout! Au Portugal j'ai eu le privilège de participer au jury du remarquable festival de cinéma de Lisbonne & Estoril, c'était une expérience passionnante.

Tu t'habilles de façon unique. Costume de trois pièces, cravate, broche en forme araignée tous les jours . . . pas du tout l'habituel look d'un mathématicien! Pourquoi?

Parce que c'est moi. Sérieusement, il n'y a pas de raison particulière. J'ai adopté mon look vers 20 ans, c'était un besoin, c'était instinctif. J'ai cherche le costume qui semblait le mieux correspondre a moi.

Tu as dit que tu étais timide en garçon. C'est fort difficile à croire, en te voyant aujourd'hui . . . tu veux commenter?

Un « monument humain a la gloire de la timidité » : c'est ainsi qu'un journaliste m'a qualifié dans un article de journal, alors que j'avais 17 ans. (J'étais interviewé pour mes bons résultats au baccalauréat.) Pendant mon enfance on m'a répété que j'étais trop timide : les parents, les enseignants, les camarades . . . J'ai plus souffert de ces rengaines, que d'être timide . . . Quant a mon évolution vers mon caractère actuel plutôt extraverti, elle s'est faite naturellement; les humains évoluent au fur et a mesure qu'ils changent d'environnement.

Tu as des garçons et on voit dans ton livre que tu es un père très affectueux. Ont-ils la conscience que Papa est l'un des plus grands scientifiques du monde et une étoile planétaire? Qu'en pensent-ils?

Une étoile planétaire, il ne faut pas exagérer, je suis sur que quelqu'un comme Justin Bieber, quoi que l'on pense de sa musique, a beaucoup plus de succès médiatique que moi !! Quant a mes enfants, je crois qu'ils sont fiers de me voir intervenir un peu partout, et ils lisent mes ouvrages avec attention; en même temps, ils aiment bien se moquer gentiment de moi; et ils sont surtout heureux quand je suis a la maison, a coup sur !

Representations, Higgs bundles and components: an overview

by André Oliveira*

1 Introduction

Compact oriented surfaces, without boundary, are among the most classical objects in geometry. Their topological classification has been completely achieved for quite a long time: such a surface must be equivalent to one, and precisely one, of the types Σ_g, where Σ_0 is the sphere, Σ_1 is the torus and, for $g \geq 2$, Σ_g is the connected sum of g copies of the torus Σ_1. The integer g is called the *genus* of Σ_g. (See figure 1 for an example of Σ_2.)

Despite being a classical object, there are spaces naturally associated to Σ_g, whose geometry and topology is unknown and which lie on the edge of current mathematical research. One instance of such spaces are the so-called *character varieties* of Σ_g, also known as *spaces of surface group representations*. These are natural objects to consider, occurring in several areas of geometry, topology and even physics.

The study of character varieties is a motivation for introducing *Higgs bundles* over compact Riemann surfaces and their moduli spaces, the main subject of the present article. Higgs bundles and their moduli were introduced by Nigel Hitchin in the outstanding paper [17] almost thirty years ago. It is truly amazing the research that has been carried out based on that paper. However, the topology of moduli spaces of Higgs bundles on Riemann surfaces is far from being understood.

The aim of this article is to give an overview of the problem of studying the connected components of moduli spaces of Higgs bundles and of character varieties. Half of the article deals with the definition of character varieties, of Higgs bundles and with the relation between them. The goal is that the reader acquires a feeling of this exciting area of mathematics, also of the problems we address and (hopefully) of some techniques to handle them. The interested reader may find more details in the references.

Although we introduce Higgs bundles as a motivation for the study of character varieties, we stress the fact that they play a crucial role in many other different areas including gauge theory, Kähler and hyperkähler geometry, integrable systems, mirror symmetry, Langlands duality and more. We do not touch any of these topics.

2 Two moduli spaces

2.1 Representations and character varieties

Let us start with a fixed closed oriented surface Σ_g. Assume that $g \geq 2$. The fundamental group of Σ_g is a finitely generated group, with $2g$ generators, such that the product of

* CMUP, Faculdade de Ciências–Univ. do Porto [andre.oliveira@fc.up.pt]
On leave from: Dep. de Matemática, Univ. de Trás-os-Montes e Alto Douro, UTAD [email: agoliv@utad.pt]

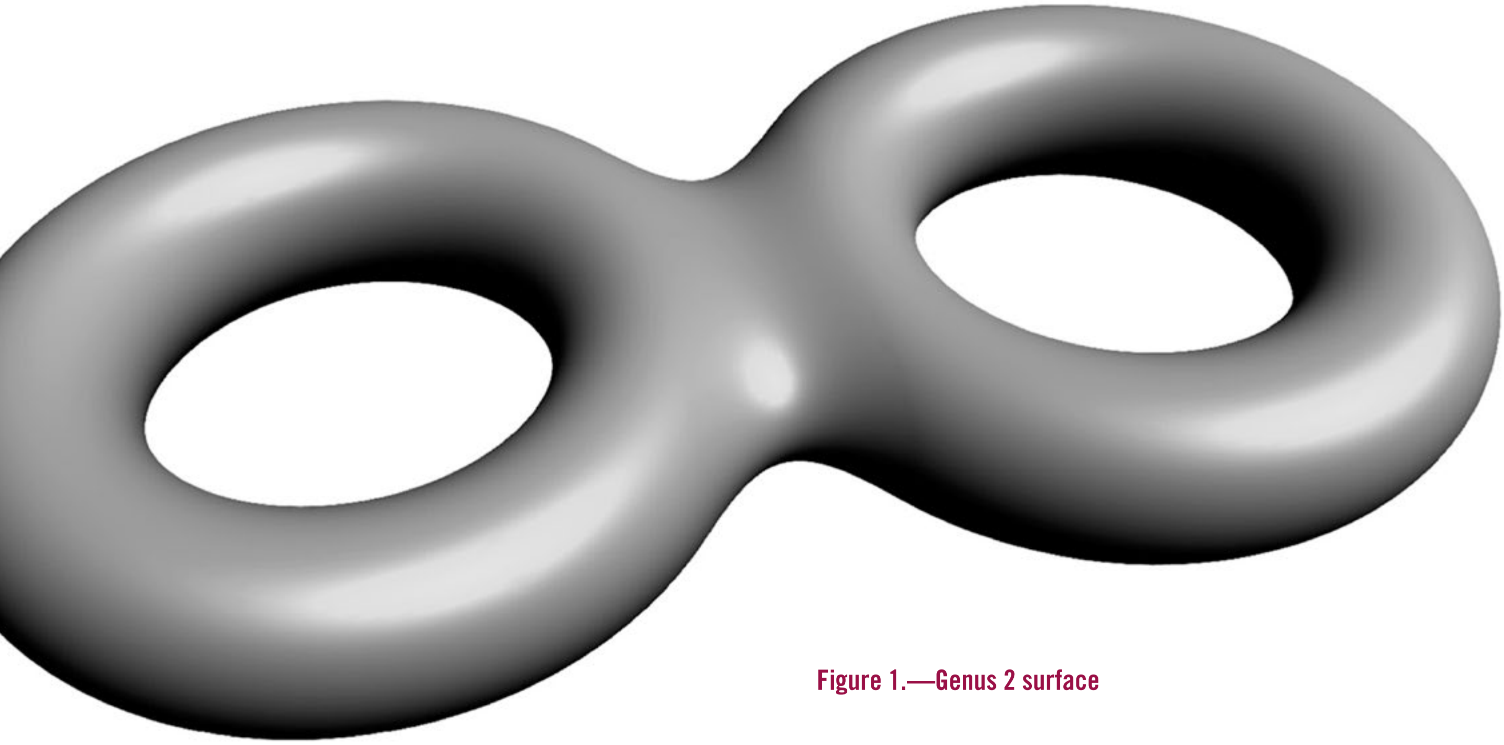


Figure 1.—Genus 2 surface

their commutators is trivial:

$$\pi_1(\Sigma_g) = \left\langle a_1, b_1, \dots, a_g, b_g \;:\; \prod_{i=1}^{g} [a_i, b_i] = 1 \right\rangle.$$

Let G be a connected real semisimple Lie group, which we assume admits a complexification $G^{\mathbb{C}}$. Consider the set $\mathrm{Hom}(\pi_1(\Sigma_g), G)$ of all group homomorphisms from $\pi_1(\Sigma_g)$ to G. Such a homomorphism $\rho : \pi_1(\Sigma_g) \to G$ is also called a *representation* of $\pi_1(\Sigma_g)$ in G. The name comes from the fact that G is often a linear Lie group, acting naturally on a vector space, so yielding a representation of $\pi_1(\Sigma_g)$ on that vector space. Any representation is determined by its values on the $2g$ generators, so $\mathrm{Hom}(\pi_1(\Sigma_g), G)$ is contained in G^{2g} as the subset of those $2g$-tuples $(A_1, B_1, \dots, A_g, B_g)$ satisfying the equation $\prod_{i=1}^{g} [A_i, B_i] = 1$. So we take the induced topology on $\mathrm{Hom}(\pi_1(\Sigma_g), G)$, which coincides with the compact-open topology, hence does not depend on the choice of the generators of $\pi_1(\Sigma_g)$.

It is natural to consider two representations equivalent when they lie in the same orbit of the G-action on $\mathrm{Hom}(\pi_1(\Sigma_g), G)$ by conjugation: $g \cdot \rho = g\rho g^{-1}$. Indeed if ρ and ρ' are in the same orbit under this action, and if G acts on a vector space $\mathbb{V}$ through a linear representation, say α, then the difference between the representations $\alpha \circ \rho$ and $\alpha \circ \rho'$ of $\pi_1(\Sigma_g)$ in $\mathbb{V}$ is just given by a change of basis. Hence we are interested on the quotient space $\mathrm{Hom}(\pi_1(\Sigma_g), G)/G$. However, it may not be Hausdorff due to the existence of non-closed orbits whose closures intersect. A way to solve this is to take only *reductive* representations, meaning the ones that become a sum of irreducible representations when composed with the adjoint representation of G on its Lie algebra $\mathfrak{g}$, i.e., with $\mathrm{Ad} : G \to \mathrm{GL}(\mathfrak{g})$. In any case, reductive representations are dense in $\mathrm{Hom}(\pi_1(\Sigma_g), G)$. Denote the space of such representations by $\mathrm{Hom}^{\mathrm{red}}(\pi_1(\Sigma_g), G)$.

Definition 1.— The G-*character variety* of Σ_g is the quotient space

$$\mathcal{R}(G) = \mathrm{Hom}^{\mathrm{red}}(\pi_1(\Sigma_g), G)/G.$$

One natural question is about the existence of discrete invariants of such classes of representations. Indeed we can define them in an easy way. Take a class $[\rho]$ in $\mathcal{R}(G)$ and choose a representative $\rho : \pi_1(\Sigma_g) \to G$. Let $(A_1, B_1, \dots, A_g, B_g) \in G^{2g}$ be the images through ρ of the generators of $\pi_1(\Sigma_g)$. Consider the universal covering $p : \widetilde{G} \to G$, whose kernel is isomorphic to $\pi_1(G)$, the fundamental group of G. Choose a lift $(\tilde{A}_1, \tilde{B}_1, \dots, \tilde{A}_g, \tilde{B}_g) \in \widetilde{G}^{2g}$ of $(A_1, B_1, \dots, A_g, B_g)$ under p and define the element

$$c([\rho]) = \prod_{i=1}^{g} [\tilde{A}_i, \tilde{B}_i] \in \pi_1(G). \tag{1}$$

This $c([\rho]) \in \pi_1(G)$ is an invariant of the class $[\rho]$. It does not depend on the choices made because $\pi_1(G)$ is contained in the centre of $\widetilde{G}$.

