Feature Article

How Do Fish-Food Pellets Float?

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Abstract

One growth industry over recent years has been fish farming. Fish are raised in large cages kept within sea inlets, estuaries or lakes. The fish are fed with food pellets which are scattered onto the water above the cages. Ideally the pellets sink within fifteen seconds and can then be eaten by the caged fish. Sometimes there have been problems at fish farms with the fish food floating for too long, whereas in simple laboratory experiments with the same batch of pellets, throwing a handful onto water in a bucket, the pellets were observed to sink quickly. The aim is to understand why.

1 Introduction

An agricultural industry which has expanded rapidly in recent decades has been fish farming. The fish, which are being reared for food, are kept in large cages in inlets from the sea or fresh-water lakes. The fish, in turn, need to be fed and this is commonly done by scattering food pellets, each of which has a shape approximately that of a circular cylinder, onto the surface of the water above the caged fish. Ideally the pellets sink quickly – as might be expected because their density is greater than that of water, although the two densities are comparable. As the pellets sink they can be eaten by the fish. Sometimes, however, batches of pellets have been prone to prolonged floating, allowing them to drift away from the fish cages and/or be eaten by birds. In such cases the pellet manufacturers might run tests with pellets of the same batch, throwing them onto the surface of water in a bucket. The pellets in such tests might proceed to sink rapidly, even if the water used is identical (in terms of dissolved impurities and temperature) to that at the fish farm. It is important to understand the difference between the two cases, and hence to be able to conduct better tests which can more accurately represent what is done in practice – and improve the quality of the manufacturers' products.

One hypothesis has been that the surface tension of the water plays a key role. This short article is aimed at seeing how this effect can help determine whether or not an object, such as a fish-food pellet, will float on the surface of a liquid. For simplicity, and also motivated by the shape of the pellets – circular cylinders with lengths greater than their diameters – the object is imagined to be an infinitely long cylinder with circular cross-section, rather than the true, approximately cylindrical shape (with one concave and one convex end), as sketched in Fig. 1.

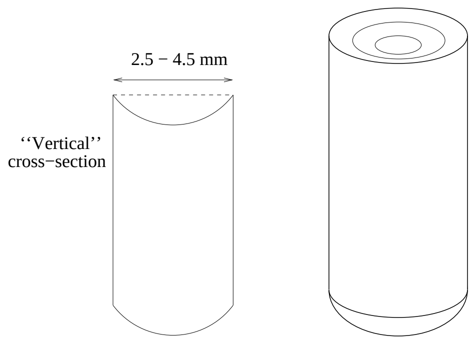

Figure 1: *Schematic diagram of a food pellet.*

It is then possible to to consider a two-dimensional situation, with a circular object floating at the surface of an infinite expanse of liquid (which would occupy a half plane if the object had not been present); see Fig. 2.

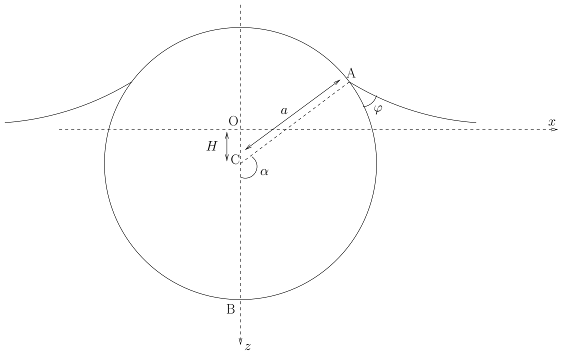

Figure 2: *The two-dimensional case (or a cross-section of a long cylindrical pellet), with a circular object of radius a. Its centre lies a distance H below the undisturbed level of the water while angle α is subtended at the centre, C, by the point on the circle vertically below, B, and the point where the water surface meets the pellet, A.*

We start by writing down the standard equations for the free surface of water, $z = h(x)$, where z is measured vertically downwards from the undisturbed water level, *i.e.* the water surface is $z = h = 0$ at $x = \pm\infty$, and x is the horizontal azis, in the plane of the pellet cross-section, measured from the centre of the circle. (This choice makes things symmetric about the z axis, $x = 0$.) Next the equilibrium conditions for a floating pellet are obtained and used to get equations relating H, the depth of the pellet centre line below the undisturbed surface (so that the circle centre is at C, $(x, z) = (0, H)$), and the points at which the water surface meets the circle. These points can be specified by the angle α between CA, with A the meeting point with $x > 0$, and the downward vertical (see Fig. 2). The **contact angle**, φ in Fig. 2 plays an important role these equations. This particular angle is between the water/air interface and pellet surface, measured through the water. The contact angle depends on the nature of the solid surface: φ will be small for "hydrophilic" materials (it is energetically favourable for the water and the pellet to be in contact) and close to π for "hydrophobic" materials (as would be the case for a waxy or oily substance). Special cases include $\varphi = \pi$, $\varphi = \pi/2$, which for convenience will be particularly looked at here, and $\varphi = 0$, which also deserves comment and corresponds to "wetting" of the pellet surface.

We shall end by considering the implications of the results for floating, and for the experiments.

