JOSÉ ANASTÁCIO DA CUNHA (1744–1787)

by **Luis Saraiva***

Introduction

José Anastácio da Cunha is one of the few Portuguese mathematicians that can be said to have made significant contributions to the world of mathematics. He was born in Lisbon, in 1744. On his life see [Ferro, 1987]. Earlier in his youth (some sources mention the period 1760–1763) he was educated by the priests of the *Oratorian Congregation*. In 1764 he was appointed first lieutenant of the Company of Artillerymen of the *Porto Artillery Regiment*. Although the regiment is said to be from Porto, in fact it was stationed at Valença, where it remained during the time da Cunha was a member. The majority of the officers of the regiment were foreigners, many from protestant countries, and so da Cunha contacted authors, ideas and books that normally a Portuguese officer of that time would not have access. While in this Regiment, he wrote the *Essay on the Mines* (*Ensaio sobre as Minas*") [da Cunha, 1994], which is said to have been decisive for the Marquis of Pombal noticing him, and also, in 1769, the *Physical-Mathematical Letter, on the Theory of Powder in general, etc* (*Carta Fisico-Mathematica, sobre a Theorica da Polvora em Geral, etc*). None was published during his lifetime.

In 1772 took place the *Reform of Coimbra University*, the first reform of the University since 1612, aiming at bringing Portugal up to the level of the best European Universities. In particular, it was created the first Faculty of Mathematics in Portugal. In September of that year three teachers for the new faculty were appointed: Miguel Franzini, Miguel António Ciera and José Monteiro da Rocha. In October of the following year Anastácio da Cunha was appointed lecturer of the Faculty. He taught there during four academic years: 1774/75 to 1777/78.

The King D. José died in 1777, the *Marquis of Pombal* was dismissed, there was a political change, the backward forces that Pombal had subdued came back to power, and the arrests by the Inquisition started. In January 1778 da Cunha was arrested on the grounds of religious heterodoxy, following a denunciation. He was taken to the Inquisition tribunal and condemned in October. He was expelled from the university, and was sentenced to stay three years with the *Oratorian Congregation*, to be followed by four years of exile in Évora. He was said to never again return either to the University or to Valença. In January of 1781, the last year of the stay at the *Oratorian Congregation* and the four years of Exile in Évora were pardoned. On his Inquisition Trial, see [Ferro, 1987]. It is not known the exact date he was appointed to teach mathematics at the Colégio de S. Lucas of Casa Pia de Lisboa. It is probable that the year he started to teach there was 1782. We know for sure that in 1783 he was already teaching there, The Casa Pia is an institution founded in 1780 by Queen D. Maria I and initially organized by the Police Superintendent Pina Manique, which aimed to protect and educate young children either orphans or with very poor parents, who had not enough money to be able to offer their children the minimum for surviving and having an education. In this way da Cunha avoided to be unemployed, but his salary at the S. Lucas college was one quarter of the one he had at the University. We do not know exactly when he left the S. Lucas College, we know by the time he died, on January 1, 1787, he was no longer a teacher there. He completed his masterwork, *Principios Mathematicos* while he was teaching at the Casa Pia, although the book was only published in 1790. three years after his death. In 1807 Domingos de Sousa Coutinho (1760–1833)[1] published in

[1] Brother of Rodrigo de Sousa Coutinho. He obtained a degree in Law awarded by the University of Coimbra. He was a professional diplomat, and started in Denmark in 1788. He was in Turim from 1796 to 1803. He was at the Portuguese Embassy in London from 1803 to 1814.

