Math in the Media

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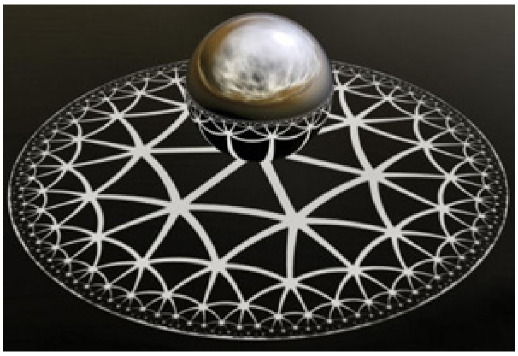
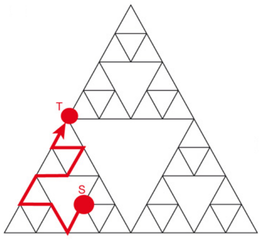
Cosmic geometries.



The cover art from *Science News*, November 17, 2007. Design by Anders Sandberg (Future of Humanity Institute, Oxford), used with permission.

This elegant image means to illustrate "the link between laws of physics as they are perceived in universes with different geometries, even different numbers of dimensions" (from the caption in *Science News Online*, at www.sciencenews.org/articles/20071117/bob9.asp). The accompanying article, by David Castelvecchi, sketches some recent developments related to Juan Maldacena's 1997 ideas about string-particle duality: "Just as a hologram creates the illusion of the third dimension by scattering light off a 2-D surface, gravity and the however many dimensions of space could be a higher-dimensional projection of a drama playing out in a flatter world." Castelvecchi quotes Maldacena to the effect that recently "very strong evidence" has been found that the conjecture is true. But then we read: "Unfortunately, the equations ... seem a good match only for the mathematics of strings living in a contracting universe." So what about this universe here? A semi-theological argument has it that "It would be too much of a coincidence ... if such a seemingly miraculous mathematical duality were to apply to a particular kind of abstract universe but not to our own." On the other hand Abhay Ashketar (Penn State) reminds us, as Castelvecchi puts it, that "In the 1860s, Kelvin pointed out that many of the known properties of chemical elements could arise naturally if atoms were knotted vortices in the fabric of the ether. The uncanny coincidence went away once physicists demonstrated that the ether probably didn't exist."

First encounters in strange places.



Candamin *et al.* give the Sierpinski gasket as an example of the kind of fractal for which they can compute the mean first-passage time from one point S to another T. A typical random path is shown. Image reprinted by permission from McMillan Publishers Ltd: *Nature* (Vol. 450, 1 November 2007, p. 77), copyright (2008).

"First-passage times in complex scale-invariant media" by a team (S. Candamin, O. Bénichou, V. Tejedor, R. Voituriez, J. Klafter) at Paris-VI and Tel-Aviv University appears in the November 1, 2007 *Nature*. It leads off with the definition of first-passage time (FPT): "How long does it take a random walker to reach a given target point?" and continues: "Our analytical approach provides a universal scaling dependence of the mean FPT on both the volume of the confining domain and the source-target distance." In all cases the mean FPT $\langle T \rangle$ from point S to point T scales linearly with the volume N of the medium; and scales with a power of the distance r from S to T, according to the relative size of the "walk dimension" d_w and the fractal dimension d_f. The walk dimension is defined so that the first time a random walk reaches a point at distance r from its start scales as r^{d_w}; the fractal dimension so that the number of sites within a sphere of radius r scales as r^{d_f}. For the Sierpinski gasket illustrated above, $d_f = \ln 3/\ln 2$ and $d_w = \ln 5/\ln 2$ so we are in the $d_f < d_w$ regime for which their general result gives $\langle T \rangle$ scaling as $r^{d_w - d_f}$; a prediction the authors buttress with numerical simulations. In an interview with *Nature*, Bénichou explains how his team worked around the problem of boundary conditions: "... we use a mathematical trick to isolate and replace the confinement effect. Then, we relate the mean FPT in confined conditions to properties of random walks in infinite space, which are easier to estimate." Nature also published an appraisal of this work, by M. Shlesinger, in their "News and Views" section.

Euclid in China, in 1607.

400 years ago, the first six volumes of Euclid's *Elements* were published in China, in Chinese. Last October the Partner Institute for Computational Biology (Shanghai) marked the anniversary with a meeting, reported on by Richard Stone under the title "Scientists Fete China's Supreme Polymath" (*Science*, November 2, 2007). Stone is referring to Xu Guangqi, a prominent Ming-dynasty scholar/administrator, who along with the Jesuit missionary Matteo Ricci carried out the translation.

