

Didactics of Mathematics as a Mathematical Discipline:

A Portuguese reflection upon a Kleinean challenge

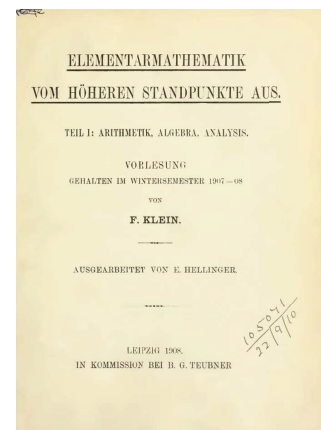
Elfrida Ralha, CMAT, Universidade do Minho
 Jaime Carvalho e Silva, CMUC, Universidade de Coimbra
 José Manuel Castanheira, DME, Universidade da Madeira
 Suzana Nápoles, CMAF, Universidade de Lisboa

The famous German mathematician Felix Klein (Düsseldorf, 25th April, 1849 - Göttingen, 22nd June, 1925) is well known both as a distinguished researcher (non-euclidean geometry, group theory and theory of functions) and as a remarkable teacher. According to Hedrick and Noble⁵, *he combined familiarity with all the fields of mathematics and the ability to perceive the mutual relations of these fields; and he made it his notable function, as a teacher, to acquaint his students with mathematics, not as isolated disciplines, but as an integrated living organism... He endeavored to reduce the gap between the school and the university, to rouse the schools from the lethargy of tradition, to guide the school teaching into directions that would stimulate healthy growth; and also to influence university attitude and teaching toward a recognition of the normal function of the secondary school.*

Klein was appointed professor at Erlangen⁶ in 1872 and he joined the Technische Hochschule, at Munich, in 1875, and the University of Leipzig, in 1880. By 1886 he accepted a chair at the University of Göttingen and there he established a research center which became a role model for the best mathematical research centers all over the world: he promoted weekly discussion meetings, he arranged for a mathematical reading room with a mathematical library and he even managed to give fame to the journal *Mathematische Annalen* and to convince David Hilbert to join in his research center. By then the distinction between pure and applied mathematics began to fade away and the intellectual partnership between Klein and Hilbert proved to be, in Göttingen, a decisive orientation for others in exploring the relationship among mathematics, science in general and technological disciplines. From then on, Klein became also interested in mathematical in-

struction in schools and in 1905 he was to play a decisive role in formulating a plan recommending that the mathematical concept of function as well as the basics of differential and integral calculus should be taught in secondary schools. Many countries around the world gradually implemented this recommendation and, by 1908, Felix Klein became elected the first chairman of the International Commission on Mathematical Instruction (ICMI), at the Rome International Mathematical Congress.

By 1908, Klein's famous notes "Elementarmathematik vom höheren Standpunkte aus" (Tome I) were published, in Leipzig, for the first time.



In this volume dedicated to "Arithmetik, Algebra and Analysis" Klein would then write on its preface that:

The new volume which I herewith offer to the mathematical public, and specially to the teachers of mathematics in our secondary schools, is to be looked upon as a first continuation of the lectures "Über den mathematischen Unterricht an den höheren Schulen"... At that time our concern was with the different ways in which

⁵E. R. Hedrick (Vice President and Provost) and C. A. Noble (Professor of Mathematics, Emeritus), from the University of California, were the translators (from German to English) and the authors of the preface of Klein's books on "Elementary Mathematics from an advanced standpoint".

⁶Felix Klein is particularly linked to his "Erlangen Program" which he first presented, to a restricted audience, when he was appointed as professor for the Faculty of Philosophy and for the Senate of the University of Erlangen, in Germany.

the problem of instruction can be presented to the mathematician. At present my concern is with developments with subject matter of instruction. I shall endeavor to put before the teacher, as well as the maturing student, from the view-point of modern science, but in a matter as simple, stimulating and convincing as possible, both the content and the foundations of the topics of instruction, with due regard for the current methods of teaching.

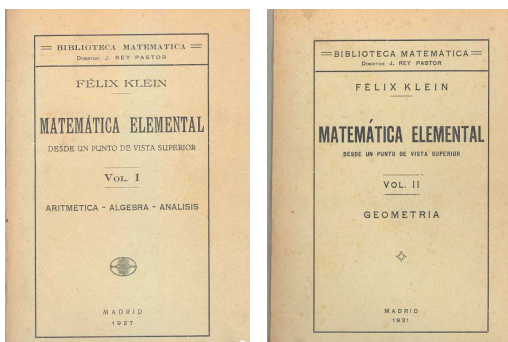
Two other volumes on the same theme of “Elementarmathematik vom höheren Standpunkte aus” were to follow: Part II, on “Geometry” (1909) and Part III, published posthumously by Springer, on “Precision and Approximation Mathematics” (1928).



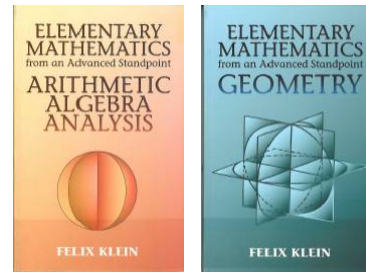
Several German editions of parts I and II were published and for the 3rd edition, Felix Klein, himself, would incorporate more material to the original text, adding the following justification to its preface:

Two volumes on “Elementary Mathematics from an Advanced Standpoint”... were to bring the attention of secondary school teachers of mathematics and science the significance for their professional work of their academic studies... But during the sixteen years which have elapsed since the first publication, science has advanced, and great changes have taken place in our school system, changes which are still in progress. This fact is provided for in the appendices which have been prepared.