Given $c \in \pi_1(G)$, denote by $\mathcal{R}_c(G)$ the subspace of $\mathcal{R}(G)$ whose invariant defined by (1) is c. Each $\mathcal{R}_c(G)$ is a union of connected components of $\mathcal{R}$, and we have a disjoint union $\mathcal{R}(G) = \bigsqcup_{c \in \pi_1(G)} \mathcal{R}_c(G)$.

2.2 Higgs bundles and their moduli spaces

We defined character varieties in a purely topological context. In contrast, to introduce Higgs bundles we have to impose a Riemann surface structure on Σ_g. So let X be a compact Riemann surface of genus $g \geq 2$, whose underlying 2-dimensional manifold is the surface Σ_g. Hence X is a complex manifold of (complex) dimension 1. It will be fixed throughout.

The definition of Higgs bundles on X requires some basic knowledge of holomorphic algebraic geometry, which we provide next. We emphasise that these are not the rigorous definitions (which can be found in [15, 23, 25]).

A *holomorphic vector bundle V of rank n* on X is a family of vector spaces V_x, for each $x \in X$, varying holomorphically with the point x. Each of these vector spaces V_x, called the *fibre* of V at x, is non-canonically biholomorphic to $\mathbb{C}^n$. Moreover, V is required to be locally trivial, that is, for every point $x \in X$, there is an open neighbourhood U_x of x such that V restricted to U_x is biholomorphic to the product $U_x \times \mathbb{C}^n$. The *trivial vector bundle* over X is just the cartesian product $X \times \mathbb{C}^n$ (hence the name "locally trivial" above). When $n = 1$ we say that V is a *line bundle*.

Any operation over vector spaces, such as tensor product, direct sum or duality, can be extended to the vector bundle setting. In particular if we consider the wedge product and have a rank n vector bundle V, we construct the line bundle $\bigwedge^n V$, whose fibres are biholomorphic to the top wedge power of the fibres of V. This is called the *determinant of V*.

A *section* of a vector bundle V over X is a holomorphic map $s : X \to V$ such that $s(x) \in V_x$. The vector space of sections of V is denoted by $H^0(X, V)$.

If G is a complex Lie group, a *holomorphic principal G-bundle* or, for short, *G-bundle*, E is a family of (non-canonical) copies of the group $G_x \simeq G$, for each $x \in X$, varying holomorphically with the point x. Again, E is required to be locally trivial, that is, every point $x \in X$ has an open neighbourhood U_x such that E restricted to U_x is biholomorphic to the product $U_x \times G$. The *trivial G-bundle* on X is again the product $X \times G$.

If G acts on some vector space $\mathbb{V}$ and if E is a G-bundle, then there is a canonical way to construct a vector bundle, with fibres isomorphic to $\mathbb{V}$, out of the action $G \to \mathrm{GL}(\mathbb{V})$ and of E. Denote this vector bundle by $E(\mathbb{V})$.

Now we pass to some definitions of Lie theory, which again are not given in complete detail. These may be found for instance in [6]. Let $H \subseteq G$ be a maximal compact subgroup of G and $H^{\mathbb{C}} \subseteq G^{\mathbb{C}}$ be its complexification. If $\mathfrak{h}^{\mathbb{C}} \subseteq \mathfrak{g}^{\mathbb{C}}$ are the corresponding Lie algebras, there is a *Cartan decomposition*

$$\mathfrak{g}^{\mathbb{C}} = \mathfrak{h}^{\mathbb{C}} \oplus \mathfrak{m}^{\mathbb{C}}, \tag{2}$$

where $\mathfrak{m}^{\mathbb{C}}$ is a complex vector space. An example of (2) is the fact that any complex square matrix can be uniquely written as a sum of a symmetric and a skew-symmetric matrix. Now, the adjoint representation $\mathrm{Ad} : G^{\mathbb{C}} \to \mathrm{GL}(\mathfrak{g}^{\mathbb{C}})$ induces a representation of $H^{\mathbb{C}}$ on $\mathfrak{m}^{\mathbb{C}}$. So, if E is an $H^{\mathbb{C}}$-bundle over X, let $E(\mathfrak{m}^{\mathbb{C}})$ be the vector bundle associated to E and to the action of $H^{\mathbb{C}}$ on $\mathfrak{m}^{\mathbb{C}}$, as explained in the preceding paragraph.

Let $K = T^*X$ be the canonical line bundle of X. By definition this is the holomorphic cotangent bundle of X, that is, the dual of its tangent bundle.

Definition 2.— A *G-Higgs bundle* over X is a pair (E, φ) where E is a (holomorphic) $H^{\mathbb{C}}$-bundle and φ is a section of $E(\mathfrak{m}^{\mathbb{C}}) \otimes K$, called the *Higgs field*.

We now give some examples of G-Higgs bundles (E, φ). Whenever $H^{\mathbb{C}}$ acts in $\mathbb{C}^n$ in a standard way, we take the corresponding vector bundle associated to E.

Examples 1.—

1. If G is compact, a G-Higgs bundle is simply a (holomorphic) $G^{\mathbb{C}}$-bundle, hence $\varphi \equiv 0$.
2. If G is complex with maximal compact H, then $H^{\mathbb{C}} = G$ and also $\mathfrak{m}^{\mathbb{C}} = \mathfrak{g}$. Thus a G-Higgs bundle is a pair (E, φ) where E is a G-bundle and $\varphi \in H^0(X, E(\mathfrak{g}) \otimes K)$ where $E(\mathfrak{g})$ denotes the adjoint bundle of E, obtained from E under the adjoint action $\mathrm{Ad} : G \to \mathrm{GL}(\mathfrak{g})$. As an example, an $\mathrm{SL}(n, \mathrm{C})$-Higgs bundle is a pair (V, φ) with V a holomorphic rank n vector bundle with trivial determinant and $\varphi \in H^0(X, \mathrm{End}_0(V) \otimes K)$, where $\mathrm{End}_0(V)$ denotes the bundle of traceless endomorphisms of V; so we can think of φ as a map $\varphi : V \to V \otimes K$ (linear on each fibre) such that $\mathrm{tr}(\varphi) \equiv 0$. These are the "original" Higgs bundles, introduced by Nigel Hitchin in the seminal paper [17].
3. Let $G = \mathrm{Sp}(2n, \mathbb{R})$ — the group of automorphisms of $\mathbb{R}^{2n}$ preserving a symplectic form. An $\mathrm{Sp}(2n, \mathbb{R})$-Higgs bundle is a triple (V, β, γ) where V is a holomorphic rank n vector bundle, and the Higgs field splits as $\varphi = (\beta, \gamma)$, with $\beta : V^* \to V \otimes K$ such that $\beta^t \otimes \mathrm{Id}_K = \beta$ and likewise for $\gamma : V \to V^* \otimes K$.

There is a natural notion of isomorphism between two G-Higgs bundles over X. Further, these being complex algebraic objects, one can construct their *moduli spaces*; cf. [23]. Roughly speaking, these moduli spaces are algebraic varieties whose points parametrize isomorphism classes of G-Higgs bundles. Yet, in order to have a nice algebraic structure on these moduli, we cannot consider all G-Higgs bundles, but only the ones which are called *polystable*. Since this point is quite technical, we will not even define the meaning of this word. Just to give an example, a holomorphic vector bundle is polystable if it can be written as a direct sum of vector bundles (with certain conditions on their degrees) which are indecomposable, meaning that they cannot be further decomposed as direct sums. A polystable G-Higgs bundle behaves in a way which generalises the example of vector bundles. We can see here a certain parallelism between the notion of polystability and the notion of reductivity of a representation presented before Definition 1. Anyway, the reader just has to keep in mind that polystable G-Higgs bundles are the objects we have to consider in order to construct a moduli space of G-Higgs bundles on X, having the structure of an algebraic variety.

DEFINITION 3.— If G is a semisimple Lie group, the *moduli space of G-Higgs bundles* over the compact Riemann surface X is the variety whose points are given by isomorphism classes of polystable G-Higgs bundles over X. We denote it by $\mathcal{M}(G)$.

REMARK 1.— $\mathcal{M}(G)$ is a finite dimensional complex algebraic variety. It can be defined more generally for real reductive Lie groups. Although we are focusing our attention on semisimple Lie groups, all this theory generalises to real reductive Lie groups, with some slight modifications. In particular, when G is complex reductive, the complex dimension of $\mathcal{M}(G)$ is

$$\dim_{\mathbb{C}}(\mathcal{M}(G)) = (2g-2)\dim_{\mathbb{C}}(G) + 2\dim_{\mathbb{C}}(Z(G)), \quad (3)$$

where $Z(G)$ is the centre of G. When G is semisimple then $\dim_{\mathbb{C}}(Z(G)) = 0$.

As in the case of representations, we can define discrete invariants of (isomorphism classes of) G-Higgs bundles, which distinguish them in the C^∞ category, but not in the holomorphic one. Given a G-Higgs bundle (E, φ), we associate to it the topological invariant of the underlying $H^{\mathbb{C}}$-bundle E. This is well-known [21] to be given by an element

$$c(E) \in \pi_1(H^{\mathbb{C}}) = \pi_1(H) = \pi_1(G).$$

For an element $c \in \pi_1(G)$, let $\mathcal{M}_c(G)$ be the subspace of $\mathcal{M}(G)$ such that the corresponding G-Higgs bundles have topological invariant given by c. Again, we have a disjoint union $\mathcal{M}(G) = \bigsqcup_{c\in\pi_1(G)} \mathcal{M}_c(G)$, and each $\mathcal{M}_c(G)$ is a union of connected components.

2.3 The correspondence

Although apparently unrelated, the spaces $\mathcal{R}(G)$ and $\mathcal{M}(G)$ are tightly connected, by the following fundamental theorem.

THEOREM 4 ([17, 24, 8).—] If G is semisimple, then there is a natural correspondence between $\mathcal{M}_c(G)$ and $\mathcal{R}_c(G)$, which induces a homeomorphism $\mathcal{M}_c(G) \cong \mathcal{R}_c(G)$, for any $c \in \pi_1(G)$. This correspondence comes from the fact that a G-Higgs bundle over X is polystable if and only if it corresponds to a reductive representation of $\pi_1(\Sigma_g)$ in G.

