On GAeL: Géometrie Algébrique en Liberté XVIII

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Abstract

Géometrie Algébrique en Liberté is a school and conference organized by and for algebraic geometers in the beginning of their scientific careers. The 18th edition of GAeL took place in the Mathematics Department of the University of Coimbra, Portugal, on June 7-11 2010. It gathered together about 70 participants coming from whole parts of the world who got the opportunity to learn and discuss together and "en Liberté" the most recent developments in this area of research. The senior speakers for this year were the Professors Olivier Debarre (C.N.R.S.-Paris), Gerard van der Geer (Amsterdam) and Bernd Sturmfels (Berkeley).

Algebraic Geometry

Algebraic Geometry is the branch of mathematics that consists of the study of algebraic varieties: geometric incarnations of solutions of systems of polynomial equations. The simplicity and generality of this idea, that involves a big number of mathematical objects, allowed the concept to grow in many different directions developing an amazingly rich theory. It is nowadays a wide area of mathematics that combines tools from many different disciplines as Abstract Commutative Algebra, Number Theory, Complex Analysis, Differential and Complex Geometry, Algebraic Topology, Category Theory, Homological Algebra, Algebraic Combinatorics and Representation Theory to study problems arising from Geometry. The following words by the fields medalist David Mumford reflect how rich and complex are the ideas and problems that appear in this area of pure mathematics.

Algebraic geometry seems to have acquired the reputation of being esoteric, exclusive, and very abstract, with adherents who are secretly plotting to take over all the rest of mathematics. In one respect this last point is accurate. David Mumford

Even if it is one of the most classical areas of pure mathematics, it is certainly one of the most active as well. The importance of this area of research in the global context of mathematical sciences is clear, for instance, by the strong presence of algebraic geometers among the speakers of the next ICM conference (http://www.icm2010.org.in/about-icm-2010/) that will take place in Hyderabad, India on August 19-27 2010. The great developments of Algebraic Geometry in the last decades are also clear by the list of the field medalist winners: about a quarter of the total number of field medals so far was delivered to algebraic geometers. The field medalists Kodaira, Serre, Atiyah, Grothendieck, Hironaka, Bombieri, Mumford, Deligne, Yau, Donaldson, Faltings, Drinfel'd, Mori, Witten, Kontsevich, Lafforgue and Okounkov got their award by their contributions in algebraic geometry or closely related areas. To these we should also add the name of Andrew Wiles, who got a "Special Tribute" of the Fields Institute for his proof of the famous "Fermat's last theorem" which relies upon methods from Algebraic Geometry, namely elliptic curves and modular forms.

One can say that the roots of Algebraic Geometry date back to the arabian and greek mathematicians, who often used the geometry of plane curves and their intersection properties to solve algebraic equations. The same kind of ideas were also used much later by some mathematicians in the renaissance period like Cardano and Tartaglia while studying the cubic equation. However, after a long period of great development of analytical methods in Geometry, the systematic use of algebraic methods in Geometry was established only at the end of the 19th century by the italian school of Algebraic Geometry composed by mathematicians like Enriques, Chisini, Castelnuovo and Segre. The foundations of Algebraic Geometry using notions from Commutative Algebra like the theory of ideals were established in the beginning of the 20th century by mathematicians like van der Waerden, Oscar Zariski and André Weil. In the 1950's and 1960's Serre and

Grothendieck used sheaf theory and techniques from Homological Algebra to introduce in Algebraic Geometry the notion of scheme. This led to an enormous transformation in the whole theory: classical Projective Geometry was more concerned with the geometric notion of point while the later emphasizes the concepts of regular function and regular map. Moreover, this new point of view provided Algebraic Geometry with tools to treat a wide number of problems from other areas of mathematics like Commutative Algebra and Algebraic Number Theory (recall for instance elliptic curve cryptography and Wiles' celebrated proof of Fermat's last theorem). It also allowed to solve classical problems on algebraic varieties like moduli problems or resolution of singularities. Nowadays, Algebraic Geometry is again in great transformation after the introduction of stacks by Grotendieck, Deligne, Mumford and Artin and, more recently, by the development of the theory of Derived Algebraic Geometry.

Even if Algebraic Geometry is fundamentally a rather abstract discipline, recently it has been heavily used in other rather applied areas of mathematics like statistics, control theory, robotics and also in other sciences. For instance, it is fundamental in the development of the physics' theory of strings and it has deep connections with the Phylogenetics theory of biology.