2 The Model

2.1 Equations for the water surface

Throughout, for simplicity, we shall subtract off atmospheric pressure from all pressures appearing in the model. This means that at large distances the pressure p can be taken to be zero at the water surface, so $p = 0$ at $z = 0$. Because there is equilibrium, pressure is hydrostatic in the water:

$$p = \rho g z \quad \text{for} \quad z > h\,. \tag{2.1}$$

At an interface between two fluids there is a jump in pressure. Here

$$p = p_{\text{water}} - p_{\text{air}} = \sigma\kappa\,, \tag{2.2}$$

where σ is the **surface tension** between water and air and κ is the curvature of the surface, taken to be positive if the surface curves towards the water. To be precise,

$$\kappa = -\frac{\mathrm{d}\theta}{\mathrm{d}s}\,, \tag{2.3}$$

with θ = angle of slope (positive for a surface rising as x increases) and s = distance along the surface (in the $x - z$ plane), so

$$\cos\theta = \frac{\mathrm{d}x}{\mathrm{d}s}\,, \quad \tan\theta = -\frac{\mathrm{d}h}{\mathrm{d}x}\,, \quad \sin\theta = -\frac{\mathrm{d}h}{\mathrm{d}s}\,, \tag{2.4}$$

see Fig. 3.

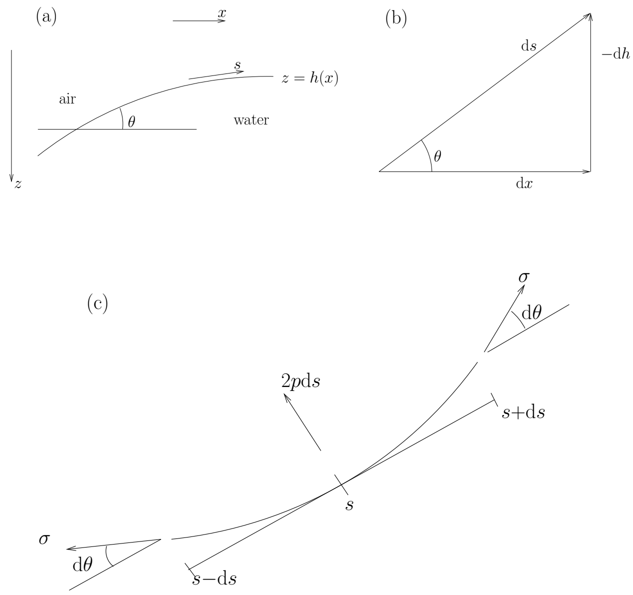

Figure 3: *(a) The water surface away from the pellet. (b) A small element of a curve along the surface in the $x - z$ plane. (c) Forces which must balance for a small section of the surface between $s - \mathrm{d}s$ and $s + \mathrm{d}s$ (shown for a case of $\kappa < 0$).*

These are standard results. Eqn. (2.2) comes from requiring that normal forces balance on a small section of the surface: force from water pressure (towards the air) is equal to

$$\text{pressure} \times \text{length} = 2p\mathrm{d}s,$$

is the same as normal force from surface tension (towards the water), namely

$$-2\sigma \sin \mathrm{d}\theta \approx -2\sigma \mathrm{d}\theta = -2\sigma \frac{\mathrm{d}\theta}{\mathrm{d}s}\mathrm{d}s = 2\sigma\kappa \mathrm{d}s,$$

since $\mathrm{d}\theta$ is small. See Fig. 3(c).

Note that it follows from (2.4) that

$$\kappa \cos\theta = \frac{\mathrm{d}^2 h}{\mathrm{d}s^2}\,. \tag{2.5}$$

Using the known hydrostatic pressure, there is then a differential equation for the depth of the water surface:

$$\begin{aligned} p &= \rho g h &= -\sigma \frac{\mathrm{d}\theta}{\mathrm{d}s} \\ &= -\sigma \frac{\mathrm{d}\theta}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}s} &= \sigma \frac{\mathrm{d}\theta}{\mathrm{d}h}\sin\theta \end{aligned} \tag{2.6}$$

or

$$h\frac{\mathrm{d}h}{\mathrm{d}\theta} = \frac{\sigma}{\rho g}\sin\theta\,. \tag{2.7}$$

The quantity $\sigma/\rho g$ has dimensions of area and we choose to write it as ℓ^2:

$$\ell = \sqrt{\sigma/\rho g} \approx 3 \text{ mm} \tag{2.8}$$

for water with normal gravity. This distance is the length scale characteristic of a water meniscus. Now

$$h\frac{\mathrm{d}h}{\mathrm{d}\theta} = \ell^2 \sin\theta\,. \tag{2.9}$$

(The problem can be made somewhat simpler by scaling. On writing $\hat{h} = h/\ell$, (2.9) becomes $\hat{h}\,\mathrm{d}\hat{h}/\mathrm{d}\theta = \sin\theta$.)