REMARKS 1.—

1. This theorem is known as the *non-abelian Hodge correspondence*, since it generalises usual Hodge theory obtained when $G = \mathbb{C}^*$. In fact, the moduli space of $\mathbb{C}^*$-Higgs bundles (with fixed topological type $d \in \mathbb{Z}$) is isomorphic to the cotangent bundle to the Jacobian variety $\mathrm{Jac}(X)$ of X. This cotangent bundle is trivial, so the moduli space is $\mathrm{Jac}(X) \times \mathbb{C}^g$. The Jacobian variety of a compact Riemann surface is one of the most classical objects in algebraic geometry [15]. Topologically, it is a $2g$-dimensional real torus $(S^1)^{2g}$, hence
$$\mathcal{M}_d(\mathbb{C}^*) \cong \mathrm{Jac}(X) \times \mathbb{R}^{2g} \cong (S^1 \times \mathbb{R})^{2g} \cong (\mathbb{C}^*)^{2g}.$$
In particular, $\dim_{\mathbb{C}}(\mathcal{M}(\mathbb{C}^*)) = 2g$ as in formula 3. On the other hand, since $\mathbb{C}^*$ is abelian, $\mathcal{R}_d(\mathbb{C}^*) = (\mathbb{C}^*)^{2g}$, so we see here explicitly an homeomorphism $\mathcal{M}_d(\mathbb{C}^*) \cong \mathcal{R}_d(\mathbb{C}^*)$. Note that $\mathbb{C}^*$ is reductive but not semisimple. However, although Theorem 4 is stated for semisimple groups, there is a similar result which holds, more generally, for reductive groups.
2. Theorem 4 generalises the Narasimhan and Seshadri Theorem [19], which implies that $\mathcal{R}_0(\mathrm{SU}(n))$ is homeomorphic to $\mathcal{M}_0(\mathrm{SU}(n))$. This theorem was then generalised in [21] for any compact connected group. In these cases Higgs bundles are not really into the picture, because the groups in question are compact.
3. Recall also that in order to define Higgs bundles, we had to consider a Riemann surface structure X on Σ_g. The structure of $\mathcal{M}_c(G)$ as an algebraic variety depends on this choice, but Theorem 4 shows that its topological structure is independent of it.
4. Although the spaces are homeomorphic, their geometric structures tend to be very different. For example $\mathcal{M}_c(G)$ has always the complex structure coming from the one of X whereas, if G is real, $\mathcal{R}_c(G)$ is not a complex variety.

2.4 The Hitchin equations and their relation with Higgs bundles and representations

There is a third space $\mathcal{H}_c(G)$, homeomorphic to $\mathcal{M}_c(G)$, very important on its own and also fundamental in the proof of the theorem. This space $\mathcal{H}_c(G)$ is the space of equivalence classes of solutions to the so-called *Hitchin equations*. These are partial differential equations on the infinite-dimensional space of connections on a fixed C^∞ vector (or principal) bundle, coming from the Yang-Mills equations [1, 17]. We roughly explain it in a few lines, referring to [15, 25] for basic definitions of differential geometry, such as connection or curvature.

Given a G-Higgs bundle (E, φ), denote the C^∞-objects underlying E and φ by the same symbols. Then the Higgs field may be viewed as a $(1,0)$-form on X with values in $E(\mathfrak{m}^{\mathbb{C}})$: $\varphi \in \Omega^{1,0}(X, E(\mathfrak{m}^{\mathbb{C}}))$. Let $H \subseteq G$ be a maximal compact subgroup, so that its Lie algebra $\mathfrak{h}$ is a compact form of $\mathfrak{g}$. Given a C^∞ reduction of structure group h of E to H (thus h is a metric in E), let F_h be the curvature of the unique H-connection compatible with the holomorphic structure on E. Let also τ_h be the involution on $\Omega^{1,0}(X, E(\mathfrak{m}^{\mathbb{C}}))$ given by the compact conjugation on $\mathfrak{g}^{\mathbb{C}}$ (which determines its compact form), and which is given fibrewise by the metric h. It is a fundamental result that (E, φ) is polystable if and only if there is a metric h of E that satisfies the *Hitchin equation*

$$F_h - [\varphi, \tau_h(\varphi)] = 0. \tag{4}$$

This so-called *Hitchin-Kobayshi correspondence* yields a homeomorphism $\mathcal{H}_c(G) \cong \mathcal{M}_c(G)$, first proved for $G = \mathrm{SL}(2, \mathbb{C})$ by Hitchin in [17]. A proof in full generality can be found in [8]. The Hitchin-Kobayshi correspondence comprises half of the proof of Theorem 4. The other half, also done in [17], yields a homeomorphism $\mathcal{R}_c(G) \cong \mathcal{H}_c(G)$ and relies on theorems of Donaldson and Corlette.

The proof of Theorem 4 involves deep existence results of solutions of partial differential equations on manifolds. Hence it is not at all clear which polystable G-Higgs bundle corresponds to a given reductive representation ρ and vice-versa. It would be very interesting to find a way to see explicitly the correspondence of Theorem 4.

Remark 2.— A natural question is to ask if the terminology *Higgs bundles* somehow reveals some relation with the Higgs boson or with the Higgs field in the standard model of particle physics (the name *Higgs* is in both cases after the theoretical physicist Peter Higgs). The author's lack of competence to answer this question in a satisfactory way, is solved by referring to Remark 7.1 in [26], where some indications are provided by E. Witten, using the Hitchin equations (4).

3 Their topology

The spaces $\mathcal{R}_c(G)$ and $\mathcal{M}_c(G)$ are hence topologically equal. In the next sections we give some ideas on how to study their topology. We do it on the side of $\mathcal{M}_c(G)$, since there we have the powerful tools of complex algebraic geometry at our disposal.

3.1 The Hitchin functional

It is known that the moduli space of G-Higgs bundles $\mathcal{M}_c(G)$ ought to have an extremely rich topology. However, its Poincaré polynomial — which encodes the dimensions of the compactly supported cohomology groups of $\mathcal{M}_c(G)$ — has been computed only for some low rank cases for the group $\mathrm{SL}(n, \mathbb{C})$ and with topological type $d \in \mathbb{Z}$ coprime with n, so that the moduli $\mathcal{M}_d(\mathrm{SL}(n, \mathbb{C}))$ is smooth; cf. [17, 13, 19]. The key tool is the following real functional, which we define here for linear groups:

Definition 5.— The *Hitchin functional* on $\mathcal{M}_c(G)$ is the real function $f : \mathcal{M}_c(G) \to \mathbb{R}$ defined as

$$f(E, \varphi) = \|\varphi\|^2_{L^2} = \int_X \mathrm{tr}(\varphi\varphi^*)\omega,$$

where φ^* is the adjoint of φ with respect to h (the metric that solves the Hitchin equations (4)) and ω is the volume form on X.

3.2 The smooth case and Morse theory

The functional f is proper [17]. In fact, in the few cases where $\mathcal{M}_c(G)$ is smooth, f is a perfect Morse-Bott function, so the Poincaré polynomial can, in theory, be computed using Morse theory and by studying the topology of the critical subvarieties of f. The identification of these subvarieties uses the crucial fact that the moduli spaces $\mathcal{M}_c(G)$ carry a non-trivial $\mathbb{C}^*$-action by multiplication on the Higgs field,

$$\lambda \cdot (E, \varphi) = (E, \lambda\varphi). \tag{5}$$

The critical subvarieties of f coincide with the subvarieties of fixed points under this $\mathbb{C}^*$-action. The problem is that these subvarieties also have in general a very intricate topology, whose complete study is quite difficult. This is the basic reason why only a few low rank cases have been successfully addressed, even in the smooth case.

On the other hand, recent developments [10, 9, 22] were achieved on the study of $\mathcal{M}_c(\mathrm{SL}(n, \mathbb{C}))$, which seem to confirm some fascinating conjectures [16].

3.3 Connected components

In general, however, $\mathcal{M}_c(G)$ are singular spaces so the Morse theory picture breaks down. The topology of $\mathcal{M}_c(G)$ is hence basically unknown, with the honourable exception of the most basic topological invariant: the number of connected components.

Actually, the properness of f is enough to draw conclusions on the components of these moduli spaces. Since f is bounded below and proper, it attains a minimum on every component. The number of components of $\mathcal{M}_c(G)$ is thus bounded above by the number of components of the subvarieties $\mathcal{N}_c(G) \subset \mathcal{M}_c(G)$ of local minima of f.

The idea pioneered by Hitchin in [17, 18] to study the connected components of $\mathcal{M}_c(G)$ is to identify the minimum subvarieties $\mathcal{N}_c(G)$, among the fixed points of the $\mathbb{C}^*$-action (5), study the components of $\mathcal{N}_c(G)$ and then see what we can conclude about the components of $\mathcal{M}_c(G)$. This procedure has been extensively studied for many families of real semisimple Lie groups G [14, 3, 4, 20, 7, 11, 5, 12]. In the next section we describe some results in this direction.

4 Components and real forms

4.1 Compact Lie groups

When the group G is compact, we are really in the world of (holomorphic) $G^{\mathbb{C}}$-bundles. Then it is known for a long time that $\mathcal{M}_c(G)$ is non-empty and connected for any $c \in \pi_1(G)$ (see [21]).

4.2 Complex Lie groups

The same conclusion remains true for complex Lie groups. The most general form of the following connectedness theorem has been proved recently in [12].

Theorem 6 ([12).—] Let G be a complex reductive connected Lie group. Then $\mathcal{M}_c(G)$ is non-empty and connected for every $c \in \pi_1(G)$.

This theorem is even valid for non-connected groups. There we proved that the only local minima of f are the Higgs bundles with $\varphi \equiv 0$. Hence the subvarieties of local minima are moduli spaces of H-Higgs bundles, where $H \subset G$ is a maximal compact subgroup. Then Theorem 6 follows from the previous subsection.

These techniques generalise to the real case but become much more complicated. Indeed, if the group is real, then this story is completely different, as we will see in the remaining part of the article.

4.3 Split real forms

Assume that G is a *split real form* of $G^{\mathbb{C}}$. Roughly, this means that G is the *maximally non-compact* real form of $G^{\mathbb{C}}$ (cf. [6]). Examples are $G = \mathrm{SL}(n, \mathbb{R})$ and $G = \mathrm{Sp}(2n, \mathbb{R})$.

Theorem 7 ([18).—] For G a split real form, there is at least one $c \in \pi_1(G)$ such that $\mathcal{M}_c(G)$ is disconnected and has a connected component of $\mathcal{M}_c(G)$ isomorphic to a vector space.

The contractible component of $\mathcal{M}_c(G)$ mentioned in the theorem is called the *Hitchin component*. It carries relevant information about geometric structures on the surface X itself. For example, it is isomorphic to Teichmüller space when $G = \mathrm{SL}(2, \mathbb{R})$ [18]. Theorems 7 and 4 prove the existence of a Hitchin component in $\mathcal{R}_c(G)$, being an interesting question to characterise the representations in it.

4.4 Hermitian type groups

A different kind of phenomena occurs if G is a real non-compact group of *hermitian type*. One possible definition of such groups is that they are characterised by the fact that the centre of their maximal compact subgroup contains a circle. For instance $\mathrm{Sp}(2n, \mathbb{R})$ is a group of hermitian type since a maximal compact is $\mathrm{U}(n)$, whose centre is $\mathrm{U}(1)$. The symplectic group $\mathrm{Sp}(2n, \mathbb{R})$ is especially interesting because is the only one, up to finite covering, that is simultaneously split and hermitian.

In this hermitian case, there is a new phenomena concerning the non-emptiness of $\mathcal{M}_c(G)$. Indeed, the free part of $\pi_1(G)$ is isomorphic to $\mathbb{Z}$, giving rise to an integer d. There is a bound for $|d|$, called the *Milnor-Wood inequality*, above which $\mathcal{M}_d(G)$ is empty (see [4, 2]). Moreover, for some groups of hermitian type, $\mathcal{M}_d(G)$ is disconnected when $|d|$ is maximal. This is the case of $G = \mathrm{Sp}(2n, \mathbb{R})$, studied by García-Prada, Gothen and Mundet i Riera in [7]:

Theorem 8 ([7).—]The moduli space $\mathcal{M}_d(\mathrm{Sp}(2n, \mathbb{R}))$ is non-empty if and only if $|d| \leq n(g-1)$. Moreover, if $n \geq 3$, $\mathcal{M}_{n(g-1)}(\mathrm{Sp}(2n, \mathbb{R}))$ has 3×2^{2g} non-empty connected components.