GAeL origins

The origins of "GAeL: Géometrie Algébrique en Liberté" date back to France, as one can guess by its name. In fact, the first 13th editions of GAeL took place at the "CIRM: Centre International des Rencontres Mathématiques" (Marseille, France). However, nowadays all communication and talks are in English. Most recently, GAeL has taken place in in Bedlewo (Poland), Istanbul (Turkey), Madrid (Spain), and Leiden (The Netherlands). This year edition was the 18th GAeL conference and it took place in the Mathematics Department of the University of Coimbra, Portugal. GAeL has been, since its beginning, a reference meeting for young algebraic geometers specially from European countries.

As the name indicates, the aim of GAeL is to give young algebraic geometers the opportunity to learn and discuss the most recent developments of Algebraic Geometry in a relaxed atmosphere with no fear to ask questions of any type. Young participants have the opportunity to lecture, often for the first time, in front of an international audience. At the same time, senior experts deliver courses in selected topics at the cutting edge of modern and classical Algebraic Geometry. Among the speakers of previous editions of GAeL are Batyrev, Beauville, Bridgeland, Campana, Ciliberto, Colliot-Thélène, Corti, Demailly, Faber, Fantechi, Farkas, Hassett, Huybrechts, Itenberg, Izvoskiy, Lehn, Manivel, Mukai, Muller-Stäch, Mustaţa, Okonek, Oort, Oxbury, Peskine, Reid, Siebert, Sottile, Teissier, Thomas, Tyurin, Vakil, Van Straten, Vistoli, Voisin, and Zak.

Between the senior speakers' mini-courses and the junior speakers talks there is also time for discussions and for exercise sessions. The junior participants who do not give a talk must present their work on the poster session at the beginning of the conference. Posters remain available for the rest of the week so that at the end everybody gets the opportunity to learn about each others own research. For that reason the number of participants of GAeL is quite limited: each person can participate at most in two editions of GAeL.

The organizing committee is also made of young researchers in Algebraic Geometry. Each organizer will organize GAeL for two years: in each edition organizers with some experience help the beginning ones. This year organizing committee was Víctor González Alonso (Universitat Politècnica de Catalunya, Nathan Ilten (Freie Universität, Berlin), Pedro Macias Marques (Universidade de Évora), Margarida Melo (Universidade de Coimbra), Kaisa Taipale (University of Minnesota) and Filippo Viviani (Università Roma Tre and Universidade de Coimbra).

There is also a scientific committee that helps the organizing committee in aspects like the choice of topics or funding advice. The actual scientific committee for GAeL is composed by Professor Frances Kirwan (Oxford), Professor Yuri Manin (Max-Planck Institute für Mathematik) and by Professor Farns Oort (Utrecht).

This year's GAeL

This year edition of GAeL, the XVIII, took place in the Mathematics Department of the University of Coimbra, Portugal, on June 7-11. It was possible thanks to the financial support of the Foundation Compositio Mathematica, the Center for Mathematics of the University of Coimbra (CMUC), the International Center for Mathematics (CIM), the Foundation for Science and Technology of Portugal (FCT), the FCT project "Geometria Algébrica em Portugal" and of the Mathematics Department of the University of Coimbra.

As in the previous editions of GAeL, the program of the conference consisted of 3 mini-courses of 4 hours each delivered by selected experts in different areas of Algebraic Geometry and of 25 contributed talks by the junior participants. The poster session was organized in the first afternoon of the program: we had this year almost 30 posters presented by participants at different stages of their careers. The talks and posters presented

by the junior participants where quite various: there were talks on moduli spaces of curves and sheaves, moduli spaces of surfaces, moduli spaces of abelian varieties, toric geometry, tropical geometry, derived algebraic geometry, arithmetic geometry, deformation theory, singularity theory, minimal resolutions among others. The program of the conference as well as a list of participants and abstracts can be found in http://severian.mit.edu/gael/files/quickschedule.pdf. This diversity of subjects allowed the young participants to learn from their colleagues several recent achievements and ongoing projects in this vast discipline and also favored discussions and questions in a GAeL flavor: always "en Liberté"!

The senior speakers of this edition of GAeL were the Professors Olivier Debarre (http://www.math.ens.fr/~debarre/) from the C.N.R.S. -École Normale Supérieure of Paris, Gerard van der Geer (http://www.science.uva.nl/~geer/) from the University of Amsterdam, The Netherlands, and Bernd Sturmfels (http://math.berkeley.edu/~bernd/) from the University of Berkeley, California.

