The Planet Earth System, a challenge to mathematicians

by **José Francisco Rodrigues***

In 2000 the World Mathematical Year offered an occasion for a collective reflection on the great challenges of the 21st Century, on the role of Mathematics as a key for the development and on the importance of the image of Mathematics in the public understanding. The countless repeated phrase "*the Universe is written in mathematics language*", written by Galileu in 1614, is truer than ever but it raises new challenges in the current age of data-intensive science driven, in particular, by the information and communication technologies. The "rising tide of scientific data" created by the digital revolution provides new possibilities to face some of society's great challenges of energy and water supply, global warming and healthcare.

During the last centuries, Mathematics has developed a "universal method for the study of the systems". In particular, for the Planet Earth System, Jacques-Louis Lions synthesized in his book "*El planeta Tierra. El papel de las matemáticas y de los superordenadores*" (Madrid, 1990) that universal method in three parts: the mathematical modelling; the analysis and the simulation; and the control of the systems.

In 2007 a scientific workshop on "Climate Change: From Global Models to Local Action", organized by the Mathematical Sciences Research Institute, in Berkeley, identified several mathematical research topics that might contribute to resolving problems whose solutions have a large societal impact. From high dimensional systems to model reduction, from multiscale computations to data assimilation, from uncertainty quantification to economics and societal aspects, the areas of mathematics that might have a significant role in those problems vary from dynamical systems and nonlinear differential equations to asymptotic and numerical analysis, from computational science to statistics and operations research, or from stochastic processes to game and control theories.

During the 2010 International Congress of Mathematicians, held in Hyderabad, India, at the meeting of delegates of the International Mathematical Sciences Institutes, Christiane Rousseau has presented an invitation to Institutes and Societies in Mathematical Sciences around the World: *Mathematics of the Planet Earth — 2013* www.mpe2013.org. This initiative, first launched in USA and Canada, has now many partners in Europe and around the world and consists of holding a year of activities in 2013 under that theme. The project is to hold scientific activities, research programmes and activities for the public, the media and the schools.

Some institutions already announced activities or made calls for proposals. For instance, the *Centre de Recerca Matemàtica*, Barcelona, will be organizing in the summer 2012 a special activity entitled "The Mathematics of Biodiversity". The *Centre de recherches mathématiques*, Montréal, is considering to organize a thematic semester on "Biodiversity and Sustainable Development", during the fall of 2013, and to partner with other Canadian institutes in the organization of a program on "Models and Methods in Ecology, Epidemiology and Public health". The Portuguese *Centro Internacional de Matemática* (CIM), in collaboration with two research associates of the University of Lisbon, the *CMAF* and the *Instituto Dom Luiz*, has made a first public session the 6th May 2011 to present the MPE2013 initiative and to call for collaboration and initiatives in Portugal under the theme Mathematics of the Planet Earth. In addition, several mathematical societies, including the European Mathematical Society and the Portuguese Mathematical Society are also planning to participate with initiatives related to that theme.

On the other hand, it has been suggested for the year 2013 the organization of *A Global Exhibition on Mathematics of Planet Earth* of a new type. The proposal is to have an Open Source Exhibition with modules that could be reproduced and utilized by many users around the world from science centres and museums to schools. The realization will not be centralized. It will rather be split among many partners around the world, possibly with collaborative networks of participants. Some coordination by an international committee on exhibits and museums associated to the MPE2013 initiative is under preparation. The exhibition will have a virtual part, as well as several material parts. Copies of the material parts could be recreated or travel around the world and the virtual modules could be available on the basis of creative

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commons licenses. If possible, a global opening coordinated at the same day in all countries could be planned in order to amplify the visibility of the mathematics of the planet Earth. The idea was presented by the CIM director at the annual ERCOM meeting held the 9th April 2011 in the Mathematical Institute of Oberwolfach, and is currently under development.

The Mathematics of the Planet Earth – 2013 initiative will be another great occasion for showing the essential relevance of mathematics in planetary issues at research level for resolving some of the greatest challenges of the 21st Century, as well as at the level of raising the public awareness of mathematics and at the educational and cultural level.

Four themes with potential examples of modules for a virtual exhibition on the

"Mathematics of Planet Earth"

by **Christiane Rousseau***

A planet to discover: *oceans; meteorology and climate; mantle processes, natural resources; celestial mechanics; cartography*.

1.—Crystallography: The crystallographic groups describe the different possible symmetries of the tilings of 3-dimensional space which are invariant under three independent translations. There are applications in the structure of crystals inside rocks. More generally, in chemistry, crystallography is the science of the arrangements of atoms inside a solid. Some arrangements are denser than others and the density of the packings is related to the chemical properties of the chemical elements. On the mathematical side, density of packings is linked with Kepler conjecture on the densest packing of spheres. Different densities can be studied: the densest one, the random density when spheres are packed at random. The same questions can be asked for objects with other shapes than spheres. More recently, mineralogical finding offered evidence that quasicrystals might form naturally under suitable geological conditions.

2.—Fractals provide models for the shapes of nature: Rocky coasts, ferns, the networks of brooks and rivers, for instance deltas. The fractal dimension is a measure of the "density" of a fractal which allows to compare the density of different fractals.

3. The movements of the Earth and the planets in the Solar system: The inner planets (Mercury, Venus, the Earth and Mars) have chaotic motions. Simulations show a 1% chance that Mercury be destabilized and encounters a collision with the Sun or Venus. There is a much smaller chance that all the inner planets be destabilized and that there could be a collision between the Earth and either Venus or Mars in ~3.3Gyr.

4.—The role of the Moon to stabilize the axis of the Earth. If we remove the Moon, then simulations show that the Earth's axis would undergo large oscillations and we would not experience the climates that we now have. In the same spirit there are recent studies making the link between the changes of the parameters of the Earth: angle of the axis, eccentricity of the orbit, etc. and the past climates of the Earth (glaciations periods).

5.—Why the seasons? Why the length of the day is different at different dates, depending of the latitude? Theses themes are very standard. But, in many countries, they disappear from basic science education and needs to be taught independently.

6.—The eclipses. Two types of eclipses: Sun eclipses of Moon eclipses; Explanation of the phenomenon; Previsions of the eclipses.

7.—Weather previsions: The use of models; The butterfly effect.

8.—Remote sensing for exploring the Earth. It could be the use of aerial photographs to discover resources or the use of seismic waves to discover resources in the underground.

9.—Localizing events: Earthquakes, thunderstorms, etc. This is done through triangulation when several distant stations note the time when they register the event.

10.—The Global Positioning System (GPS).

11.—Elements of cartography. It is not possible to draw a map of the Earth respecting ratios of distances.

12.—The use of tools in geography to measure the Earth: How to measure the height of a mountain? How to draw maps of a region?

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