As x and s tend to infinity, $h \to 0$ and the slope θ also goes to zero. The ODE (2.9) is therefore subject to the condition

$$h = 0 \quad \text{at} \quad \theta = 0\,. \tag{2.10}$$

The solution to (2.9) and (2.10) satisfies

$$h^2 = 2\ell^2(1-\cos\theta) = 4\ell^2 \sin^2\frac{\theta}{2}\,. \tag{2.11}$$

Two cases might be considered:

(1) The presence of the pellet raises the water level, so $h < 0$ and $\theta < 0$, in which case (2.11) gives $-h = -2\ell \sin(\theta/2)$.

(2) The presence of the pellet lowers the water level, so $h > 0$ and $\theta > 0$, in which case (2.11) gives $h = 2\ell\sin(\theta/2)$.

We see that in either case

$$h = 2\ell \sin\frac{\theta}{2}\,. \tag{2.12}$$

For more on surface tension and on curvature of curves and of surfaces, see, for example, the books [1], [2] and [3].

2.2 Equations for the pellet position

Referring back to Fig. 2, the depth of the pellet centre, H, pellet radius, a, angle α giving the location, A, of the intersection of the free surface with the pellet boundary, and the local depth of A, say h^*, are related through

$$h^* = H + a\cos\alpha\,. \tag{2.13}$$

Looking at, θ^*, the local angle of slope of the free surface, it is seen that

$$\theta^* = \alpha + \varphi - \pi\,, \tag{2.14}$$

with φ the contact angle. Note that $|\theta^*| > \pi/2$ corresponds to a free surface which turns over; see Fig. 4.

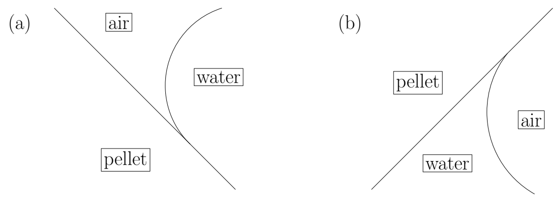

Figure 4: *A water surface which turns over. (a) The case of $h > 0$, so $\pi/2 < \theta^* < \pi$. (b) The case of $h < 0$, so $-\pi < \theta^* < -\pi/2$.*

With h^* and θ^* related through (2.12), (2.13) and (2.14) give

$$\begin{aligned} H &= 2\ell \sin\left(\frac{\alpha+\varphi-\pi}{2}\right) - a\cos\alpha \\ &= -\left(2\ell\cos\left(\frac{\alpha+\varphi}{2}\right) + a\cos\alpha\right)\,. \end{aligned} \tag{2.15}$$

A final equation relating the unknowns H and α is obtained from requiring that the total force on the pellet be zero for it to float in equilibrium. Because of symmetry it is only necessary to look at the vertical components.

The surface tension at the two sides provides an upward force of

$$2\sigma\sin\theta^* = -2\sigma\sin(\alpha+\varphi)\,.$$

There is also an upward buoyancy force from the water pressure of

$$\begin{aligned} \int_{-\alpha}^{\alpha}(p\cos\phi)(a\,\mathrm{d}\phi) &= 2\rho g a\int_0^{\alpha}(H + a\cos\phi)\cos\phi\,\mathrm{d}\phi \\ &= \rho g a\left(2H\sin\alpha + a\left(\alpha + \frac{\sin 2\alpha}{2}\right)\right), \end{aligned}$$

taking a variable of integration ϕ = angle from vertical from the pellet centre to a point on its surface, so that the local depth is $z = H + a\cos\phi$. These two upward forces must balance the weight of the pellet, $\rho_{\mathrm{p}} g \pi a^2$, with ρ_{p} = the pellet's density as πa^2 is the relevant cross-sectional area:

$$\rho_{\mathrm{p}} g \pi a^2 = \rho g a \left(2H\sin\alpha + a\left(\alpha + \frac{\sin 2\alpha}{2}\right)\right) - 2\sigma\sin(\alpha+\varphi). \tag{2.16}$$

Note that the right-hand side of (2.16) should, by Archimedes' principle, be the weight of water displaced, *i.e.* twice that in $0 < z < H + a\cos\phi$ for $0 < \phi < \alpha$ plus twice that in $0 < z < h(x)$ for $x > a\sin\alpha$, thinking of $0 \le \theta^* \le \pi/2$ for simplicity. It is clear that $\rho g \int_0^{a\sin\alpha} (H + a\cos\phi)\,\mathrm{d}x$ gives, on writing $x = a\sin\phi$, the first part of (2.16). The second part comes from

$$\begin{aligned} \sigma\sin\theta^* &= \int_{a\sin\alpha}^{\infty} \left(-\sigma\frac{\mathrm{d}\theta}{\mathrm{d}x}\cos\theta\right)\mathrm{d}x \\ &= \int_{a\sin\alpha}^{\infty} \left(-\sigma\frac{\mathrm{d}\theta}{\mathrm{d}s}\frac{\cos\theta}{\mathrm{d}x/\mathrm{d}s}\right)\mathrm{d}x = \int_{a\sin\alpha}^{\infty} \rho g h\,\mathrm{d}x \end{aligned}$$

from (2.6) and (2.4).