Professor Olivier Debarre lectured on "Rational curves on algebraic varieties", a very classical argument that had important developments on the last few years yielding important contributions to the recent spectacular advances towards the proof of the minimal model program (see [1]). One of the most classical arguments of algebraic geometry concerns the classification of algebraic varieties. The most classical case is the classification of algebraic curves, which was already understood by Riemann in the 19th century. The classification of algebraic surfaces is more intricate and was one of the biggest achievements of the italian school of the beginning of the 20th century. After several decades without significant progress towards the higher dimensional case, S. Mori proposed a program, the so-called "minimal model program", which would lead to the classification of algebraic varieties in any dimension. The presence of rational curves on algebraic varieties detects several important information on birational invariants of the same varieties (see [4]). Professor Debarre lectures were mainly on the contributions of S. Mori himself and later of J. Kollár on this part of the program. Professor Debarre notes for this course are available at http://www.math.ens.fr/~debarre/NotesGAEL.pdf.

Professor van der Geer delivered a course on "Algebraic cycles on abelian varieties". Abelian varieties are among the most studied objects in Algebraic Geometry due to their extremely interesting and rich properties: they combine the structure of a projective variety with the structure of a compact algebraic group (see, for instance, [2] or [3]). The study of algebraic cycles on varieties is closely related to the famous "Hodge conjecture", which is one of the millennium problems. Moreover, the case of abelian varieties is commonly considered to be a crucial test case towards the validity or disproval of this famous conjecture. Among abelian varieties the so called jacobian varieties are of particular interest. Recent conjectures relate classical aspects of the geometry of algebraic curves, namely the study of their linear series which is the subject of Brill-Noether theory, with the existence of cycles on their associated jacobian varieties. Professor van der Geer lectures culminated in this important aspect of this theory.

Professor Sturmfels' lectures were on a new branch of algebraic geometry that lies on the border line between Algebraic Geometry and Optimization Theory: "Convex algebraic geometry". This new area of mathematics arises from the necessity of studying certain convex objects arising from linear and/or semidefinite programing. These objects seem particularly featured to be studied with tools from algebraic geometry. Professor Sturmfels lectures were centered on several examples that are crucial to understand the foundations of this new discipline in an attempt to systematically study such convex objects. References as well as the slides of Professor Sturmfels lectures are available at http://severian.mit.edu/gael/sturmfels.html.

To relax a bit of such an intense program there were also some social activities scheduled. Monday June 7th there was a visit to "Pátio das Escolas" and the historical buildings of the University of Coimbra. There was also a small reception while the poster session was held. On June 10th afternoon a visit to "Conimbriga" was organized and, after that, the participants were transported to Figueira da Foz were the social dinner was held.

Even if algebraic geometry is a very classical subject of mathematics there were not many portuguese mathematicians working on it until recently. However things are changing rapidly and a proof of this is the fact that there were more then 10 portuguese junior participants. Most of these obtained their PhD in Algebraic Geometry either in Portugal or abroad in the last 3 years. Most algebraic geometers in Portugal participate in the project "Algebraic Geometry in Portugal", run by Margarida Mendes Lopes, that aims to promote a strong portuguese Algebraic Geometry community. Already in this century, there have been several other events in Portugal related to algebraic geometry. among which we recall the "IST courses on algebraic geometry" that are regularly organized by Margarida Mendes Lopes: there were already 5 editions, the "Oporto meeting on Geometry, Topology and Physics", the "Geometry in Lisbon Summer school", the 2010 edition of "VBAC: Vector bundles on algebraic curves", the "Lisbon Summer Lectures in Geometry" in IST (Lisbon) and the "Coimbra-Salamanca Algebraic Geometry seminar", whose first edition was held in Coimbra in February 2010. On the "GA-P: Geometria Algébrica em Portugal" web-

page, http://home.utad.pt/~ga-p/index.html, run by Carlos Rito from UTAD, is possible to find updated information on people working in Algebraic Geometry in Portugal, on events in Algebraic Geometry either in Portugal and abroad as well as several useful links and other information concerning this beautiful discipline, that is creating solid roots in Portugal as well.

Bibliography

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Figure 12: *Groupe Picture,* On GAeL: Géometrie Algébrique en Liberté XVIII.