Recall from subsection 2.2 that $\mathrm{Sp}(2n, \mathbb{R})$-Higgs bundles are given by a triple (V, β, γ). The distinctive feature of the case $d = n(g-1)$ is that $\gamma : V \to V^* \otimes K$ is an isomorphism precisely for that value of d (the case $d = -n(g-1)$ is similar but it is β that becomes an isomorphism). This uncovers 2×2^{2g} *hidden* components. A further analysis, using the Hitchin functional as before, proves the existence of the remaining 2^{2g} components, which are the Hitchin ones mentioned in the preceding subsection (recall that $\mathrm{Sp}(2n, \mathbb{R})$ is also split).

In general further difficulties arise for the study of components for non-maximal and non-zero d. In the known cases, the non-maximal subspaces are connected for each fixed topological type. We expect that the same holds true in general, but a potential proof of this conjecture seems out of reach at the moment.

4.5 Other real forms

The components of $\mathcal{M}_c(G)$, hence also of $\mathcal{R}_c(G)$, have been studied for many families of groups, not necessarily split or hermitian; see for instance [11]. Until recently, no examples were known of real groups, neither split nor hermitian, for which $\mathcal{M}_c(G)$ is disconnected. However, by considering the group $\mathrm{SO}_0(p, q)$ — the identity component of the group of automorphisms of $\mathbb{R}^{p+q}$ preserving an orthogonal structure with signature (p,q) — we recently realised that there exist many different local minima of f. This panoply of local minima may potentially give rise to new components of $\mathcal{M}_c(\mathrm{SO}_0(p,q))$ whose geometric structure differs from all the known cases. This is still work in progress. Again, it would be very interesting to characterise the representations lying in these new components of $\mathcal{M}_c(\mathrm{SO}_0(p,q))$.

Acknowledgments

Author partially supported by CMUP (UID/MAT/00144/2013), by the Project EXCL/MAT-GEO/0222/2012 and by Post-Doctoral fellowship SFRH/BPD/100996/2014, which are funded by FCT (Portugal) with national (MEC) and European structural funds (FEDER), under the partnership agreement PT2020.

The author thanks Carlos Florentino for helpful comments on a previous version of this article.

References

[1] M. F. Atiyah, R. Bott, The Yang-Mills equations over Riemann surfaces, *Philos. Trans. Roy. Soc. London* Ser. A 308 (1982), 523–615.

[2] O. Biquard, O. García-Prada, R. Rubio, Higgs bundles, Toledo invariant and the Cayley correspondence, Preprint arXiv:1511.07751.

[3] S. B. Bradlow, O. García-Prada, P. B. Gothen, Surface group representations and U(p,q)-Higgs bundles, *J. Diff. Geom.* 64 (2003), 111–170.

[4] S. B. Bradlow, O. García-Prada, P. B. Gothen, Maximal surface group representations in isometry groups of classical Hermitian symmetric spaces, *Geom. Dedicata* 122 (2006), 185–213.

[5] S. B. Bradlow, O. García-Prada, P. B. Gothen, Higgs bundles for the non-compact dual of the special orthogonal group, *Geom. Dedicata* 175 (2015), 1–48.

[6] W. Fulton, J. Harris, *Representation Theory — A First Course*, Graduate Texts in Mathematics 129, Springer-Verlag, 2004.

[7] O. García-Prada, P. B. Gothen, I. Mundet i Riera, Higgs bundles and surface group representations in the real symplectic group, *J. Topology* 6 (2013), 64–118.

[8] O. García-Prada, P. B. Gothen, I. Mundet i Riera, The Hitchin-Kobayashi correspondence, Higgs pairs and surface group representations, Preprint arXiv:0909.4487v3.

[9] O. García-Prada, J. Heinloth, The y-genus of the moduli space of PGL$_n$-Higgs bundles on a curve (for degree coprime to n), *Duke Math. J.*, 162 (2013), 2731–2749.

[10] O. García-Prada, J. Heinloth, A. Schmitt, On the motives of moduli of chains and Higgs bundles, *J. Eur. Math. Soc.*, 16 (2014), 2617–2668.

[11] O. García-Prada, A. Oliveira, Connectedness of the moduli of Sp$(2p,2q)$-Higgs bundles, *Quart. J. Math.*, 65 (2014), 931–956.

[12] O. García-Prada, A. Oliveira, Connectedness of Higgs bundle moduli for complex reductive Lie groups, *Asian J. Math.*, to appear. Preprint arXiv:1408.4778.

[13] P. B. Gothen, The Betti numbers of the moduli space of stable rank 3 Higgs bundles on a Riemann surface, *Int. J. Math.* 5 (1994), 861–875.

[14] P. B. Gothen, Components of spaces of representations and stable triples, *Topology* 40 (2001), 823–850.

[15] P. Griffiths, J. Harris, *Principles of Algebraic Geometry*, John Wiley & Sons, 1978.

[16] T. Hausel, F. Rodriguez-Villegas, Mixed Hodge polynomials of character varieties. With an appendix by Nicholas M. Katz, *Invent. Math.* 174 (2008), 555–624.

[17] N. J. Hitchin, The self-duality equations on a Riemann surface, *Proc. London Math. Soc.* (3) 55 (1987), 59–126.

[18] N. J. Hitchin, Lie groups and Teichmüller space, *Topology* 31 (1992), 449–473.

[19] M. S. Narasimhan, C. S. Seshadri, Stable and unitary vector bundles on a compact Riemann surface, *Ann. Math.* 82 (1965), 540–567.

[20] A. G. Oliveira, Representations of surface groups in the projective general linear group, *Int. J. Math.* 22 (2011), 223–279.

[21] A. Ramanathan, Stable principal bundles on a compact Riemann surface, *Math. Ann.* 213 (1975), 129–152.

[22] O. Schiffmann, Indecomposable vector bundles and stable Higgs bundles over smooth projective curves, *Ann. Math.* 183 (2016), 297–362.

[23] A. H. W. Schmitt, *Geometric Invariant Theory and Decorated Principal Bundles*, Zurich Lectures in Advanced Mathematics, European Mathematical Society, 2008.

[24] C. T. Simpson, Higgs bundles and local systems, *Inst. Hautes Études Sci. Publ. Math.* 75 (1992), 5–95.

[25] R. O. Wells, *Differential Analysis on Complex Manifolds*, Graduate Texts in Mathematics 65, Springer-Verlag, 2008.

[26] E. Witten, Mirror Symmetry, Hitchin's Equations, and Langlands Duality, 113–128, in *The many facets of geometry. A tribute to Nigel Hitchin*, Ed. O. García-Prada, J. P. Bourguignon, S. Salamon. Oxford University Press, 2010.

The 109th European Study Group with Industry

by Rui Pereira*, Senhorinha Teixeira and A. Ismael F. Vaz****

The 109th European Study Group with Industry (ESGI) took place from May 11th to May 15th, 2015, at the *Department of Production and Systems* of the *School of Engineering*, *University of Minho*. Among others, the event counted with the scientific support from the ALGORITMI (www.algoritmi.uminho.pt) and CMAT (www.cmat.uminho.pt) research centers. These ESGI meetings were created with the objective of renovating and reinforcing the links between Mathematics and Industry. More information on the study groups and related aspects is available at the International Study Groups website [2] and the European Consortium for Mathematics in Industry [1].

This particular meeting is part of the series of European Study Groups, where industries are requested to pose mathematical challenges to a set of experienced mathematical researchers. These researchers dedicate a full working week in providing a solution, or avenues to get a solution, to the posed challenges. We counted with the participation of several national experts, with a large experience in this type of events, to address six mathematical challenges submitted by local companies, but with a national and international impact, operating in the portuguese market and overseas.

* Department of Mathematics and Applications, School of Science, University of Minho
** Department of Production and Systems, School of Engineering, University of Minho

The posed challenges were: *Modelling and optimization of production scheduling*, where a textile company posed the challenge to model the full production operations; *Physical model of MDF boards*, where the challenge to provide a physical model of MDF boards was proposed; *Setting the Reserve Fleet*, where a public transportation company challenged the group to provide an optimal vehicles reserve fleet; *Surgical cases packages*, where an optimal set of surgical cases packages for use in surgical wards was to be obtained; *Prediction model to textile parameters*, where the combination of yarns and yarns types where to be obtained in order to get a textile with given properties; and *Optimization of a shoes injection moulding machine*, where the scheduling of an injection moulding machine was addressed.

To deal with such a wide set of challenges, mathematical subjects such as probability and statistics, operational research, optimization, differential equations, and finite element numerical methods were used.

A detailed report with the group achievements was delivered to the corresponding companies, where a solution for the challenge and/or avenues for future collaborations were proposed.

References

[1] ECMI European Consortium for Mathematics in Industry. Information service. http://www.ecmi-indmath.org.

[2] Mathematics in Industry. Information service. http://www.maths-in-industry.org.

Applications of Mathematics in Fluid Dynamics

by Marco Martins Afonso*

1 General considerations

Fluid dynamics [1, 2, 3] represents one of the very few fields where, in the framework of classical physics, a full comprehension of the problem is still far from being achieved, and therefore constitutes a vast subject for ongoing and future research. Even if relativistic [4] and quantum [5] hydrodynamics have their own importance, the laws of classical physics and of continuum mechanics are implemented in almost the entirety of hydrodynamic investigations.

The analytical approach faces the obstacles of nonlinearity and non-locality of the problem, and typically of non-ideality of the initial and boundary conditions or of the forcing terms. The computational approach is nowadays very common, but numerical simulations of fluid flows usually have to deal with the very huge number of active—and non-trivially interacting—degrees of freedom to be described. Experiments can only reproduce part of the interesting problems, and heavily rely on the (Bertrand-Vaschy-Buckingham) π theorem [6, 7, 8] for the appropriate geometric scaling and the introduction of nondimensional numbers. Among these latter, the best renowned is the one associated with Osborne Reynolds' famous experiment [9]:

$$\text{Reynolds number} = \mathrm{Re} \equiv \frac{LU}{\nu}.$$

Here, L and U are characteristic length and speed scales of the flow under consideration, and the kinematic viscosity is defined as the ratio between dynamic viscosity and mass density: $\nu \equiv \mu/\rho$.

Two main descriptions of the analytical problem are possible [10]. One is Lagrangian: a small region of fluid (*particle* or *parcel*) is ideally identified, isolated and followed along its evolution. All its properties are thus represented by physical quantities which are only functions of time, and obey ordinary differential equations. The other is Eulerian, where partial differential equations are derived for fields depending on space and time. The two descriptions are complementary and both relevant, and related by the fundamental statement that the velocity of a fluid particle equals by definition the local and instantaneous velocity field:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{u}(\boldsymbol{x}(t), t). \tag{1}$$

The equation for the fluid velocity is due to Claude-Louis-Marie-Henri Navier [11] and George Gabriel Stokes [12], and in its incompressible form—when free from thermal effects—reads:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \boldsymbol{u}, \tag{2}$$

endowed with appropriate initial and (Dirichlet/Neumann/Robin) boundary conditions, and possibly modified with the appearance of a forcing term on the right-hand side.

