



## An Interview with Sir Michael Atiyah

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Sir Michael Atiyah is one of the most well-known and important mathematicians of the past century. He has had a long and distinguished career, receiving both the Fields Medal in 1966 and the Abel Prize in 2004. Some of his most significant contributions include the Atiyah-Singer Index Theorem and topological K-theory, and he remains active in mathematical research to this day. His planned visit to Portugal to deliver the Pedro Nunes lectures was disrupted by the Icelandic volcano eruption last spring and is now rescheduled for the period between 27 March and 7 April 2011.

You have once said that your father thought you'd be a mathematician when he realized you were making money out of exchanging foreign currency. How/when would you say you first become interested in pursuing mathematics as a career?

I seriously focused on mathematics at the age of 16. When we moved to England in 1945 we selected the school (Manchester Grammar School) that had the reputation as the best in the country for Mathematics.

Could you tell us a bit about your undergraduate/ PhD experience, and what some of the difficulties you encountered were?

I had no difficulties as an undergraduate, coming out top of the university in my first year. As a graduate student I had a bad period at the beginning of my second year, and I took to attending classes in other subjects (architecture and archaeology). But then I got a prize and things improved. Many years later Serre told me he had a similar experience and almost decided to give up mathematics.

## GRADUATE STUDENTS

You had graduate students who became very well-known mathematicians, for example, Simon Donaldson, Nigel Hitchin and Graeme Segal. Do the benefits to a researcher of taking on PhD students outweigh the costs?

The best research students cause least work and are the most rewarding, but others cause much work and there is certainly a limit on how many students one should take on at a time.

What advice would you give to people who are doing a PhD now or planning to do one in the near future?

My advice is that one should not start research in mathematics unless one is dedicated to the subject. One has to have a passion for it in order to surmount the difficulties and disappointments that always arise.

## COLLABORATORS AND OTHER MATHEMATICIANS

You have had some notable collaborators such as Hirzebruch, Singer, and most recently Vafa and Witten. Can you tell us a bit about these collaborations, and how important they were/are to your work? Do you they really help you peer around the corner?

I find collaboration natural and very useful. Another person has a different point of view and usually a different background. Moreover collaboration often develops into friendship and makes research a less solitary undertaking.

Which mathematicians do you most admire/ respect, and why?

Among past mathematicians Hermann Weyl is the one I admire most. The breadth of his interests and the elegance of his style have been my model.

## THE INDEX THEOREM

The Atiyah-Singer index theorem is a landmark of the 20th century mathematics. Could you tell us a bit about how it all begun?

With Gelfand's conjecture that the index of an elliptic operator expressed as topological invariants or with the proof of the integrability of the  $\hat{A}$ -genus? What made you think that the  $\hat{A}$ -genus of a spin manifold was the index of a Dirac operator?

The story of the index theorem is one I have recounted often and I recommend you read the introduction to my collected works volume 3.

There are at least three different proofs of the index theorem, one involving K-theory, one involving the cobordism theory and the other using heat kernels. Why was it so important to the mathematical community to have different proofs? Do they give more insight on the theory?

Different proofs have different merits. They lead to different generalizations and they connect up with other subjects in different ways. One could say that

the number of different proofs of a theorem is one measure of its significance. Gauss is said to have had 8 proofs of the law of quadratic reciprocity.

#### AWARDS

You were awarded the Fields Medal in 1966. How do you think receiving this award affected your career? Could you also tell us about the Abel prize and other awards? What kind of impact did these have in your research?

Awards provide encouragement, but are not the motivation of mathematicians. The Fields Medal, coming when one is young (under 40) has the most effect. Others, such as the Abel Prize, come later in life, are gratifying and compensate for the passage of youth!

#### A BIT OF CONTROVERSY

People often wonder what goes on inside a brilliant mind. Do you agree with the sentence “genius is 1% inspiration and 99% perspiration?”

Brilliant ideas are rather rare and have to be backed up by detailed work, but the ratio of 99 to 1 sounds excessive. It depends on what measure one is using. It also depends on the individual and the style of mathematics.