2.3 Negligible surface tension

Taking $\sigma \to 0$, so $\ell \to 0$, (2.15) reduces to $H = -a\cos\alpha$ and floating, (2.16), gives

$$\rho_{\mathrm{p}} = \frac{\rho}{\pi}\left(\alpha - \frac{1}{2}\sin 2\alpha\right). \tag{2.17}$$

This says that, without surface tension, the centre of the pellet is at depth

$$H = -a\cos\alpha \quad \text{with} \quad 0 \le \alpha \le \pi$$

and

$$\rho_{\mathrm{p}} = \rho f(\alpha), \qquad \text{defining } f(\alpha) = \frac{1}{\pi}\left(\alpha - \frac{1}{2}\sin 2\alpha\right). \tag{2.18}$$

Note that

$$\begin{cases} \dfrac{\mathrm{d}f}{\mathrm{d}\alpha} = \dfrac{1}{\pi}(1-\cos 2\alpha) > 0 \text{ for } 0 < \alpha < \pi, \\ f(0) = 0,\ f(\pi/2) = 1/2,\ f(\pi) = 1 \\ 0 < f(\alpha) < 1 \text{ for } 0 < \alpha < \pi, \end{cases} \tag{2.19}$$

see Fig. 5.

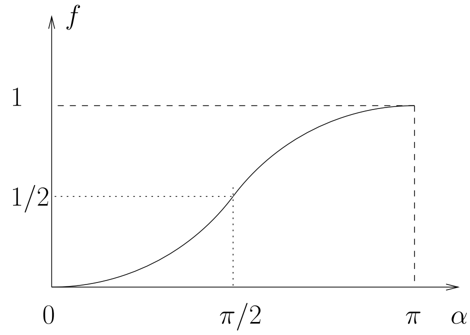

Figure 5: *Graph of the function $f(\alpha)$ (f gives the pellet density in terms of the position angle α in the absence of surface tension).*

Clearly $\mathrm{d}H/\mathrm{d}\alpha = a\sin\alpha > 0$ for $0 < \alpha < \pi$, $H(0) = -a$, $H(\pi/2) = 0$ and $H(\pi) = a$. We get the expected result that as long as the density of the pellet, which must be non-negative, is no greater than that of water, $0 \le \rho_{\mathrm{p}} \le \rho$, the pellet can float. Moreover: an increased pellet density leads to a lower floating position; $\rho_{\mathrm{p}} \to 0$ corresponds to $H \to -a$ (with zero density it sits on the surface of the water like a balloon); $\rho_{\mathrm{p}} \to \rho$ corresponds to $H \to a$ (the pellet becomes totally submerged and just reaches the surface of the water). Fig. 6 shows different cases. Note that, for this case, if, and how, the pellet floats has nothing to do with the pellet size, a.

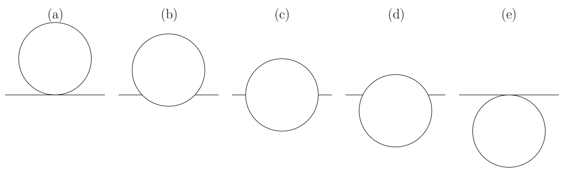

Figure 6: *Floating pellet without surface tension: (a) $\rho_{\mathrm{p}} = 0$ ($\alpha = 0$); (b) $0 < \rho_{\mathrm{p}} < \rho/2$ ($0 < \alpha < \pi/2$); (c) $\rho_{\mathrm{p}} = \rho/2$ ($\alpha = \pi/2$); (d) $\rho/2 < \rho_{\mathrm{p}} < \rho$ ($\pi/2 < \alpha < \pi$); (e) $\rho_{\mathrm{p}} = \rho$ ($\alpha = \pi$).*

3 How Flotation Depends on Surface Tension

With significant surface tension, $\sigma > 0$, the floating position, *i.e.* the angle α, is related to the pellet density through (2.16) and (2.15). These give

$$\begin{aligned} \pi\rho_{\mathrm{p}} g a^2 = {} & \rho g a \left(a\left(\alpha - \frac{\sin 2\alpha}{2}\right) - 4\ell\sin\alpha\cdot\cos\left(\frac{\alpha+\varphi}{2}\right)\right) \\ & -2\sigma\sin(\alpha+\varphi) \end{aligned}$$

which can be rewritten as

$$R = f(\alpha) - \frac{1}{\pi}\left(4L\sin\alpha\cdot\cos\left(\frac{\alpha+\varphi}{2}\right) + 2L^2\sin(\alpha+\varphi)\right), \tag{3.20}$$

where

$$\begin{cases} R & = \dfrac{\rho_\mathrm{p}}{\rho} & \text{is the ratio of the pellet and water densities,} \\ f(\alpha) & = \dfrac{\alpha - \frac{1}{2}\sin 2\alpha}{\pi} & \text{as in (2.18),} \\ L & = \dfrac{\ell}{a} = \dfrac{\sqrt{\sigma/\rho g}}{a} & \text{is the ratio of the length scale of the meniscus to the pellet size.} \end{cases}$$

The dimensionless quantity L can be thought of as a measure of the importance of surface tension. If L is small, we can regard the surface tension as being weak, or the pellet as large. If L is large, the surface tension should be thought of as strong or the pellet as small. (This latter case will be true for insects such as pond skaters which can "walk" on water.) For the pellets of interest, L is likely to be around 1, so surface tension is important but does not dominate.