Matteo Ricci and Xu Guangqi, from Kircher's *China Illustrata* (1667). Athanasius Kircher was a Jesuit colleague of Ricci's; the image evokes Xu's conversion to Catholicism.

Xu's long career spanned agriculture ("His experiments in Shanghai with yams, then a new import from South America, led to the widespread adoption of the high-energy crop."), weaponry ("Xu also trained imperial soldiers to use a newfangled device from Europe, the cannon.") and calendar reform. His most lasting contribution may have been the vocabulary he and Ricci developed for their translation. They chose the characters *ji he* for "geometry," as well as the Chinese terms for "point," "line," "parallel," etc. which remain in use today.

How complex is mathematics?

Richard Foote (University of Vermont) has a review article, "Mathematics and Complex Systems," in the October 19, 2007 *Science*. His goal is to analyze mathematics itself as a complex system. (There is in fact no exact and generally accepted definition of "complex systems," but they are usually characterized as a) made up of many interconnected elements and b) expressing emergent behaviors that require analysis at a higher level that that appropriate for the component elements. The standard example is the brain, with neurons as its component elements, and consciousness as emergent behavior.) Foote proposes "that areas of mathematics, even ones based on simple axiomatic foundations, have discernible layers, entirely unexpected 'macroscopic' outcomes, and both mathematical and physical ramifications profoundly beyond their historical beginnings."

The area he chooses to examine in detail is Finite Group Theory: he gives the axioms, defines a *simple group*, and studies the history of the classification problem for finite simple groups as one might study the evolution of a life-form, emphasizing the points where the theory underwent a transformation comparable to an emergent behavior. He distinguishes three epochs:

• From Galois to the early 1960s. It was understood how any finite group could be (essentially uniquely) decomposed into simple groups; the classification of simple groups was underway. There were 18 (infinite) families of finite simple groups and in addition 5 "sporadic" finite simple groups belonging to no family.

• The Feit-Thompson Odd Order Theorem (1962; *the only odd-order simple groups are the cyclic groups of order* > 2) was, according to Foote, "a breakthrough to the next level of complexity." Their huge paper "spawned the first 'quantum jump' in technical virtuosity that practitioners would need in order to surmount problems in this arena." The road to classification was not smooth: a sixth sporadic group was discovered in 1965, 20 more surfaced during the next few years, but by 1980 the enormous project was done.

• "The Monster and Moonshine." The Monster (the king of the sporadics, with some 10^{54} elements) is "the nexus of a new level of complexity." Starting in 1978, "striking coincidences," mysterious enough to merit the appellation Moonshine, were discovered between the structure of the Monster and the classical theory of modular functions. Finding a basis for this correspondence led to a Fields Medal for Richard Borcherds in 1998; the new level of complexity comes from the string theory methods used in Borcherds' work. These directly connect Moonshine to current research, often mathematically problematical, in theoretical physics.

Foote concludes by remarking: "... the work of scientists is inherently incremental and precise. On the other hand, it is incumbent on us all to work toward enhancing the understanding of 'big picture' issues within our own disciplines and beyond."

Hardy and Ramanujan - the novel.

Last September saw the publication of *The Indian Clerk*, David Leavitt's novelistic imagining of the Hardy-Ramanujan story. Nell Freudenberg's very positive review of *The Indian Clerk* took the front page of the *New York Times Book Review* for September 16, 2007. As she explains it, the genre here is "a novel about people who really existed, recreated by an author who plays with the facts, and especially the intriguing lacunae, of their lives." Leavitt is a specialist in gay-themed intellectual history, and this book seems to be no exception. "Hardy was a member of the Cambridge Apostles, an illustrious secret society that counted Bertrand Russell, G. E. Moore, John Maynard Keynes and Lytton Strachey among its members. Many of the Apostles were homosexuals," as, we are given to understand, was Hardy himself. "Leavitt has been praised and condemned for the explicit sex in his fiction," Freudenberg tells us. But rest assured, readers: whatever bodice-ripping (or the equivalent) takes place in the novel, it will not involve our two protagonists. As Freudenberg puts it: "... what he makes of their relationship is much more subtle than a love affair. Initially frustrated by the young genius's tendency to pursue several ideas in an associative fashion, Hardy eventually realizes he has come in contact with a mind that expands his notion of their discipline."

Math: Gift from God or Work of Man?