* Centro de Matemática da Universidade do Porto

Here $p(\boldsymbol{x}, t)$ is the pressure field, and is the source of non-locality of the problem. Indeed, the continuity equation for incompressible flows reads as a solenoidality property, $\nabla \cdot \boldsymbol{u} = 0$, and pressure is required to satisfy a Poisson equation. Therefore, even if (2) is in principle evaluated locally at one single point, actually it contains a term which represents a contribution coming from a spatial integral on the whole domain, as the propagation velocity of any disturbance is infinite. Despite this difficulty, incompressibity is a scheme widely used for the simplifications it brings about, and is usually abandoned only when compressibility effects cannot be neglected, most notably because the velocities into play are not negligible with respect to the sound speed [13]. In this latter case, the mass density varies. Also thermal effects can come into play and modify the parameters, in which case also the evolution of the temperature field must be taken into account, along with a suitable equation of state. Even more problematically, also viscosity can be different from a constant, and then the fluid under consideration is dubbed as non-Newtonian and described by a different equation.

Apart from the incompressible scheme, common simplifications in the resolution process hold if the flow is parallel (only pointing in one same direction always and everywhere), or plane (independent of at least one direction), or potential (i.e. irrotational: $\nabla \times \boldsymbol{u} = \boldsymbol{0}$). Other intrinsic properties of (2) are its non-linearity and its time irreversibility (with energy dissipation), due to the second term on the left and on the right-hand sides respectively. Three main analogues of the Navier-Stokes equation are worth mentioning: the Burgers one [14], where the pressure term is dropped and which gives rise to compressibility shocks; the Euler one [15], where the viscous contribution is neglected leading to finite-time singularities, and which is often an excellent approximation of the problem in the bulk of the fluid but requires matching asymptotic techniques developed by Ludwig Prandtl [16] to be extended to boundary layers near walls; and the Stokes one, where the left-hand side of (2) is negligible and which describes creeping flows. Dropping only one term on left-hand side is also common: the first, when one looks for steady solutions of the full equation; the second, when linearizing around a mean flow with small-amplitude fluctuations.

When Re grows past a certain threshold, the flow loses its laminar character, undergoes a series of transitions and becomes fully turbulent [17, 18, 19, 20, 21]. Fluid turbulence is a problem of paramount importance and difficulty (sometimes referred to as "the last mystery of classical physics"), as admitted by Richard Feynman and underlined by two famous historical quotes. One is from Peter Bradshaw [22]: "Turbulence was probably invented by the devil on the seventh day of creation, when the Good Lord wasn't looking". An older one, "I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic", is attributed to Horace Lamb, even if a very similar version—with *relativity* as the former matter—was reportedly pronounced by Werner Heisenberg.

It is absolutely astonishing to note the degree of uncertainty related to turbulent flows, for instance in comparison to the one typical of astronomy. Not unusual are calculations of the trajectory of a poorly-known asteroid for many decades to come, and safe conclusions that it will barely miss an impact with the Earth in more than one century, even if all we know about it comes from few possible measurements from such a far distance. On the other hand, in principle we can measure as much information as we want in our low atmosphere, but weather forecast is limited to very few days, with the practical impossibility of specifying the exact hour and location of some kind of precipitation or disruption.

Turbulent flows can simply be seen as general solutions of the Navier-Stokes equation (2) lacking any property of regularity characteristic of laminar flows. Turbulence is a phenomenon very far from equilibrium, with typically no small parameter in which to expand around a known state. Turbulent velocity fields greatly enhance mixing and dispersion, and are intermittent and usually ergodic. They are self-organized and made up of coherent structures, the so-called *eddies*, but they are chaotic [23]. Therefore—despite being far from random—they cannot be described deterministically, and a statistical approach is common [24, 25, 26, 27, 28], with the turbulent quantities (or often their deviations from the mean) considered as stochastic variables. This implies that also many tools of statistical mechanics are employed, along with several techniques borrowed from theoretical physics, such as renormalization-group formalism, diagrammatic representation, path-integral formulation, second-quantization algorithm, non-linear Schrödinger and Ginzburg-Landau equations [29, 30].

Without entering the details of this field, we leave the interested reader to the vast literature on the subject, and we only point out the enormous difference in phenomenology between two-dimensional and three-dimensional turbulence (see [31, 32] and bibliographical references therein).

The number of mathematical tools employed in the analytic investigation of fluid mechanics is immense [33, 34], which justifies the fact that this subject is often studied in centers of applied mathematics. They range from the most common ones—SO-d decomposition [35], Fourier/Laplace/Legendre transforms—to more sophisticated counterparts, among which we just mention a few here: functional analysis and Furutsu-Novikov-Donsker theorem [36, 37, 38], multifractals, multiplicative model, refined large-deviation theory, steepest-descent method, Lyapunov exponents and Cramér function, telegraph-noise model and degenerate perturbation theory [39], (time-ordered) matrix exponentials and Cayley-Hamilton theorem [40], Hermitianization [41], Ornstein-Uhlenbeck process [42], multiple-scale technique [43], variational formulation, adjoint method, continued fractions and Heun equation [44].

Of course, describing the velocity field is the principal objective in fluid mechanics. However, there are a lot of related problems which deserve the same attention and importance, also because they may appear easier for some aspects but harder for others. First of all, one should mention transport phenomena.

From a Lagrangian perspective, one can think of replacing a fluid particle with a tracer one, i.e., a particle which has the same exact properties of the fluid replaced (and therefore evolves in the same way and does not alter the effects on the surrounding fluid) but that simply acts as a marker and can be followed individually in its evolution. More complex cases arise when these inclusions are inertial particles—the subject of the next section—or polymers. These latter denote particles with an internal structure, which can be described by different models (e.g. Oldroyd-B, FENE-P, etc. [45, 46]) and produce non-trivial feedbacks on the carrier fluid, the most important of which is probably the drag reduction that can be achieved in oil ducts with efficiency improvement of even 80% by simply adding a few parts per million in mass.

From the Eulerian viewpoint, the transported quantity is a field. This latter can be a scalar or a vector, and may be passive or active depending on whether it has a feedback on the advecting velocity—by appearing as a source term on the right-hand side of (2) and thus fully coupling the system. Passive-scalar advection [47], e.g., for the concentration field of a tracer, is a paradigmatic case because, despite the linearity and locality of the problem, many aspects are reminiscent of the kinetic-energy cascade picture [48, 49, 50, 51, 52, 53, 54, 55]. In the realm of active-vector turbulence, magnetohydrodynamics still constitutes a formidable problem [56].

Large-Eddy Simulations (LES) consist in a computational resolution of these problems, different from the Reynolds-Averaged (RANS) and Direct (DNS) Numerical counterparts [57] in the sense that a coarse-graining procedure is implemented [58, 59], whose analytical foundations are still being studied. The basic difficulty relies on a closure problem, due to the fact that any filtering operation aimed at focusing on the sole large scales does not commute with the non-linear multiplicative term in (2), thus requiring a suitable parameterization of the small scales which cannot just be neglected [60, 61].

Back to the investigation of the velocity field itself, the different types of instabilities [62] represent a huge research theme. Among them, we can mention those named as Rayleigh-Bénard (fluid cooled from above and heated from below), Taylor-Couette (fluid between two counter-rotating cylinders), Kelvin-Helmholtz (fluid with internal layers moving in relative shear) and Von Kármán-Strouhal (fluid in the wake of an obstacle with vortex street).

Finally, control theory [63] plays a crucial role. By means of studies of structural sensitivity, the aim is to identify which changes in the boundary conditions or in the forcing terms are the most suitable to obtain some desirable or desired result (such as the reduction of the aerodynamic drag on cars or of acoustic noise on airplanes), in the sense that they maximize the kinetic-energy gain or engage/delay some transition.

1.1 Inertial particles

Particles that have a different mass density (σ) with respect to the surrounding fluid, and whose size—let us say radius R in the range μm ÷ mm—is small but not tiny, cannot simply be described as point tracers and have a finite relative inertia. Their trajectory thus deviates from the underlying fluid one, which can lead to preferential concentration and even clustering. Common examples are droplets in gases, bubbles in liquids and aerosols in fluids. Let us consider the simplest realistic model, where the particles are spherical and isolated, or belonging to a very dilute suspension, in order to neglect any interaction with boundaries and other particles, and to take into account their feedback on the flow in an effective simplified fashion.

Equation (1) now becomes a second-order dynamical system of the Langevin type (Itô or Stratonovich) for the particle position $\boldsymbol{x}(t)$ and velocity $\boldsymbol{v}(t)$, because Newton's law can be recast as [64, 65]:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{v}(t) \\ \dot{\boldsymbol{v}}(t) = \beta \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{u}(\boldsymbol{x}(t), t) - \dfrac{\boldsymbol{v}(t) - \boldsymbol{u}(\boldsymbol{x}(t), t)}{\tau} \\ \qquad\quad + \dfrac{\sqrt{2\kappa}}{\tau}\boldsymbol{\eta}(t) + (1 - \beta)\boldsymbol{g} \ . \end{cases} \tag{3}$$

The four terms on the right-hand side of the equation for the acceleration represent the four basic components of the force acting on the particle. The first is proportional to the non-dimensional coefficient

$$\beta \equiv \frac{3\rho}{\rho + 2\sigma} \in [0, 3] \tag{4}$$

and expresses the added-mass effect, i.e. the fact that any motion of the particle implies a movement of fluid around it: this contribution vanishes for very heavy particles ($\sigma \gg$

$\rho \Rightarrow \beta \simeq 0$) and is maximum for very light ones ($\sigma \ll \rho \Rightarrow \beta \simeq 3$), because there all inertia lies in the particle or in the fluid, respectively; for tracers ($\sigma = \rho \Rightarrow \beta = 1$) of course this acceleration is the same as if a fluid particle were there.[1] The second is the linear viscous drag for small relative slip velocity, it means that the particle relaxes towards the local and instantaneous flow configuration with a typical response time $\tau \equiv R^2/3\nu\beta$ ($\to 0$ for tracers and $\to \infty$ for ballistic objects), from which the Stokes number—a measure of the inertia-driven delay—can be constructed:

$$\mathrm{St} \equiv \frac{\tau}{L/U}. \tag{5}$$

The third takes thermal noise into account via the Brownian diffusivity κ, coupled through the standard vectorial white noise $\boldsymbol{\eta}(t)$, and gives rise to the Péclet number:

$$\mathrm{Pe} \equiv \frac{LU}{\kappa}. \tag{6}$$

The fourth represents buoyancy, parallel to gravity acceleration $\boldsymbol{g}$ for heavy particles ($\beta < 1$) and anti-parallel for light ones ($\beta > 1$), and leads to the definition of the Froude number:

$$\mathrm{Fr} \equiv \frac{U}{\sqrt{Lg}}. \tag{7}$$

Some corrective terms have been neglected in (3), namely those due to Basset–Boussinesq (time integration for memory effects), Faxén (spatial expansion for finite particle size), Oseen (non-linearity for finite relative slip velocity) and Saffman (side lift in case of rotation).

From (3), one derives the generalized Fokker-Planck equation for the phase-space density $p(\boldsymbol{x}, \boldsymbol{v}, t)$, which serves as a basis to compute the quantities of physical relevance. Typically, in the presence of a localized particle source—such as a chimney for pollutants in the atmosphere, or a syringe for powders in microchannels—one is interested in the temporal evolution of the physical-space concentration. If no source is present, the most important quantities are the particle transport properties, such as: the average terminal velocity (or more precisely its deviation with respect to the asymptotic bare value in still fluids) describing how even a zero-mean flow can modify the sedimentation process and Stokes' drift; and the effective *eddy* diffusivity, whose value also tells one whether, in the frame of reference moving according to the ballistic component, the diffusion process is standard or anomalous. All this information can be obtained from p, either by simply integrating on the velocity degree-of-freedom, or by dealing with this latter in some appropriate way in order to obtain advection-diffusion-like equations known as auxiliary cell problem [66].