Floating is now controlled by (3.20). This equation could be solved numerically to find α from known values of the dimensionless quantities L and R, and contact angle φ. Equivalently, for given L and φ, α can be varied from 0 to π to see what density ratios (and hence densities) allow floating; the results of such calculations can be plotted graphically.

It can be noted that for $\alpha = \pi$, $R = 1 + 2L^2 \sin\varphi$ so, at least for $0 < \varphi < \pi$, the presence of surface tension allows pellets with density greater than that of water ($R > 1$) to float. (Intuitively, it is to be expected that a higher contact angle φ makes the pellet more likely to float, in other words higher values of R and ρ_p are possible, as the water attracts the pellet less, or repels it more.) In fact it only makes sense to take $\alpha = \pi$ for $0 < \varphi \leq \pi/2$ because, for $\pi/2 < \varphi \leq \pi$, there will be some position angle α_c, $\pi/2 < \alpha_\mathrm{c} < \pi$, such that the water surfaces on the two sides of the pellet will overhang sufficiently to touch at some point (x, z) with $x = 0$ and $z > 0$, see Fig. 7.

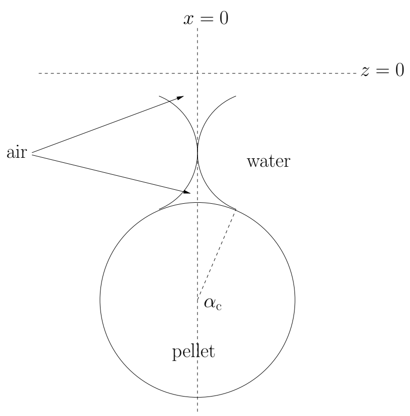

Figure 7: *A critical case, with $\pi/2 < \varphi \leq \pi$, with the free water surface touching itself when position angle $\alpha = \alpha_\mathrm{c}$, $\pi/2 < \alpha_\mathrm{c} < \pi$.*

However, it is clear from Archimedes principle that, since there is air lying below the undisturbed level $z = 0$, R is again greater than 1.

Rather than consider (3.20) in full generality, attention is focused on special cases for simplicity but some graphs illustrating the behaviour of (3.20) are plotted in Sec. 5. One such particular case is when water wets the pellet, $\varphi = 0$. Now

$$R = f(\alpha) - \frac{1}{\pi}\left(4L\sin\alpha\cdot\cos\frac{\alpha}{2} + 2L^2\sin\alpha\right).$$

Since $\sin\alpha\cdot\cos\frac{\alpha}{2} = \sin\frac{\alpha}{2}\cdot\cos^2\frac{\alpha}{2} \geq 0$ for $0 \leq \alpha \leq \pi$, it is apparent that $R \leq f(\alpha)$ and that the maximum floating density is given by $\alpha = \pi$: $R = 1$ and $\rho_\mathrm{p} = \rho$. This should be expected as, for $0 < \alpha < \pi$, *i.e.* $0 < \rho_\mathrm{p} < \rho$, the free-surface slope at the the pellet, θ^*, is negative and surface tension pulls the pellet down (see Fig. 8).

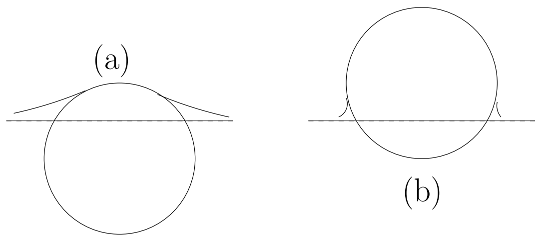

Figure 8: *A floating pellet with contact angle $\varphi = 0$: (a) $\pi/2 < \alpha < \pi$; (b) $0 < \alpha < \pi/2$.*

The other extreme is $\varphi = \pi$, when the water acts to expel the pellet, see Fig. 9.

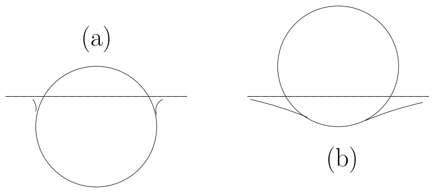

Figure 9: *A floating pellet with contact angle $\varphi = \pi$: (a) $\pi/2 < \alpha < \pi$; (b) $0 < \alpha < \pi/2$.*

Although it is expected, physically, that increasing φ, will increase the maximum pellet density which can be supported, this is only demonstrated here for $0 \leq \varphi \leq \pi/2$; this avoids any difficulty with one part of the free surface touching another part above the pellet, as noted earlier, for $\varphi \geq \pi/2$.