Do you still have to wear a bulletproof vest because of your comment on the classification of finite simple groups?

I am not afraid to speak my mind, though I may not always be diplomatic.

You are still a very active mathematician. Would you disagree with Hardy’s famous quote that “mathematics is a young man’s game”?

I agree with Hardy that the main advances and innovations in mathematics are made when people are young, but they can continue to produce worthwhile mathematics into later life.



#### INTERACTIONS WITH DIFFERENT AREAS OF MATHEMATICS, PHYSICS AND OTHER SCIENCES

You once said you were a jack of all trades. Would you say the future of mathematics lies in finding bridges between different areas or is mathematics getting so specialized that it will eventually be fragmented?

My personal interest has always been in the interactions between different areas of mathematics (and also physics). This also helps to keep mathematics together. But others have a different style and concentrate on one field. We need all types.

Could you tell us a bit more about your recent work in physics and its interactions with mathematics? Was the time ripe for topological quantum field theory?

The new interaction between topology and quantum physics came as something of a surprise. In retrospect the time seems to have been auspicious, but this is hindsight.

What would you say has been the impact of String theory on Mathematics?

The impact of physics on mathematics has been unexpectedly wide. Almost all branches have been dramatically affected. In fact it can be seen as a revolution.

Is physics the lifeblood of mathematics or is it that physics just happens to be written in the language of mathematics? Can one exist without the other?

Physics and mathematics have a long history in common, and have affected each other. It is not possible to do modern physics without the language and techniques of mathematics. The converse is not true.

Have you ever done any research in other sciences, like Biology or Engineering for example? If so, how did it affect your outlook on Mathematics?

I have a modest interest in neurophysiology (and have been part author of a paper in the field). I have also published papers on historical or philosophical themes.

## POSITIONS HELD

You had different positions in several universities around the world. How do you see the duality between a Professor's duties as a teacher and as a researcher? Do you think that teaching just "gets in the way" or do you find that there is something to be gained by teaching undergraduate students?

My undergraduate teaching career was limited to my early years, but I taught graduate courses for longer and I regard supervision of graduate students as teaching. Too much teaching can wear one down but some teaching is a good discipline and contact with students is essential.

You were involved in the creation of the Isaac Newton Institute for Mathematical Sciences in Cambridge and you were its first director. Are you happy with its development?

I was involved in setting the Newton Institute on its path and in determining its general policy of taking a very wide view of mathematical science. I think it has been a success and has influenced similar institutes in other countries.

You had other administration positions, such as Master of Trinity College. Could you tell us about that experience?

My other main administrative positions have been as Master of Trinity College, President of the Royal Society (of London) and more recently President of the Royal Society of Edinburgh. At a suitable age I felt it was my duty to take on such positions, so as to contribute to society, in return for the privilege I have had of a life mainly devoted to research. They have all been interesting and have widened my experience.

You also contributed to the Foundation of the European Mathematical Society. How important was it for Europe to have a Mathematical Society and how do you think it helps promoting the exchange between mathematicians?

The European Mathematical Society had a slow start but eventually took off at an opportune time, when Europe was getting together. It is now finding its own feet and developing in new directions.

You were also Savilian Professor of Geometry at Oxford. How do you compare Oxford to Cambridge?

To an outsider Oxford and Cambridge seem identical. They have a similar history and are unique establishments in many ways. However they do differ significantly in detail. Cambridge has always been stronger in mathematics and science, with Oxford focusing more on the humanities. But they have converged and these differences are now less marked.

## VISIT TO PORTUGAL

It was very unfortunate that you couldn't deliver the Pedro Nunes lectures in April due to the incident with the volcanic ash cloud. Would this have been your first visit to Portugal? If not, what did you think of the country in your previous visits?

40 years ago I had a family holiday in Portugal, including Lisbon and the Azores, so I was disappointed that the volcano prevented my trip this spring.

When can we expect your visit to be rescheduled?

It will now be rescheduled for next spring.