A statistical perspective on the identification of geophysical trends

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Introduction

In the diverse geoscience problems investigated at the Instituto Dom Luiz (IDL), a partner of CIM, the identification and quantification of trends is one of the most ubiquitous activities. From the analysis of the outputs of complex meteorological numerical models in a climate change context to the exploitation of geophysical resources and renewal energy sources, accurate knowledge of trends and corresponding uncertainties is fundamental for answering most scientific and societal questions.

Although the concept of trend, as a general direction and tendency, is physically intuitive and apparently simple, its mathematical formulation is far from trivial. In fact, there's no formal definition of trend, which makes trend quantification a delicate, despite common, activity. The wide range of time scales involved in most geophysical problems, and the usually very short period for which reliable data are available, further hinders the identification and quantification of geophysical trends. Here the mathematical aspects of trend assessment in a geophysical context are briefly described. Specific details can be found in Fatichi et al (2009) and Barbosa (2011).

Stochastic models

Different types of stationary and non-stationary processes can originate sequences of observations with trend-like features. Even purely random processes can generate time series exhibiting visually appealing trends, particularly for relatively short records. Some of the most common generating models assumed for geophysical time series are described below.

Autoregressive model

A first order autoregressive process X_t is defined as $X_t = c + \phi X_{t-1} + \varepsilon_t$ with $0 < \phi < 1$, c = constant and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$. It is also called a red noise since its spectrum decreases as frequency increases, similarly to red light in the range of visual radiation. This is a purely random, stationary process, with constant mean and variance, but red-noise time series can exhibit an apparent monotonic temporal structure that can be misleadingly taken as indication of non-stationary behavior.

Trend-stationary model

A trend-stationary process X_t is defined as $X_t = a + bt + \varepsilon_t$ with a, b = constant and $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$. It is a non-stationary process, since the mean evolves in time. This is the model implicitly considered in the majority of geophysical contexts, though the purely linear approximation can be inadequate (e.g. Miranda & Tomé, 2009).

Difference-stationary model

A random walk or difference stationary process X_t is defined as $X_t = c + \phi X_t - 1 + \varepsilon_t$ with $\phi = 1$ and describes a process whose value at a time t is equal to its value at the previous instant plus a random shock, similarly to the path of a drunken man whose position at a given time is its position at the previous time plus a step in a random direction. It corresponds to a 1st order autoregressive process with $\phi = 1$ and is also called an integrated process of order 1, since its 1st derivative is stationary. This is a non-stationary process, since both the mean and the variance evolve in time.

Long-memory model

A process X_t is a stationary long range dependent or long memory process if its autocovariance function γ_X decays as a power law, such that observations widely separated in time can still have a non-negligible covariance: $\lim_{\tau\to\infty} \gamma_X(\tau) = C\tau^{-\alpha-1}$, where C and α are constants satisfying $C > 0$ and $-1 < \alpha < 0$. Long-memory time series are characterised in the time domain by persistent-

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in-time autocorrelations, decaying as a power law, and in the frequency domain by high spectral content at frequency zero.

Trend assessment

A fundamental aspect in the study of geophysical trends is the possible underlying mechanism generating the observed sequence of observations. Though inherently unobtainable, understanding the underlying generating process is the ultimate aim of any trend analysis.

The conventional approach in the study of geophysical trends is to assume a trend-stationary model, estimate the parameters a, b of the regression, and then test the adequacy of the model from the statistical significance of the resulting estimates. However, even if the slope of a linear regression model is statistically significant, the underlying stochastic model may not be a reasonable assumption. In fact, all of the stochastic models mentioned in section 2 are able to generate finite sequences with statistically significant linear trends.

Assessment of whether the monotonic behavior exhibited by a geophysical time series is better characterised by a trend-stationary model, a difference-sttaionary model or a long-memory model has both conceptual and practical implications. For example, while both trend-stationary and difference-stationary time series exhibit a tendency behavior, the former is characterised by a deterministic trend tendency with stable variance, while the latter is characterized by a stochastic tendency with increasing variance. The distinction between the two kinds of nonstationary behavior has not only practical implications (e.g. forecasting) but more importantly on the physical interpretation of the identified trend: in the case of a trend-stationary model the trend can be interpreted as deterministic and due to some forcing factor, while in the case of a difference-stationary model the apparent trend is the result of stochastic fluctuations.

A possible approach to trend assessment is based on parametric statistical tests, developed in econometrics contexts for discriminating between difference-stationary and trend-stationary time series. The PP test (Phillips & Peron, 1988) tests the null hypothesis of a difference stationary random walk process against a trend stationary alternative. It is based on the model $X_t = \eta + \beta_t + \pi X_{t-1} + \psi_t$ where ψ_t is a stationary noise process and η and β are the parameters of a first-order polynomial regression. The null hypothesis is expressed by $H_0 : \pi = 1$ against the alternative $H_1 : \pi < 1$ The KPSS test (Kwiatkowski et al, 1992) tests the null hypothesis of a trend-stationary process against a difference-stationary alternative. The KPSS test is based on the model $X_t = \beta t + r_t + \nu_t$, where r_t is a random walk, $r_t = r_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and ν_t is a stationary noise process.

The joint application of the KPSS and PP allows to assess whether a deterministic linear trend is a reasonable assumption for the data considered. If only the null hypothesis of the KPSS test is rejected, the time series is difference stationary. Then its long-term variability should not be characterized by the slope of a linear regression model (even if it is statistically significant), since the assumption of a deterministic trend is not itself plausible. Conversely, if only the null of the PP test is rejected, the time series is trend-stationary. If both tests reject the respective null hypothesis, alternative behaviors (such as long range dependence) should be considered.

Concluding remarks

The identification of trends is one of the most common activities in geosciences and one with the highest societal implications, since policy makers require information on tendencies to sustain environmental policies, for example in a climate change context. Different kind of stochastic processes can originate finite temporal sequences with visually appealing (and statistically significant!) trends. Trend assessment is therefore a fundamental activity, that can be performed by the joint application of parametric statistical tests of hypothesis.

References

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