* CIUHCT• CMAF-CIO Universidade de Lisboa

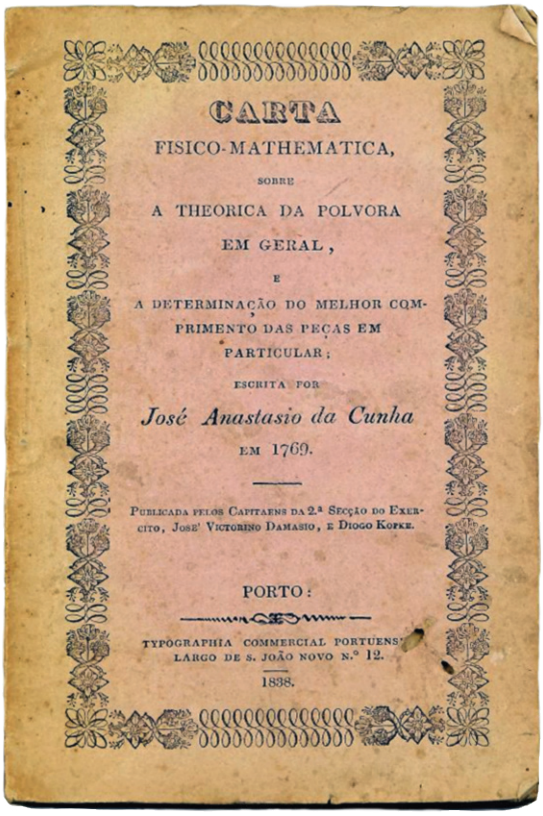
CARTA
FISICO-MATHEMATICA,
SOBRE
A THEORICA DA POLVORA
EM GERAL,
E
A DETERMINAÇÃO DO MELHOR COMPRIMENTO DAS PEÇAS EM PARTICULAR;
ESCRITA POR
José Anastasio da Cunha
EM 1769.
PUBLICADA PELOS CAPITAENS DA 2.ª SECÇÃO DO EXERCITO, JOSÉ' VICTORINO DAMASIO, E DIOGO KOPKE.
PORTO:
TYPOGRAPHIA COMMERCIAL PORTUENSE
LARGO DE S. JOÃO NOVO N.º 12.
1838.

Figure 1. Physical-Mathematical Letter


ENSAIO
sobre
AS MINAS
Por
JOZE ANASTACIO
da
CUNHA.

Figure 2. Essay on the Mines

London his *Essay on the Principles of Mechanics* (*Ensaio sobre os Princípio da Mecânica*). In 1811 it is the turn of the French translation of his *Mathematical Principles* by his student and friend João Manuel de Abreu (1757–1815)[2] to be published in Bordeaux, with a so called second edition, but probably just the leftovers of the first one with a different front cover, appearing in Paris in 1816.

The Mathematical Principles[3]

The main work of da Cunha is his *Mathematical Principles*, a 302-page book divided in 21 chapters, which he calls "books". It covers a wide range of subjects, and there are different levels of both rigour and depth in this work, from the elementary to the highly specialized and innovative. Subjects go from Euclidean to analytic geometry, from differential calculus to differential equations, from algebraic equations to the theory of number series.

There are no references in his book, but that might be because he died before completing it. Nevertheless there are clear influences of Newton, D' Alembert, Lagrange and Euler. Da Cunha uses the Greek method of unfolding his subjects, so he uses systematically a sequence of axioms-definitions-propositions-proofs, he tries to be the most concise possible. This was most unusual in his time, and so it had the consequence of becoming a difficult book for the University students of his time, as its understanding implied the reader to be able to reason mathematically.

Internationally, the first historian who had an impact in calling the attention to da Cunha's work in the 20th century was the Soviet historian Adolph Pavlovich Youschkevitch (1906–1993) who wrote an important paper in *Revue d'Histoire des Sciences* [Youschkevitch, 1973]. Youschkevitch concludes his paper saying [Youschkevitch, 1973; p. 22]

> nous avons le droit et le devoir de ranger J. A. da Cunha parmi les éminents prédécesseurs de la reforme du calcul infinitesimal réalisée peu après sa mort prématurée par Bolzano, Gauss, Cauchy, Abel et d'autres géomètres du XIXème siècle

[2] He was at the *Porto Artillery Regiment* with da Cunha. He was first a student, then a friend of da Cunha. He graduated in mathematics at the Coimbra University. He was also arrested by the Inquisition in 1778, and condemned to three years of confinement

[3] In this chapter I will follow my paper [Saraiva, 2012]

We will go into more detail in books IX and XV, respectively on the theory of series and on differential calculus, where DA CUNHA was as innovator.[4]

Book XV is a 15 page chapter [da Cunha, 1790; pp. 196–120] where the author provides a rigorous study on the main properties of number series. His definition of a convergent series is

> Mathematicians call convergent series a series whose terms are initially determined, each one by the preceding terms, such that the series can be continued, and finally it makes no difference whether it is continued or not, because we can disregard without any noticeable error the sum of any number of terms that we wish to add to those already written or designated; and these last terms are denoted by writing &c. after the first two or three terms, or any number of terms we want; however it is necessary that either the written terms show how we could continue the series, or that this is known in some other way.[5]

This definition can be said to be formally equivalent to CAUCHY's although there are some subtle differences in attitude, which do not affect the practical use of both definitions, as it is expressed in [Giusti, 1990; p. 45]:

> ce qui pour Cauchy sera choisi arbitrairement, pour da Cunha est arbitrairement fixé

For CAUCHY we chose an arbitrary quantity, and we prove that from a certain order onwards the sum of any number of consecutive terms is less than the arbitrary quantity chosen. But for DA CUNHA, he assumes an agreement has been reached on what can be considered negligible, that is, this must be established beforehand, although this quantity must be arbitrary.