It is known that these phenomena critically depend on the interplay of several control quantities, i.e., the full details of both flow and particles contribute to establish whether, e.g., the activation of a fluid velocity field increases or decreases the settling of a suspended particle. Among the key factors is the list of non-dimensional numbers β (4), St (5), Pe (6) and Fr (7), to which we must add a few others. Most importantly, the compressibility degree (an analogue of the Mach number)

$$\mathscr{P} \equiv \frac{\langle(\nabla \cdot \boldsymbol{u})^2\rangle}{\langle||\nabla\boldsymbol{u}||^2\rangle} \in [0, 1]\,, \tag{8}$$

where the average can be on the space-time periodicity, or on the statistical ensemble for random flows. Then, the space dimension d, where only the two- and three- dimensional cases can be investigated if the flow is incompressible, but also $d = 1$ if $\mathscr{P} \neq 0$.

Also the geometric and temporal details of the flow are extremely relevant, and to fix the ideas let us focus on two examples. First, a laminar 2D incompressible flow with a cellular structure,

$$\boldsymbol{u} = U \begin{pmatrix} \sin(2\pi k x_1/L)\cos 2\pi[x_2/L + \sin(\omega t)] \\ -k\cos(2\pi k x_1/L)\sin 2\pi[x_2/L + \sin(\omega t)] \end{pmatrix},$$

where k is the vertical-to-horizontal aspect ratio, and ω is the angular frequency of synchronous vertical oscillation of the cells. Second, a zero-mean, stationary, homogeneous, isotropic, Gaussian random flow, with two-point correlation

$$\langle u_i(\boldsymbol{x}, t) u_j(\boldsymbol{0}, 0)\rangle = U^2 f_{ij}(\mathscr{P}) \mathrm{e}^{-x^2/2L^2} \mathrm{e}^{-t^2/2T^2} \cos(\omega t)$$

(the tensorial structure f_{ij} simply enforces the desired compressibility degree); here, T is the characteristic life time of turbulent vortices and ω is an angular frequency which takes into account the presence of recirculation— i.e., areas with negatively-correlated velocity—that lead to the definition of two additional non-dimensional numbers:

$$\text{Kubo number} = \mathrm{Ku} \equiv \frac{T}{L/U}, \tag{9}$$

$$\text{Strouhal number} = \mathrm{Sr} \equiv \frac{\omega}{2\pi U/L}\,. \tag{10}$$

It can be shown that also the parameters $\mathscr{P}$ (8), Ku (9), Sr (10), along with d and k, have a huge impact on the transport properties, not only directly, but indirectly too, by changing or even reversing the direct role of other parameters.

Acknowledgements

The author was partially supported by CMUP (UID/MAT/00144/2013), which is funded by FCT (Portugal) with national (MEC) and European structural funds (FEDER), under the partnership agreement PT2020.

[1] It is debated whether the material derivative $\mathrm{d}/\mathrm{d}t$ should be computed along the particle trajectory as $\partial/\partial t + \boldsymbol{v}(t)\cdot\nabla$ or along the corresponding fluid path as $\partial/\partial t + \boldsymbol{u}(\boldsymbol{x}(t), t)\cdot\nabla$

References

[1] Lamb, H. 1932 *Hydrodynamics*. Cambridge University Press.

[2] Batchelor, G.K. 1967 *An introduction to fluid dynamics*. Cambridge University Press.

[3] Kundu, P.K. & Cohen, L.M. 1990 *Fluid mechanics*. Academic Press.

[4] Lichnerowicz, A. 1967 *Relativistic hydrodynamics and magnetohydrodynamics*. Benjamin.

[5] Wyatt, R.E. 2005 *Quantum dynamics with trajectories: introduction to quantum hydrodynamics*. Springer.

[6] Bertrand, J. 1878 Sur l'homogénéité dans les formules de physique. *Comptes Rendus* 86 (15), 916–920.

[7] Vaschy, A. 1892 Sur les lois de similitude en physique. *Annales Télégraphiques* 19, 25–28.

[8] Buckingham, E. 1914 On physically similar systems: illustrations of the use of dimensional equations. *Physical Review* 4 (4), 345–376.

[9] Reynolds, O. 1883 An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels. *Philosophical Transactions of the Royal Society of London* 174, 935–982; *Proceedings of the Royal Society of London* 35, 84–99.

[10] Falkovich, G., Gawędzki, K. & Vergassola, M. 2001 Particles and fields in fluid turbulence. *Review of Modern Physics* 73, 913–975.

[11] Navier, C.L.M.H. 1823 Mémoire sur les lois du mouvement des fluides. *Mémoires de l'Académie Royale des Sciences de l'Institut de France* 6, 389–440.

[12] Stokes, G.G. 1843 On some cases of fluid motion. *Transactions of the Cambridge Philosophical Society* 8, 105–165.

[13] Acheson, D.J. 1990 *Elementary Fluid Dynamics*. Clarendon.

[14] Burgers, J.M. 1940 Application of a model system to illustrate some points of the statistical theory of free turbulence. *Proceedings of the Royal Academy of Sciences at Amsterdam* 43 (2).

[15] Euler, L. 1757 Principes généraux de l'état d'équilibre d'un fluide. *Académie Royale des Sciences et des Belles-Lettres de Berlin, Mémoires* 11, 217–273.

[16] Prandtl, L. 1952 *Essentials of fluid dynamics*. Hafner, Blackie.

[17] Tennekes, H. & Lumley, J.L. 1972 *A first course in turbulence*. MIT Press.

[18] Frisch, U. 1995 *Turbulence*. Cambridge University Press.

[19] Pope, S.B. 2000 *Turbulent flows*. Cambridge University Press.

[20] Lesieur, M. 2008 *Turbulence in fluids*. Springer.

[21] Tsinober, A. 2014 *The essence of turbulence as a physical phenomenon*. Springer.

[22] Bradshaw, P. 1994 Turbulence: the chief outstanding difficulty of our subject. *Experiments in Fluids* 16, 203–216.

[23] Benzi, R., Paladin, G., Parisi, G. & Vulpiani, A. 1984 On the multifractal nature of fully developed turbulence and chaotic systems. *Journal of Physics* A 17 (18), 3521–3531.

[24] Monin, A.S. & Yaglom, A.M. 1975 *Statistical fluid mechanics: mechanics of turbulence*. MIT Press.

[25] Gardiner, C.W. 1985 *Handbook of stochastic methods: for physics, chemistry and the natural sciences*. Springer.

[26] Risken, H. 1989 *The Fokker–Planck equation: methods of solutions and applications*. Springer.

[27] Klyatskin, V.I. 2005 *Dynamics of stochastic systems*. Elsevier.

[28] Van Kampen, N.G. 2007 *Stochastic processes in physics and chemistry*. Elsevier.

[29] Schönberg, M. 1952 Application of second quantization methods to the classical statistical mechanics. *Il Nuovo Cimento* 9 (12), 1139–1182.

[30] Doi, M. 1976 Second quantization representation for classical many-particle system. *Journal of Physics* A 9 (9), 1465-1477.

[31] Eyink, G.L. & Sreenivasan, K.R. 2006 Onsager and the theory of hydrodynamic turbulence. *Reviews of Modern Physics* 78 (1), 87-135.

[32] Boffetta, G. & Ecke, R.E. 2012 Two-dimensional turbulence. *Annual Review of Fluid Mechanics* 44, 427–451.

[33] Bensoussan, A., Lions, J.L. & Papanicolaou, G. 1978 *Asymptotic analysis of periodic structures*. North-Holland.

[34] Bender, C.M. & Orszag, S.A. 1978 *Advanced mathematical methods for scientists and engineers*. McGraw–Hill.

[35] Biferale, L. & Procaccia, I. 2005 Anisotropy in turbulent flows and in turbulent transport. *Physics Reports* 414 (2-3), 43–164.

[36] Furutsu, K. 1963 On the statistical theory of electromagnetic waves in a fluctuating medium. *Journal of Research of the National Bureau of Standards* D 67, 303–323.

[37] Novikov, E.A. 1965 Functionals and the random-force method in turbulence theory. *Soviet Journal of Experimental and Theoretical Physics* 20, 1290–1294.

[38] Donsker, M.D. 1964 On function space integrals. *Proceedings of Conference on the Theory and Applications of Analysis in Function Space* 2, 17–30, MIT Press.

[39] Falkovich, G. & Martins Afonso, M. 2007 Fluid-particle separation in a random flow described by the telegraph model. *Physical Review* E 76 (2), 026312:1–5.

[40] Martins Afonso, M. & Meneveau, C. 2010 Recent Fluid Deformation closure for velocity gradient tensor dynamics in turbulence: time-scale effects and expansions. *Physica* D 239 (14), 1241–1250.

[41] Martins Afonso, M., Celani, A., Mazzino, A. & Olla, P. 2009 Renormalized transport of inertial particles. *Advances in Turbulence* XII, Springer Proceedings in Physics 132, 505–508.

[42] Martins Afonso, M., Mazzino, A. & Muratore-Ginanneschi, P. 2011 Inertial-particle dispersion and diffusion. *Advances in Turbulence* XIII, *Journal of Physics: Conference Series* 318 (5), 052014:1–4.

[43] Boi, S., Martins Afonso, M. & Mazzino, A. 2015 Anomalous diffusion of inertial particles in random parallel flows: theory and numerics face to face. *Journal of Statistical Mechanics*, P10023:1–21.

[44] Martins Afonso, M. & Vincenzi, D. 2005 Nonlinear elastic polymers in random flow. *Journal of Fluid Mechanics* 540, 99–108.

[45] Peterlin, A. 1966 Hydrodynamics of macromolecules in a velocity field with longitudinal gradient. *Journal of Polymer Science* B*: Polymer Letters* 4 (4), 287–291.

[46] Bird, R.B., Curtiss, C., Armstrong, R. & Hassager, O. 1987 *Dynamics of polymeric liquids*. John Wiley & Sons.

[47] Celani, A., Martins Afonso, M. & Mazzino, A. 2007 Point-source scalar turbulence. *Journal of Fluid Mechanics* 583, 189–198; Mixing of a Passive Scalar Emitted from a Random-in-Time Point Source. *Advances in Turbulence* XI, Springer Proceedings in Physics 117, 206–208; Punctual emission of a passive scalar in turbulent flows. *Physics of Particles and Nuclei, Letters* (in press).

[48] Richardson, L.F. 1922 *Weather prediction by numerical process*. Cambridge University Press.

[49] Kolmogorov, A.N. 1941 The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *Doklady Akademii Nauk* SSSR 30, 299–303; On the degeneration of isotropic turbulence in an incompressible viscous fluid. *ibid*. 31, 538–541; Dissipation of energy in locally isotropic turbulence. *ibid*. 32, 16–18.

[50] Kolmogorov, A.N. 1962 A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number. *Journal of Fluid Mechanics* 13 (01), 82–85.

[51] Obukhov, A.M. 1949 The structure of the temperature field in a turbulent flow. *Izvestiya Akademii Nauk* SSSR*: Ser. Geogr. Geofiz*. 13, 58–69.

[52] Corrsin, S. 1951 On the spectrum of isotropic temperature fluctuations in isotropic turbulence. *Journal of Applied Physics* 22 (4), 469–473.

[53] Kraichnan, R.H. 1968 Small-scale structure of a scalar field convected by turbulence. *Physics of Fluids* 11 (5), 945–953.

[54] Kraichnan, R.H. 1994 Anomalous scaling of a randomly advected passive scalar. *Physical Review Letters* 72 (7), 1016–1019.