It has already been observed that for $0 < \varphi < \pi$, $R > 1$ for some α: for $0 < \varphi < \pi/2$ there is some α_m with

$$R_\mathrm{m}(\varphi) = \max_{0 \leq \alpha \leq \pi}\{R(\varphi, L, \alpha)\} = R(\varphi, L, \alpha_\mathrm{m}) > 1$$

(clearly $\pi/2 < \alpha_\mathrm{m} < \pi$ here). From (3.20), $R_\mathrm{m} > 1$ requires, since $f(\alpha) \leq 1$,

$$4L\left(\sin\alpha_\mathrm{m} + L\sin\left(\frac{\alpha_\mathrm{m} + \varphi}{2}\right)\right)\cos\left(\frac{\alpha_\mathrm{m} + \varphi)}{2}\right) < 0 \tag{3.21}$$

Because $0 < \alpha_{\mathrm{m}} < \pi$ and $0 < (\alpha_{\mathrm{m}} + \varphi)/2 < \pi$, $\sin \alpha_{\mathrm{m}} > 0$ and $\sin((\alpha_{\mathrm{m}} + \varphi)/2) > 0$, and (3.21) gives $\cos((\alpha_{\mathrm{m}} + \varphi)/2) < 0$ and hence $\alpha_{\mathrm{m}} + \varphi > \pi$. Then

$$\begin{aligned}\frac{\mathrm{d}R_{\mathrm{m}}}{\mathrm{d}\varphi} &= \left.\frac{\partial R_{\mathrm{m}}}{\partial \varphi}\right|_{\alpha=\alpha_{\mathrm{m}}} \\ &= \frac{2L}{\pi}\left(\sin\alpha_{\mathrm{m}} \cdot \sin\left(\frac{\alpha_{\mathrm{m}}+\varphi}{2}\right) - L\cos(\alpha_{\mathrm{m}}+\varphi)\right) > 0,\end{aligned}$$

since $0 < \alpha_{\mathrm{m}} < \pi$, $0 < (\alpha_{\mathrm{m}} + \varphi)/2 < \pi$, and $\pi < \alpha_{\mathrm{m}} + \varphi \leq \alpha_{\mathrm{m}} + \pi/2 < 3\pi/2$.

This means that for a given L, the largest density of pellet which can be supported with $0 \leq \varphi \leq \pi/2$ is that for $\varphi = \pi/2$.

We now concentrate our attention on this special value of $\pi/2$ for the contact angle. This special case avoids any possibility of having self-intersecting water surfaces, either above the pellet (noted above) for $\pi/2 < \varphi \leq \pi$ or below for $0 \leq \varphi < \pi/2$ (which would need a negative density for the pellet!).

4 Right-Angled Contact Angles

With $\varphi = \pi/2$, (3.20) becomes

$$R = f(\alpha) - \frac{2L}{\pi}\left(2\sin\alpha \cdot \cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) + L\cos\alpha\right). \tag{4.22}$$

The change from the surface-tension-free density ratio is then given by the function

$$g(\alpha; L) = 2\sin\alpha \cdot \cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) + L\cos\alpha \tag{4.23}$$

and which is of the form sketched in Fig. 10.

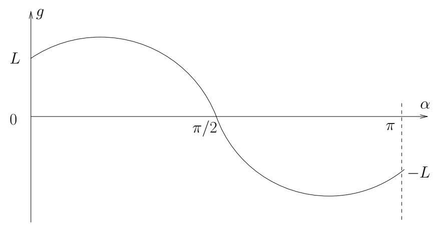

Figure 10: *Graph of g as a function of α.*

Note that:

- $g > 0$ for $0 \leq \alpha < \pi/2$; surface tension pulls the pellet down in this case (Fig. 11(a));
- $g = 0$ for $\alpha = \pi/2$; surface tension has no effect, with the water surface being flat ($h \equiv 0$) (Fig. 11(b));
- $g < 0$ for $\pi/2 < \alpha \leq \pi$; surface tension now pulls the pellet up (Fig. 11(c));
- $\mathrm{d}g/\mathrm{d}\alpha > 0$ for $\alpha = \pi$.

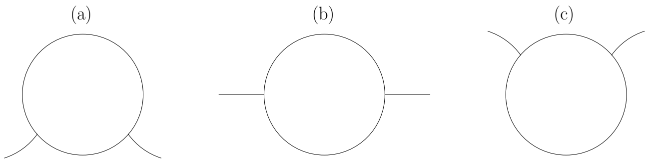

Figure 11: *Sketches of the free surface near the pellet with contact angle $= \pi/2$.*

Then the maximum of R is achieved at some α_{m}, with $\pi/2 < \alpha_{\mathrm{m}} < \pi$, since

$$\begin{cases} R \leq f(\alpha) \leq \dfrac{1}{2} \quad \text{for} \quad 0 \leq \alpha \leq \dfrac{\pi}{2}, \\ \left.\dfrac{\partial R}{\partial \alpha}\right|_{\alpha=\pi} = f'(\pi) - \dfrac{2L}{\pi}\left.\dfrac{\partial g}{\partial \alpha}\right|_{\alpha=\pi} < f'(\pi) = 0 \\ \left.R\right|_{\alpha=\pi} = f(\pi) - \dfrac{2L}{\pi}\left.\dfrac{\partial g}{\partial \alpha}\right|_{\alpha=\pi} < f(\pi) = 1. \end{cases}$$

It is clear that this maximum, R_{m}, has value $R_{\mathrm{m}} = R(\alpha_{\mathrm{m}}) > 1$.