Then using his definition, DA CUNHA proves correctly that when we have $r > 0$, then the series $\sum_{n\geq 1} r^n$ is convergent if $r < 1$. From this he obtains that the series $\sum_{n\geq 1} a^n/n!$ is convergent for any [positive] a. He then goes on to define a^b: let c be such that $\sum_{n\geq 1} c^n/n! = a$, that is, $e^c = a$. Then a^b means $\sum_{n\geq 1}(bc)^n/n!$, that is, in today's terms, $e^{bc} = (e^b)^c = a^b$. After putting this definition, DA CUNHA proves that c always exists, giving its explicit value as a sum of a series.

He goes on and proves some properties of exponentials, and ends the chapter by defining logarithm as the inverse of the exponential, and using the definition and properties of the exponential he has just proved, he shows that $\ln(a^n) = n(\ln a)$.

The subject on Book XV is differential calculus. It is a short chapter, it has only 12 pages. DA CUNHA uses Leibnizian calculus with Newtonian notations. [Mawhin, 1990] says he is the first mathematician to formulate a modern analytic definition of the differential of a real function of real variable, anticipating CAUCHY's definition. He starts the chapter stating some basic definitions: *variable* (an expression which can have more than one value), *constant* (an expression which only has a single value), *infinite variable* (a variable that admits values greater than any given magnitude), *infinitesimal variable* (a [positive] variable that can have smaller values than any given magnitude) and *function* (if the value of a certain expression A depends on the value of another expression B, then A is said to be a function of B, and B is said to be the root of A) [da Cunha, 1790; p. 193]. He defines differential as follows:

> Let dx be a quantity homogeneous [that is, of the same kind] to the root [that is , the argument] x, which we call the fluxion of the root; we call the fluxion of Γx, and we write $d\Gamma x$, the quantity that will make $d\Gamma x/dx$ constant and
>
> $$\frac{\Gamma(x+dx)-\Gamma x}{dx} - \frac{d\Gamma x}{dx}$$
>
> an infinitesimal or zero, if dx is an infinitesimal and all that does not depend on x remains constant.[6]

[Mawhin, 1990: pp. 99–100] immediately concludes that this definition implies that $d\Gamma x$, is linear with dx, that is, there is a constant k such that $d\Gamma x = k \cdot dx$, and that

$$\Gamma(x+dx) - \Gamma(x) - d\Gamma x = dx \cdot A(dx),$$

with $A(dx) \to 0$ when $dx \to 0$ that is,

$$\Gamma(x+dx) - \Gamma(x) = k \cdot dx + dx \cdot A(dx).$$

This corresponds to the modern way of defining differential of a function.[Grattan-Guinness, 1990; p. 57] expresses doubts concerning the clarity of the definitions of dx and of the fluxion of x, but this does not detract from the innovation of this definition.

DA CUNHA defines differentials of higher order and proves correctly that if A, B, C, D, &c., are constants, and if x is an infinitesimal, then $Ax + Bx^2 + Cx^3 + Dx^4 +$ &c.

[4] There are many papers on da Cunha's *Mathematical Principles*. In English see [Giusti, 1990], [Queiró, 1988] and [Oliveira, 1988] ; in Portuguese, among others, see [Rodrigues, 1999; mainly pp. 78–83]

[5] "Serie convergente chamam os Mathematicos àquella, cujos termos saõ semelhantemente determinados, cada hum pelo numero dos termos precedente[s], de sorte que sempre a serie se possa continuar, e finalmente venha a ser indiferente o continua-la ou naõ, por se poder desprezar sem erro notável a somma de quantos termos se quisesse ajuntar aos já escritos ou indicados: e estes ultimos indicam-se escrevendo &c. depois dos primeiros dois, ou três, ou quantos se quiser: he porem necessário que os termos escritos mostrem como se poderia continuar a serie, ou que isso se saiba por outra via" [da Cunha, 1790; p. 106]

[6] "Escolhida qualquer grandeza, homogenea a uma raiz *x*, para se chamar fluxaõ dessa raiz, e denotada assim *dx*; chamar-se há fluxaõ de Γ*x*, e se denotará assim *d*Γ*x*, a grandeza que faria constante, e infinitessimo ou cifra, se dx fosse infenitessimo [sic] e constante tudo o que naõ depende de *dx*."