[55] Gama, S.M.A., Vergassola, M. & Frisch, U. 1994 Negative eddy viscosity in isotropically forced two-dimensional flow: linear and nonlinear dynamics. *Journal of Fluid Mechanics* 260, 95–126.

[56] Chertovskih, R., Gama, S.M.A., Podvigina, O. & Zheligovsky, V. 2010 Dependence of magnetic field generation by thermal convection on the rotation rate: A case study. *Physica* D 239 (13), 1188–1209.

[57] Linkès, M., Martins Afonso, M., Fede, P., Morchain, J. & Schmitz, P. 2012 Numerical study of substrate assimilation by a microorganism exposed to fluctuating concentration. *Chemical Engineering Science* 81, 8–19.

[58] Meneveau, C. & Katz, J. 2000 Scale-invariance and turbulence models for large-eddy simulation. *Annual Review of Fluid Mechanics* 32, 1–32.

[59] Sagaut, P. 2006 *Large eddy simulation for incompressible flows: an introduction*. Springer.

[60] Antonelli, M., Martins Afonso, M., Mazzino, A. & Rizza, U. 2005 Structure of temperature fluctuations in turbulent convective boundary layers. *Journal of Turbulence* 6 (35), 1–34.

[61] Celani, A., Martins Afonso, M. & Mazzino, A. 2005 Coarse-grained scalar transport: closures and large-eddy simulations. *Progress in Turbulence* II, Springer Proceedings in Physics 109, 229–233.

[62] Chandrasekhar, S. 1961 *Hydrodynamic and hydromagnetic stability*. Oxford-Clarendon Press.

[63] Grilo, T., Pereira, F.L. & Gama, S.M.A. 2013 Optimal control of particle advection in Couette and Poiseuille flows. *Hindawi Conference Papers in Mathematics*, 783510:1–4.

[64] Maxey, M.R. & Riley, J.J. 1983 Equation of motion for a small rigid sphere in a nonuniform flow. *Physics of Fluids* 26, 883–889.

[65] Gatignol, R. 1983 The Faxen formulae for a rigid particle in an unsteady non-uniform Stokes flow. *Journal de Mécanique Théorique et Appliquée* 1, 143–160.

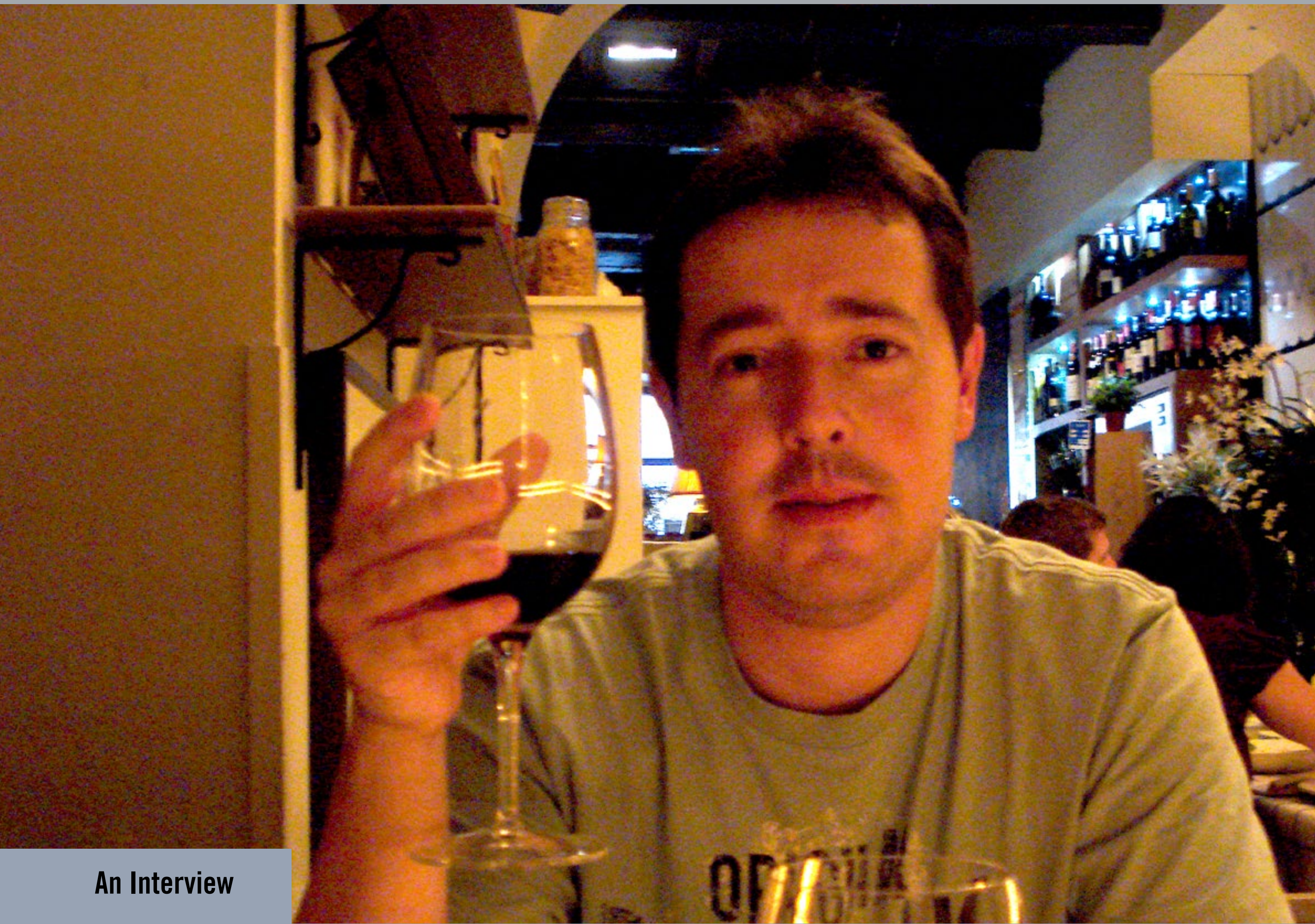
[66] Martins Afonso, M. 2012 Flow-driven renormalization of transport and sedimentation for inertial particles. *Mathematical Analysis and Applications in Engineering, Aerospace and Sciences*, Mathematical Problems in Engineering, Aerospace and Sciences 5 (13), 187–201, Cambridge Scientific Publishers.

UTOPIA — Literature and Mathematics

22 September–2 October, 2016

Óbidos — Portugal

The CIM, in partnership with the Portuguese Mathematical Society (SPM) https://www.spm.pt/ and the Science Museum of the University of Coimbra (MCUC) http://www.museudaciencia.org/, will join the **FOLIO — Óbidos International Literature Festival** http://foliofestival.com/ that will take place at the medieval town of Óbidos this autumn. The initiative will include an international workshop in **Literature and Mathematics** and much more . . . All interested may contact the organisers, Carlota Simões (UCoimbra) carlota@mat.uc.pt, António Machiavelo (UPorto) ajmachia@fc.up.pt or Pedro Jorge Freitas (ULisboa) pjfreitas@ciencias.ulisboa.pt



with Julio Rebelo

by **Alberto Pinto, Helena Reis**, and **Renato Soeiro**

Julio Rebelo is an expert in dynamical systems and foliations. He has held several prestigious positions such as Clay Mathematics Institute fellow as well as long term visiting positions at *Université d'Orsay* (Paris, France) and IMPA (Rio de Janeiro, Brazil). Currently he is Professor at Institute de Mathématiques de Toulouse — Université de Toulouse (France).

He was a member of the scientific committee for the conference *Geometric Aspects of Modern Dynamics*, together with M. Abate, A. Glutsyuk, M. Lyubich, and H. Reis. This conference took place in Porto on January 2016 and was partially sponsored by CIM.

DISCLAIMER: the questions presented here are based on several interviews; in particular, the interviews published in previous CIM's bulletins.

This was a very successful conference featuring a number of field leaders in complex dynamics and related topics. Is there something you would like to highlight?

This is likely to have been the first meeting where interactions between complex dynamics of a single map (automorphism and/or endomorphism) and dynamics of foliations took the center stage. Both theories have the same origin and the corresponding problems share some important basic properties as well as well-known differences. It is an old dream to have both theories fitting together in a unified and fruitful framework. Some important first steps in this direction were made during this conference and hopefully they will be continued in the near future.

How important do you think that events like this are for students and researchers?

It is of paramount importance for students to be exposed to a large number of problems to go along with ideas and techniques. As mentioned above, the structure of this meeting made it especially rich in terms of ideas and problems that are likely to motivate and inspire students. To some extent, this same principle applies to researchers as well.

On your research: How did you start working in this area? What was the motivations? Could you tell us about your mathematical beginnings and subsequent career development?

Dynamical systems is an area strongly represented in the Brazilian mathematical community so I think my early curiosity on the topic was raised by the fact that so many people around me was talking about it. When I started looking for a thesis problem, I had the opportunity to attend an advanced course that E. Ghys lectured in Brazil at IMPA. Pretty quickly I got fascinated by the mixture of dynamics and geometry that was ubiquitous in those lectures and then I convinced him to supervise my thesis in France.

After completing my thesis, I returned to Brazil (PUC-Rio) and three years later I moved to Stony Brook as a Clay Mathematics Institute fellow. I remained at Stony Brook for almost four years, then I returned to PUC-Rio for a couple of years before I was appointed professor at the Institut de Mathématiques de Toulouse.

How would you describe the essence of your own research to a young student?

Many phenomena from everyday life to experimental and theoretic Physics evolve with time and these evolutions are described by differential equations. Knowledge about the solutions of these equations allows us to predict the future of the corresponding systems. The relevant equations however can hardly be explicitly solved so that the subject is all about gathering enough information on their solutions without necessarily looking for a closed form for them. It is the fact that the nature of these differential equations can be extremely varied that accounts for the impressive wealth of methods and points of view in the field.

Do you have a preferred result? More generally what in your opinion makes a great paper?

I like papers with nice results but I have especial appreciation for those that also introduce a new method and/or enhance your understanding of the topic so that the reader feels more able to go on and further advance the theory by obtained still more results. The same principle applies also to books. Naturally Milnor's books and papers constitute fantastic examples of deep insight and clarity that often – or maybe always – improve the reader's own understanding of the corresponding material. Other remarkable examples of theorems whose method of proof turned out to be more important than the original problems are provided by KAM theory and by Sullivan's non-wandering theorem in one-dimensional complex dynamics.

As to my personal results, my construction of a dynamical Lie algebra associated with locally non-discrete subgroups of diffeomorphisms of the circle has proven to be very useful and is quoted in several important papers.

How do you see the relation between traveling and research?

There are three different countries in which I have worked during a significant period (say longer than three years) and I still travel on a very regular basis. For young researchers, I think it is absolutely fundamental to have contact with different research groups not only to widen horizons but also to find new collaborators. In particular I think that all graduates in Mathematics should look to a two-year post-doc overseas before seeking a more permanent job.

Do you have hobbies?

I enjoy wine very much. I have a fair amount of knowledge about French wines as well as Portuguese wines. Among Portuguese wines, those from Douro are my favorite and I also love Port.

Do you have a connection to Portugal?

Despite being born in Brazil, I have inherited Portuguese citizenship from my parents and I really love the country, especially the Douro area. In fact, for personal reasons you all know, I can regularly be found in Porto.