Meanwhile several translations of this work were taking place, from which we enhance the Spanish version (Madrid, 1927 and 1931) which was coordinated under the direction of Rey Pastor:



And we also enhance the English version, translated from the 3rd German Edition by E. R. Hedrick and C. A. Noble (Tome I, in 1932, and Tome II, in 1939) and published by The MacMillan Company, New York. By 2004 Dover Publications offered us an unabridged republication of this translation.



On the one hand, we keep in mind that, after one hundred years, this unique work is still a remarkable text for both mathematicians and mathematics educators and we are pleased to acknowledge that the Centro de Álgebra and the Centro de Matemática e Aplicações Fundamentais, from the University of Lisbon, are preparing the first Portuguese translation which is to be presented as the next three issues (the first of them, on *Aritmética*, is now completed) of the collection “Leituras em Matemática”, from Sociedade Portuguesa de Matemática.

LEITURAS EM MATEMÁTICA

MATEMÁTICA ELEMENTAR DE UM PONTO DE VISTA SUPERIOR

Felix Klein

VOLUME I
PARTE I

ARITMÉTICA



spm
SOCIEDADE PORTUGUESA DE MATEMÁTICA

On the other hand if, in the very words of Klein himself, many changes, in our school systems, had occurred in sixteen years, what can we say about such a reality in this new century? A century that has, for example, enthusiastically embraced the so-called Modern Mathematics and, not long after that, has completely banned such teaching principles and mathematical content from our schools.

Wishing to reflect upon some ideas for the teaching and learning of mathematics in the XXI century we share - **through the organization of a Workshop on Didactics of Mathematics as a Mathematics Discipline, next October, at the University of Madeira** - our worries with a specialized panel of mathematicians, historians of mathematics and mathematics educators who celebrating the 100th anniversary of Klein’s publications, in a partnership between IMU

(International Mathematical Union) and ICMI, have decided to re-visit Klein's ideas for, in the words of Michèle Artigue:

(It is intended as) a stimulus for mathematics teachers, so to help them make connections between the mathematics they teach, or can be asked to teach, and the field of mathematics, while taking into account the evolution of this field over the last century.

And, according to Bill Barton:

The objectives are (...) to produce a readable but professional book that conveys the connectedness, growth, relevance, and beauty of the discipline of mathematics from its big ideas to the cutting edge of research and applications.

As referred in the ICMI web site

<http://www.mathunion.org/index.php?id=805>, the Klein Project will have three outputs: a book simultaneously published in several languages, a resource DVD to assist teachers wishing to bring some of the ideas to realisation in their classes, and a Wikipedia-based web-site seen as a vehicle for the many people who will wish to contribute to the project in an on-going way.

Within the prospective languages of the Klein project, a Portuguese version of those outputs from its beginning will serve directly the community of mathematics teachers in Portugal, in Brazil, in Africa or in the rest of the world that use Portuguese as their primary language,

making it the 5th language in the world by number of native speakers.

The Mathematics Education of the present times is, undoubtedly, affected by many factors, some of which Klein himself might had never dreamed of, but the partnership among those who deal with mathematics and with education is, 100 years after Felix Klein defended it, still in need of some debate. For example it might be useful to reflect upon:

- Which "advanced" mathematics is more prone to be introduced in our actual school curricula?
- Which mathematics is, nowadays, more exciting and more alive?
- Which influences, both within mathematics and externally, are we getting in our mathematics classes?
- Which ways do contemporary mathematics educators prefer?
- Within a universal discipline such is mathematics, which educational factors are national and which are international?
- What kind of knowledge can we withdraw from past experiences?

In summary: what are the new mathematical challenges to be faced by our school systems in a global society where, according to MSC (Mathematical Subject Classification), ninety seven categories/mathematical fields (with several thousands of sub-fields) are nowadays to be taken into consideration?

The screenshot shows the ICMI website interface. At the top, there is a search bar and a navigation menu with links for 'About ICMI', 'Conferences', 'Publications', 'Other Activities', and 'Useful Links'. The 'Other Activities' sidebar on the left lists several projects, with 'Klein Project' highlighted. The main content area is titled 'The Klein Project' and contains the following text:

The Klein Project is inspired by Felix Klein's famous book *Elementary Mathematics from an Advanced Standpoint*, published one century ago. It is intended as a stimulus for mathematics teachers, so to help them to make connections between the mathematics they teach, or can be asked to teach, and the field of mathematics, while taking into account the evolution of this field over the last century.

The project will have three outputs: a book simultaneously published in several languages, a resource DVD to assist teachers wishing to bring some of the ideas to realisation in their classes, and a wiki-based web-site seen as a vehicle for the many people who will wish to contribute to the project in an on-going way.

The project is managed by a design team consisting of:

- Michèle Artigue, Université Paris Diderot, France
- Ferdinando Arzarello, Università degli Studi di Torino, Italy
- [Bill Barton](#), The University of Auckland, New Zealand (chair)
- Graeme Cohen, University of Technology, Sydney, Australia
- William (Bill) McCallum, University of Arizona, USA
- Tomas Recio, Universidad de Cantabria, Spain
- Christiane Rousseau, Université de Montréal, Canada
- Hans-Georg Weigand, Universität Würzburg, Germany

The first meeting of the design team will take place at the Université Paris Diderot – Paris 7, France, on May 30 – June 2, 2009, and the first associated conference is planned to be held in Madeira, next October.