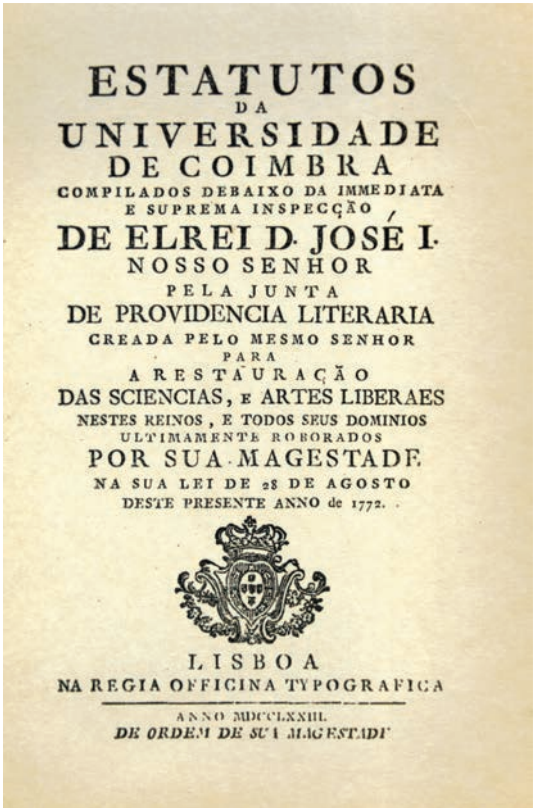
ESTATUTOS
DA
UNIVERSIDADE
DE COIMBRA
COMPILADOS DEBAIXO DA IMMEDIATA
E SUPREMA INSPECÇÃO
DE ELREI D. JOSÉ I.
NOSSO SENHOR
PELA JUNTA
DE PROVIDENCIA LITERARIA
CREADA PELO MESMO SENHOR
PARA
A RESTAURAÇÃO
DAS SCIENCIAS, E ARTES LIBERAES
NESTES REINOS, E TODOS SEUS DOMINIOS
ULTIMAMENTE ROBORADOS
POR SUA MAGESTADE
NA SUA LEI DE 28 DE AGOSTO
DESTE PRESENTE ANNO de 1772.
LISBOA
NA REGIA OFFICINA TYPOGRAFICA
ANNO MDCCLXXIII.
DE ORDEM DE SUA MAGESTADE

Figure 3. Statutes of the University of Coimbra, 1772

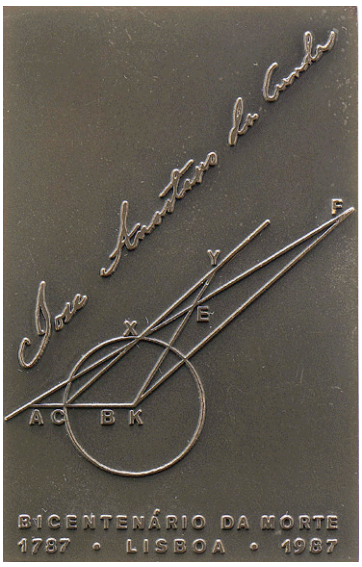

Figure 4. Signature of DA CUNHA


ENSAYO
SOBRE OS
PRINCIPIOS DE MECHANICA.
OBRA POSTHUMA
DE
JOZÉ ANASTACIO DA CUNHA.
DADA A LUZ
Por D. D. A. de S. C.
POSSUIDOR DO MS. AUTOGRAPHO.
London:
PRINTED BY COX, SON, AND BAYLIS,
GREAT QUEEN-STREET;
FOR J. BUDD, BOOKSELLER TO H. R. H. THE PRINCE OF WALES,
AT THE CROWN AND MITRE, PALL-MALL.
1807.

Figure 5. Essay on the Principles of Mechanics

is also an infinitesimal. Then some of the properties of differentials are proved, including the differentials of the sum, of the product, of the power and of the logarithm.