Editor-in-Chief
Jorge Milhazes Freitas (jmfreita@fc.up.pt)

Editoral Board
Assis Azevedo (assis@math.uminho.pt)
António Fernandes* (amfernandes@netcabo.pt)
Carlos Florentino (caflorentino@ciencias.ulisboa.pt)
Jorge Rocha (rocha@fc.up.pt)

* Responsible for the production and graphic design of the bulletin

Address
Departamento de Matemática
Faculdade de Ciências da Universidade do Porto
Rua do Campo Alegre, 687
4169-007 Porto
Portugal

The CIM Bulletin is published twice a year. Material intended for publication should be sent to one of the editors. The Bulletin is available at www.cim.pt. The CIM acknowledges the financial support of FCT–Fundação para a Ciência e a Tecnologia

Joint International Meeting of the American, European and Portuguese Mathematical Societies, Porto, June 2015

by Samuel Lopes*

The Joint International Meeting of the American, European and Portuguese Mathematical Societies (AMS, EMS and SPM, respectively) took place in Porto on June 9–13, 2015. The idea to host a joint meeting of the Portuguese and the American Mathematical Societies arose in 2010, through informal conversations with Georgia Benkart. The Portuguese Mathematical Society welcomed the idea and made a formal proposition to the AMS to host this meeting in Portugal. In 2012 the EMS also showed an interest in joining this event, which resulted in the first and, as of yet, the only joint meeting of the American and the European Mathematical Societies. In 2015, the SPM celebrated its 75th anniversary, the EMS its 25th and the AMS its 126th. All information about the conference, including the presentation slides, is accessible online at http://aep-math2015.spm.pt.

The Scientific Committee was chaired by Georgia Benkart (Univ. Wisconsin-Madison) and included also Jorge Almeida (Univ. Porto), Nils Dencker (Lunds Univ.), Gustavo Granja (Univ. Lisbon), Alexey Parshin (Steklov Inst. Moscow), Carlos Rocha (Univ. Lisbon), Jean Taylor (Courant Inst. New York Univ. and Rutgers Univ.), Susanna Terracini (Univ. degli Studi di Torino) and Luís Nunes Vicente (Univ. Coimbra).

The invited plenary speakers were: Marcus du Sautoy (Univ. Oxford), Rui Loja Fernandes (Univ. Illinois), Irene Fonseca (Carnegie Mellon Univ.), Annette Huber (Albert-

* Faculdade de Ciências da Universidade do Porto

Ludwigs Univ.), Mikhail Khovanov (Columbia Univ.), André Neves (Imperial College), Sylvia Serfaty (Univ. Pierre et Marie Curie Paris 6), Gigliola Staffilani (MIT) and Marcelo Viana (IMPA). Unfortunately due to unforeseen health problems, Mikhail Khovanov could not come to Porto. Sylvia Serfaty, winner of the 2012 Henri Poincaré prize and 2014 EMS Prize, delivered her lecture as the EMS Distinguished Speaker for 2015.

The general scientific standards of the conference were very high and included a Fields medalist and former MAA and SIAM presidents speaking in and organizing special sessions. One of the invited speakers was also a former SIAM president; another a former vice-president of the International Mathematical Union, current president of the Brazilian Mathematical Society (SBM) and first winner of the Ramanujan Prize; and yet another a member of the American Academy of Arts and Sciences. Three of the invited lecturers had spoken recently at an ICM. The presentation slides of the plenary talks can be downloaded at

http://aep-math2015.spm.pt/IAprogram

and some of the slides for the special sessions can also be viewed in http://aep-math2015.spm.pt/ListOfSessions, by selecting the corresponding session.

The conference generated great interest in the mathematical community worldwide. There were 831 abstracts submitted and the accepted papers were scheduled in one of the 53 Special Sessions approved by the Scientific Committee, or in one of the 7 Contributed Paper Sessions. More than 1 100 participants from 59 different countries attended the conference, filling the classrooms and corridors of the Faculty of Sciences of the University of Porto (FCUP) as well as its surrounding garden areas.

Although there was an informal reception on June 9th, and the first talks started at 9am on June 10th, the official opening ceremony took place in the afternoon with words by Georgia Benkart (AMS Associate Secretary), Pavel Exner (EMS President), Fernando Pestana da Costa (SPM President), José Manuel Martins Ferreira (Vice Dean of the University of Porto) and Nuno Crato (Minister of Education and Science at the time of the meeting). Being unable to attend the conference, Robert Bryant (AMS President) welcomed all participants in a video which can be accessed through the meeting website at http://tv.up.pt/premiums/80. On that evening, Marcus du Sautoy delivered the public lecture *The Secret Mathematicians* at *Casa da Música*, followed by a concert by the Classical Orchestra of the Faculty of Engineering of the University of Porto, where du Sautoy joined the trumpet players.

There were three satellite meetings related to the meeting:

1. *Higgs Bundles and Character Varieties* — a workshop sponsored by the NSF GEAR (GEometric structures And Representation varieties) Network in conjunction with Special Session #27 — by the same title;
2. Workshop on *Groups and Semigroups* on the occasion of the 60th birthday of Mikhail Volkov in conjunction with Special Session #4 — Algebraic Theory of Semigroups and Applications;
3. *Summer School on Representation Theory* at the University of Coimbra in conjunction with Special Session #41 — New Trends in Representation Theory.

An Interview

with Samuel Lopes*

by **Peter Gothen****

A meeting on this scale is only possible with the support and hard work of many people and organizations. Could you give an idea of who was involved, and is there any contribution which was particularly important?

The main actors were the American Mathematical Society (AMS), in particular Georgia Benkart, from whom the idea of such a meeting stemmed, the Portuguese Mathematical Society (SPM), which was very enthusiastic about the idea, and the local organizers, namely Adérito Araújo (Univ. Coimbra), Mário Bessa (Univ. Beira Interior), André Oliveira (Univ. Trás-os-Montes e Alto Douro), and Sílvio Gama,

* Chair of the Organizing Committee of the Joint International Meeting of the American, European and Portuguese Mathematical Societies, Porto, June 2015

** Faculdade de Ciências da Universidade do Porto

José Abílio Matos, Leonor Moreira, João Nuno Tavares and myself (Univ. Porto).

The SPM secretariat was outstanding and a pleasure to work with. All the support from the Faculty of Sciences of the University of Porto (FCUP) and the Public Relations Department of the University of Porto was also extremely helpful. Lastly, the student volunteers we had during the conference were tireless, committed and absolutely invaluable.

What was the most difficult task in organizing the meeting?

For a meeting of this dimension and considering that almost everything was done locally without resorting to external professional services, all tasks were difficult. Perhaps the most difficult one was predicting everything that would be needed, all that could go wrong (power failures, plumbing accidents, etc.) and ways to resolve potential problems.

Another difficult issue was related to timings and deadlines. The conference had been in the planning since 2010, and in general the AMS expects concrete details on space and facilities well in advance, as well as solid plans for the organization and activities prepared. But on the Portuguese and even the European side, most services and facilities are not prepared to plan that far in advance. For example, due to changes in administrations and other issues, we had to re-plan and re-negotiate with most services several times, and all agreements had to be checked and re-checked often, which considerably affected our stress levels!

How did the scientific level of the meeting compare with your expectations?

The Scientific Committee did a great job with selecting the invited addresses and ensuring we would have top quality special sessions, so I had high expectations to begin with. In the end, and in spite of the unfortunate absence of several great mathematicians, like for instance Mikhail Khovanov, who had plans to attend the conference but, mostly for health reasons, could not, I think the conference achieved a remarkable quality. Processing registrations daily and going through the speakers lists to compile the program, I would invariably recognize important names in all fields of mathematics.

Was it worthwhile for a relatively small society like the Portuguese Mathematical Society to be involved in the organization of a major event of this kind?

I cannot speak for the SPM, but I believe they were quite satisfied with the outcome of the meeting. The SPM embraced the idea of the meeting from the very beginning, which I think reveals the society's forward thinking. The podcast http://tv.up.pt/videos/wz8453bp by SPM's President, Fernando Pestana da Costa, shows some of the society's motives for hosting such a meeting.

What were the main benefits of the meeting for the Portuguese Mathematical Society and, more broadly, the Portuguese mathematical community?

The meeting gave a lot of visibility to the research done by mathematicians in Portugal, as the community was very well represented among the speakers and organizers of special sessions. From what I could gather, the meeting potentiated several new collaborations, and I know of several cases of co-authors who met for the first time at the meeting. It was also important for the public awareness of mathematics, undoubtedly one of SPM's goals, mostly thanks to the excellent job done by the University of Porto's Public Relations Department: the conference was featured in the television news, newspapers and radio.

How and to what extent did the meeting contribute to the visibility of mathematical research, and mathematics more broadly, in Portuguese society?

I think it had a very important role, by showing the general public that mathematics is an exciting field of research gathering people from all around the world to discuss their latest results and conjectures. Too often, mathematics is viewed as a dead subject, created, developed and closed centuries ago, which couldn't be further from the truth. Fortunately, I believe this misconception is getting more and more challenged with the help of a lot of public awareness actions by the mathematical community, and I hope this conference also helped in this respect.

Did you involve students in the organization? If so, how did you involve them, and did they benefit from taking part?

Peter Gothen

Yes, we had a group of about 40 students helping out with registration, computer facilities, giving directions and walking with the participants between venues. As I mentioned above, their help was invaluable, and I was very impressed by the level and quality of their contribution. We had students from Coimbra and Lisbon volunteer to join the group of student organizers. Although it was very hard work for them, I believe this was a great experience for them: as volunteers they got to speak to top-tier mathematicians whom they would otherwise be to shy to talk to, and to attend their lectures. Hopefully, this will impact their future lives as mathematicians.

What kind of feedback have you received from participants and the mathematical societies involved?

The feedback was overwhelmingly positive, which to me was a very rewarding surprise. Some things did not go as well as had been planned, but either no one noticed or, more likely, the participants were understanding and generous enough to overlook these faults. The organizing committee was very careful and meticulous, but with such a big meeting I was definitely expecting a lot of problems and maybe a catastrophe or two. Fortunately I was wrong!

Do you know if there are any meetings of a similar kind planned for the future - or was this a one-off event?

As far as I know, there are no future joint AMS-SPM, EMS-SPM or AMS-EMS meetings planned. In some countries, like Israel and South Africa, there has been a second joint meeting with the AMS although, to my knowledge, these meetings have been smaller than the one in Porto.

Was it worth all the hard work?

One year ago I would have had to pause before answering such a question but—and I think I speak for all the organizers—the answer has been absolutely clear since June 14th, 2015: yes, it was well worth it!

Ciências ULisboa
Faculdade de Ciências da Universidade de Lisboa

20-24 June 2016
FCUL, LISBON

CSA 2016

INTERNATIONAL CONFERENCE ON SEMIGROUPS AND AUTOMATA 2016

Celebrating the 60th birthday of Jorge Almeida and Gracinda Gomes

Faculty of Sciences (building C6), University of Lisbon, Campo Grande, Lisbon

Jorge Almeida (University of Porto) and Gracinda Gomes (University of Lisbon) have played a major role in the development of semigroups in Portugal. They will both be 60 years old in 2016, and a conference is being organized to celebrate their birthday.

csa2016@fc.ul.pt
http://ciencias.ulisboa.pt/en/conferencia/csa-2016

Workshop — 2nd Porto Meeting in Mathematics and Biology

15–17 June, 2016

i3S — Instituto de Investigação e Inovação em Saúde
Auditório A (Corino de Andrade)
Rua Alfredo Allen, 208
4200-135 Porto, Portugal

João Nuno Tavares [**jntavar@fc.up.pt**]
http://cmup.fc.up.pt/cmup/biomath/2016/