It is easily seen, in this case, that increasing surface tension or decreasing pellet size (making L larger) raises the maximum density which can be supported. For α near α_{m}, $\pi/2 < \alpha < \pi$ so the terms in (4.22) due to surface tension, $\frac{4L}{\pi}\sin\alpha \cdot \left(-\cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)\right) + \frac{2L^2}{\pi}(-\cos\alpha)$ are positive, and are then clearly increasing functions of L. Conversely, decreasing L, which corresponds to increasing the pellet size (or decreasing surface tension) leads to a smaller density of pellet which is able to float.

5 Numerical Calculations of Supported Densities

We conclude by plotting some graphs of pellet density, R, against floating-position angle α.

The first set of graphs, Fig. 12, shows graphs for the case of the normal contact angle, $\varphi = \pi/2$, as discussed in Sec. 4, for different values of L. Note how the maximum density ratio R increases with dimensionless surface tension L. The dashed line in Fig. 12 is a blow-up of the zero-surface-tension case. In this figure, and others below, negative densities are indicated in places. These are not of physical interest here and would correspond to a pellet being pulled up by some external force.

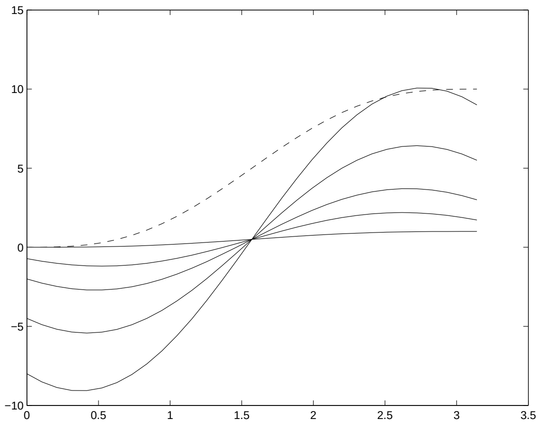

Figure 12: *Plots of the floating density ratio, R, (vertical axis) vs. position angle, α, (horizontal axis) for contact angle* $\varphi = \pi/2$. *Values of dimensionless surface tension, L, are* 0, 0.6, 1, 1.5 *and* 2 *(increasing L corresponds to increasing amplitude). The broken curve shows* $10R$ *for* $L = 0$.

The second set of graphs, Fig. 13, again shows graphs of the density ratio, but taking five different values of contact angle with a fixed value of $L = 1$. Because of having a self-intersecting water surface, below or above the pellet, the extreme left-hand parts of curves 1 and 2 (contact angle = 0 and $\pi/4$) and the extreme right-hand parts of curves 4 and 5 (contact angle = $3\pi/4$ and π) have no physical significance. Note that, with the exception of the zero-contact-angle case, curve 1, $\varphi = 0$, the maximum of the density, *i.e.* the maximum of R, is raised above 1 by the effect of the surface tension. We can also observe that this maximum floating density is raised by having an increased contact angle.

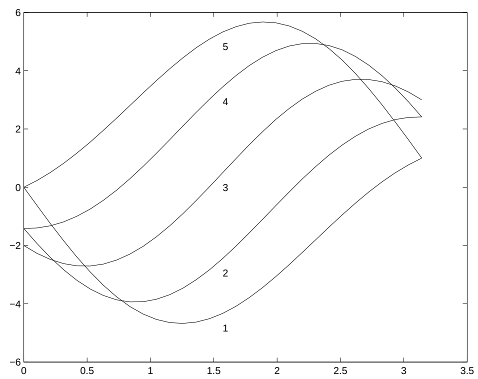

Figure 13: *Plots of the floating density ratio, R, (vertical axis) vs. position angle, α, (horizontal axis) for dimensionless surface tension* $L = 1$. *Values of contact angle* φ *are* 0 *(curve 1)*, $\pi/4$ *(curve 2)*, $\pi/2$ *(curve 3)*, $3\pi/4$ *(curve 4) and* π *(curve 5).*

The final pair of graphs, Fig. 14, show how surface tension affects pellet buoyancy for the two special cases of $\varphi = 0$ and $\varphi = \pi$. The left-hand graphs show how with $L = 1$ the pellet is pulled down when wetting occurs ($\varphi = 0$): R for $L = 1$ is below that for $L = 0$ (zero surface tension) and the maximum floating density ratio is $R = 1$ for both cases. The right-hand graphs show how with $L = 1$ the pellet is pushed up for a hydrophobic surface ($\varphi = \pi$): R for $L = 1$ is above that for $L = 0$ (zero surface tension) and the maximum floating density ratio for $L = 1$ is now much bigger than 1.