DA CUNHA defines differential of any order of a function of more than one variable [da Cunha, 1790; p.199]:

$$\frac{d^n\Gamma(u,x,z,\&c.)}{du\,dx\,dz\,\&c.}$$

denotes what results from dividing the fluxion of $\Gamma(u,x,z,\&c.)$ in order to the root u by du; then we divide the fluxion of this new quotient in order to the root x by dx; and so on.[7]

He proves incorrectly TAYLOR's theorem, as he assumes that all functions are equal to the sum of their power series.

Then he proceeds to prove that the mixed differentials of a function of two variables are equal. For his proof he uses TAYLOR's expansion of the function. Nevertheless, as is remarked in [Grattan-Guiness, 1990; p. 57] this is worthy of note, as this is a result that seemed obvious for many of DA CUNHA's contemporaries, so they did not think that a proof was required.

RECEPTION OF MATHEMATICAL PRINCIPLES

During his life DA CUNHA created and maintained a dedicated circle of students and friends that would after his death continue to defend their teacher and friend, whenever they felt he had an unfair critic. The Scottish mathematician JOHN PLAYFAIR (1748–1719) published in the July-November 1812 issue of the *Edinburgh Review* (volume XX) a review of DA CUNHA's *Mathematical Principles*, after the publication of ABREU's French translation. This is included in [Proceedings, 1990; pp. 415–423]. A Portuguese translation of PLAYFAIR's review appeared in 1813 in volume VII of the Portuguese journal published in London *O Investigador Portuguêz em Inglaterra* (*The Portuguese Researcher in England*). The review was considered among the DA CUNHA circle to be completely missing the importance of the book: therefore both JOÃO MANUEL DE ABREU and ANASTÁCIO JOAQUIM RODRIGUES (?–1818), another student and friend of DA CUNHA, wrote answers to PLAYFAIR. Both were published in volume VIII of *Investigador*

[7] "denota o que resulta de dividir por *du* a fluxaõ de Γ(*u*,*x*,*z*,&c.) tomada relativamente á raiz *u*; e por *dx* a fluxaõ do novo quociente, tomada relativamente á raiz *x*; e assim por diante." I corrected a couple of printing mistakes in the definition. But there is no doubt in DA CUNHA's definition, as the practical example that is solved after the definition clearly shows.


PRINCIPIOS MATHEMATICOS
PARA INSTRUCÇAÕ
DOS ALUMNOS DO COLLEGIO
DE
SAÕ LUCAS,
DA REAL CASA PIA DO CASTELLO
DE
SAÕ JORGE:
OFFERECIDOS
AO SERENISSIMO SENHOR
D. JOAÕ,
PRINCIPE DO BRAZIL:
COMPOSTOS PELO DOUTOR
JOSE' ANASTACIO DA CUNHA,
DE ORDEM DO DESEMBARGADOR DO PAÇO
DIOGO IGNACIO DE PINA MANIQUE,
Intendente Geral da Policia da Corte,
e Reino, &c, &c, &c.
LISBOA
NA OFFIC. DE ANTONIO RODRIGUES GALHARDO.
Impreſſor do Eminentiſſimo Senhor Cardeal Patriarca.
ANNO M. DCC. XC.
Com licença da Real Meza da Commiſſaõ Geral
ſobre o Exame, e Cenſura dos Livros.

Figure 6. Mathematical Principles by José Anastácio da Cunha (1790)

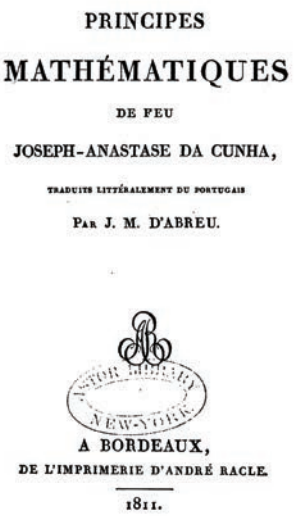
PRINCIPES
MATHÉMATIQUES
DE FEU
JOSEPH-ANASTASE DA CUNHA,
TRADUITS LITTÉRALEMENT DU PORTUGAIS
PAR J. M. D'ABREU.
A BORDEAUX,
DE L'IMPRIMERIE D'ANDRÉ RACLE.
1811.

Figure 7. The French translation of João Manuel de Abreu (1811)

Portuguez and both are included in [Proceedings, 1990: pp. 449–486 and pp. 425–448]. But of course, as the two answers were written in Portuguese and published in a journal published in England, but whose public was essentially for the emigrants of the Portuguese cultured community in England, their impact was minimal.

Rodrigues also wrote a review of da Cunha's *Principios Mathematicos* in the August 8, 1811 issue of *Moniteur Universel*, transcribed in [Proceedings, 1990; pp. 399–404].

There were other reviews: besides the above mentioned Playfair review, there was an anonymous review in the November 14, 1911 of *Göttingishche gelehrte Anzeigen*, but thought to be by Johann Tobias Mayer. Gauss, in a letter to Bessel, criticized this review. This was analysed in [Youschkevitch, 1978]. We have already mentioned the review of the *Mathematical Principles* that appeared in *Moniteur Universel*. Recently João Caramalho Domingues found an anonymous review (but thought to be of Vincenzo Brunacci), in the March/April 1816 issue of the Italian journal, *Giornale di Fisica, Chimica, Storia Naturale, Medicina ed Arti* [Domingues, 2011].

Da Cunha also influenced mathematicians in Portugal. On this matter see [Duarte and Silva, 1990]

Final Notes

In this brief paper we just give some information on da Cunha, and mainly on his *Mathematical Principles*, but all his other works are worthy of analysis. And in the last twenty five years more works by da Cunha were found: first it was the *Essay on the Mines* (*Ensaio sobre as Minas*), a work already mentioned by da Cunha in his 1769 work *Physical-Mathematical Letter*. It was found in the Archives of the Braga District by Maria Fernanda Estrada, and published in book form in 1994 [da Cunha, 1994]. In 2005, in the same Archive of the Braga District, two other researchers, Maria do Céu Silva and Maria Elfrida Ralha found another seven da Cunha manuscripts. A team of researchers was formed to study those manuscripts, and an year later a Conference took place in Braga, where the results of their research was displayed, and two books were published, one with the da Cunha papers [da Cunha, 2006a], the other one with the papers written about those manuscripts, together with others on different aspects of da Cunha's life and work and his time [da Cunha, 2006b]. Today the research continues to be done on other manuscripts that were recently found.

Anastácio da Cunha has been a much celebrated mathematician, first by his contemporaries, like João Manuel de Abreu, Anastácio Joaquim Rodrigues and other close friends, but also by people that in some way at a certain moment opposed him, as Francisco de Borja Garção Stockler[8] (1759–1829), a mathematician who wrote the first history of mathematics in Portugal and in fact it is the first history of mathematics in a single country that was published: *Historical Essay on the Origins and Developments of Mathematics in Portugal* (*Ensaio Histórico sobre as origens e progressos das matemáticas em Portugal*) published in London in 1819, thirty two years after da Cunha's death. Stocker limits his research up to 1779, the year of the foundation of Lisbon's Academy of Scienc-

[8] On Stockler see [Saraiva, 1993].

es. By that time DA CUNHA had no works published. However STOCKLER felt obliged to write a 6-page note on DA CUNHA, only second in length to what he wrote on PEDRO NUNES (1502–1578), the Portuguese leading mathematician of the 16th century.

FRANCISCO GOMES TEIXEIRA (1851–1933) the leading Portuguese mathematician of the second half of the 19th century, and the founder of the first Portuguese international journal, the *Journal of Mathematical and Astronomical Sciences* (*Jornal de Sciencias Mathematicas e Astronomicas*),[9] wrote about DA CUNHA in his *History of Mathematics in Portugal* something similar to what YOUSCHKEVITCH would write forty years later [Teixeira, 1934; p. 260]:

> In the XVIIth century, Anastácio da Cunha is one of the forerunners of the geometers who in the XIXth century did this considerable work of organizing logically the new domains that were started in the world of numbers, and his works and his name must be included in the splendid history of this organization.[10]

We hope that this introduction to JOSÉ ANASTÁCIO DA CUNHA makes some of its readers interested in exploring more in depth the mathematician and free thinker that DA CUNHA was.