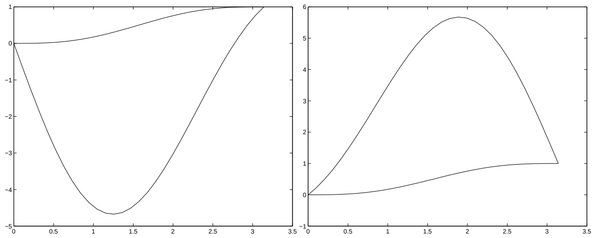

Figure 14: *Plots of the floating density ratio, R, (vertical axis) vs. position angle, α, (horizontal axis) for contact angle* = 0 *(left-hand graphs) and contact angle* = π *(right-hand graphs). In the left-hand graphs the upper curve is for* $L = 0$ *and the lower one is* $L = 1$. *In the right-hand graphs the lower curve is for* $L = 0$ *and the upper one is* $L = 1$.

6 Consequences and Discussion

The results of Secs. 4 and 5 indicate that surface tension can play an important role helping objects which are heavier than water to float. To ensure that pellets of size of a few millimetres sink, if their density is comparable with but larger than that of water, their composition would ideally be such that the contact angle is small. Of course control of contact angle is unlikely to be practical as the make-up of the pellets will be determined by the nature of the fish food.

The significance of surface tension is also suggested by the sizes of quantities involved. Typically, with water density $\rho = 10^3\,\mathrm{kg\,m^{-3}}$, acceleration due to gravity $g = 10\,\mathrm{m\,s^{-2}}$, surface tension $\sigma = 7\times10^{-2}\,\mathrm{N\,m^{-1}}$, and a pellet radius $a = 3\times10^{-3}\,\mathrm{m}$, the dimensionless surface tension is $L \approx 1$. Note that a pellet can be expected to have density fairly close to, but rather greater than, that of water, say $R = \rho_\mathrm{p}/\rho \approx 1.2$. Fig. 13 indicates that for such a pellet to sink its contact angle should be significantly less than $\pi/4$.

The results show (see Fig. 12) that the maximum density depends crucially upon the dimensionless surface tension: small objects can float with greater density than larger ones. This indicates a possible reason for problems with testing the floatation of pellets in small bodies of water, such as buckets, compared with their use on large expanses. If several pellets are put together onto a water surface they can act like a single large object, giving a small value of L and hence a good chance of sinking. With widely scattered pellets, each acts a single individual, with a large value of L, and they can be prone to floating.

Of course all the work here has been for the effectively two-dimensional case of a long cylindrical object. However, the same qualitative behaviour is to be expected for more general shapes: the larger the body (and the smaller the contact angle), the smaller the density has to be for it to float. It will be possible to do similar calculations as here, and to get to the same qualitative conclusions, for spherical pellets when there is axial symmetry about the vertical (z) axis.

References

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CIM Early History

The near origins of CIM can be traced to the end of 1990 at the foundation of the European Mathematical Society (EMS) and to an initiative of the Portuguese Mathematical Society (SPM, Sociedade Portuguesa de Matemática), which was one of the EMS founding members.

Since then, the need of a forum of European Research Centres in the Mathematical Sciences was recognized and the SPM had promoted the idea of creating a Portuguese Center. In particular, it would have the aim to cooperate with similar centres and to enhance the development and promotion of research in Mathematics in Portugal, as well as to assist mathematicians in developing countries, priority being given to the Portuguese speaking countries in Africa (Angola, Mozambique, Cape Verde, Guinea-Bissau, São Tomé and Príncipe).

Several mathematicians gave their personal and institutional support to the idea, such as J. M. Lemaire, at the time director of the CIMPA from Nice (France) who came to Portugal for a visit in 1991, Angelo Marzollo from UNESCO, and F. Hirzbruch, the first president of the EMS, who expressed his support on behalf of the Society.

During 1992 a national discussion took place among the Portuguese mathematical community and the Department of Mathematics of the University of Coimbra offered to house the future Centre on the campus of its Astronomical Observatory. Delegates from the Mathematics Departments of all public Portuguese Universities, the president of the Portuguese Mathematical Society and a representative of the Academy of Sciences of Lisbon were invited to participate in the constitutive meetings. Indeed almost all of them had participated in the two meetings that have created the consensus that the new Centre should promote activities to encourage the development of Mathematical Sciences in general and to foster international cooperation, as well as to help the improvement of the level of Mathematics and its Applications in Portugal.

CIM was legally incorporated on December 3, 1993. Until the election of its first direction, on July 1996, CIM was run by an organizing committee formed by the president of the Portuguese Mathematical Society and other mathematicians from the Universities of Coimbra, Lisbon, Porto and Minho. Since then CIM is managed by a Board of Directors elected by the associates in the General Assembly.

CIM started to publish its Bulletin in December 1996, the first meetings were organised in the following year and the first Thematic Term was held in 1998. It has been regularly in operation as can be seen in the list of events, in particular, with sponsorships from the Calouste Gulbenkian Foundation and from the Portuguese Foundation for Science and Technology. In March 2008, CIM has hosted the annual ERCOM meeting in Coimbra, Portugal.