REFERENCES

da Cunha, José Anastácio, 1790. *Principios Mathematicos*, Lisboa: Officina de António Rodrigues Galhardo

da Cunha, José Anastácio, 1811. *Principes Mathématiques de feu Joseph-Anastase da Cunha*. Bordeaux: de l'Imprimerie de André Racle.

da Cunha. José Anastácio, 1838. *Carta Fisico-Mathematica sobre a Theoria da Polvora em Geral, e a Determinação do Melhor Comprimento das Peças em Particular*, Porto: Typographia Commercial Portuense (manuscrito de 1769).

da Cunha, José Anastácio, 1987a. *Principios Mathematicos*, Fac-simile edition. Coimbra: Departamento de Matemática da Universidade de Coimbra.

da Cunha, José Anastácio, 1987b. *Principes Mathématiques de feu Joseph-Anastase da Cunha*. Fac-Simile edition. Coimbra: Departamento de Matemática da Universidade de Coimbra.

da Cunha, 1994. *Ensaio sobre as Minas*, Arquivo Distrital de Braga/Universidade do Minho

da Cunha, José Anastácio, 2006a. *Os inéditos*. Arquivo Distrital de Braga/Universidade do Minho

da Cunha, José Anastácio, 2006b. *O Tempo, as Ideias, a Obra . . .* Arquivo Distrital de Braga/Universidade do Minho

Domingues, João Caramalho. 2011. Uma recensão italiana dos *Principios Mathematicos* de José Anastácio da Cunha, *Boletim da SPM* 65, pp. 89–98.

Duarte, António Leal, and Silva, Jaime Carvalho e, 1990. Sobre a influência da obra matemática de José Anastácio da Cunha, in [*Proceedings*, 1990], pp. 133-145.

Ferro, João Pedro, 1987. *José Anastácio da Cunha (1744–1787)*, Catálogo da Exposição José Anastácio da Cunha (1744–1787) Matemático e Poeta, Lisboa: Biblioteca Nacional, pp. 25–38.

Ferro, João Pedro, 1987. *O Processo de José Anastácio da Cunha na Inquisição*. Lisboa: Palas Editores

Giusti, Enrico, 1990. Quelques reflexions sur les *Principios* de da Cunha, in [*Proceedings*, 1990], pp. 33–52

Grattan-Guinness, Ivor, 1990. Da Cunha's Calculus in its Time, in [*Proceedings*, 1990], pp. 53–62.

Oliveira, Augusto Franco de, 1988. Anastácio da Cunha and the Concept of Convergent Series, *Archive for History of Exact Sciences*, 39, Number 1, pp. 1–12.

Proceedings of the International Colloquium Anastácio da Cunha 1744/1797 o matemático e o poeta. 1990. Lisboa: Imprensa Nacional/Casa da Moeda

Queiró, João Filipe, 1988. José Anastácio da Cunha: a Forgotten Forerunner, *The Mathematical Intelligencer*, 10 (1), pp. 38–43

Rodrigues, José Francisco, 1988. Cultura e ciência em Portugal no século das luzes, a Obra Matemática de José Anastácio da Cunha, *Colóquio/Ciências* 23, pp. 74-86

Saraiva, Luis M R, 1993. On the first history of Portuguese Mathematics, *Historia Mathematica* 20, pp. 415-427.

Saraiva, Luis M R, 2012. Mathematical Analysis in Portugal in the Period 1772–1822, in *Aventure de l'analyse de Fermat a Borel- Mélanges en l'honneur de Christian Gilain* (Editor Suzanne Féry), Editions Universitaires de Lorraine, Presses Universitaires de Nancy, pp. 301–331.

Saraiva, Luis M R, 2014. A Decisive Journal in Portuguese Mathematics: Gomes Teixeira's *Jornal de Sciencias Mathematicas e Astronomicas* (1877–1905), in *L'émergence de la presse mathématique en Europe au 19ème siècle. Formes éditoriales et études de cas*, Christian Gerini & Norbert Verdier (Eds.), Collège Publication, Oxford, pp. 67–95.

Teixeira, Francisco Gomes, 1934. *História das Matemáticas em Portugal*, Academia das Ciências de Lisboa

Youschkevitch, Adolph P. 1973. J. A. da Cunha et les fondements de l' Analyse Infinitésimale, *Revue d´Histoire des Sciences* 26, pp. 3–22.

Youschkevitch, Adolph P. 1978. C. F. Gauss et J. A. da Cunha, *Revue d'Histoire des Sciences* 31, pp. 327–332

[9] On Gomes Teixeira and his journal, see [Saraiva, 2014]

[10] "Anastácio da Cunha é no século XVIII um dos precursores dos geómetras que no século XIX realizaram esta obra considerável da organização lógica dos novos domínios que se tinham aberto no Mundo dos números e os seus trabalhos e o seu nome devem figurar na história brilhante desta organização"