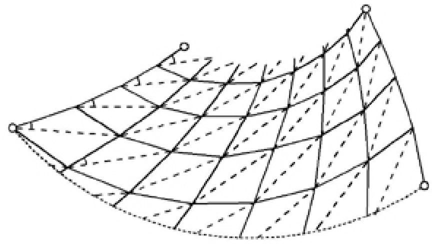
This is John Allen Paulos' column, posted September 2, 2007 on the ABC news website abcnews.go.com/Technology/WhosCounting/story?id=3543453&page=1; the subtitle: "Mathematics, Religion and Evolution in School Curricula." The insertion of religion into science courses (under the guise of "intelligent design," etc.) has now begun to spread to mathematics. So far, it does not seem too worrisome. Most of the examples Paulos shows us are merely peculiar: a standard mathematics curriculum with clumsily interpolated references to a higher being. "The study of the basics of geometry through making and testing conjectures regarding mathematical and real-world patterns will allow the students to understand the absolute consistency of God as seen in the geometric principles he created." (Many of us have done worse in trying to justify pure mathematical research to federal funding agencies). The staff at Maharishi University are more creative: "Infinity: From the Empty Set to the Boundless Universe of All Sets – Exploring the Full Range of Mathematics and Seeing its Source in Your Self." Still OK, as long as that Boundless Universe is not itself a set.

Next we take on the transcendentalists in our midst; like Eugene Wigner who believes, Paulos tells us, that the "ability of mathematics to describe and predict the physical world is no accident, but rather is evidence of a deep and mysterious harmony." For these people Paulos has a nice statement of the natural history of mathematics:

"The universe acts on us, we adapt to it, and the notions that we develop as a result, including the mathematical ones, are in a sense taught us by the universe. ... evolution has selected those of our ancestors (both human and not) whose behavior and thought are consistent with the workings of the universe. The usefulness of mathematics is thus not so unreasonable."

Origami pinecones.

Nature, on July 26, 2007, ran a "News and Views" piece (www.nature.com/nature/journal/v448/n7152/edsumm/e070726-05.html) by Ian Stewart about a new breed of mathematically inspired origami. Stewart begins by reminding us of the mathematical complexity hidden in this ancient Japanese art. "The basic problem of origami is the flat-folding problem: given a diagram of fold lines on a flat sheet of paper, can the paper be folded into a flat shape without introducing any further creases? ... [T]his question is ... an example of an NP-hard problem." Taketoshi Nojima (Department of Aeronautics and Astronautics, Kyoto) has recently published a series of papers where, among other things, he shows how to crease a sheet of paper so that it folds flat, but can also be uncompressed into a conical structure presenting equiangular spirals analogous to those produced by phyllotaxis. For example, the following fold diagram, with the dotted lines interpreted as "ridges" and the solid lines as "valleys," gives a flat object which, after stretching to bring the opposite vertical edges into coincidence, produces a cone:



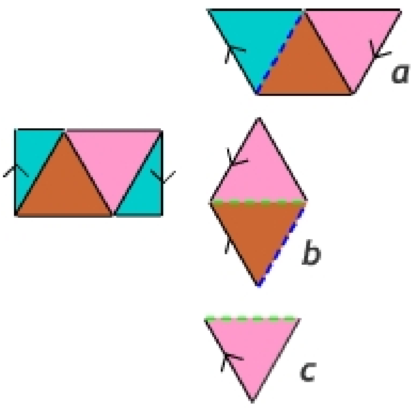
Folding diagram for origami pinecone. The angles and lengths are carefully calculated so as to satisfy the local flat folding criterion (around each vertex, the sum of every other angle must be π), to ensure that the edges and the diagonals form piecewise equiangular spirals (with respect to an origin at the center of the circle implied by the lower dotted edge), and finally to ensure that the free vertical edges match up properly. Image by Taketoshi Nojima (Origami Modelling of Functional Structures based on Organic Patterns, impact.kuaero.kyoto-u.ac.jp/pdf/Origami.pdf), used with permission.

This cone was assembled from the diagram above, enlarged by a factor of 3. Image by Taketoshi Nojima, used with permission.

Note that unlike the cones produced by phyllotaxis, this one has all three sets of equiangular spirals turning in the same direction. More "natural" configurations are also possible (see "Origami-Modellings of Foldable Conical Shells Consisting of Spiral Fold Lines," by Nojima and Takeuki Kamei, *Trans. JSME* 68 (2002) 297-302, in Japanese).

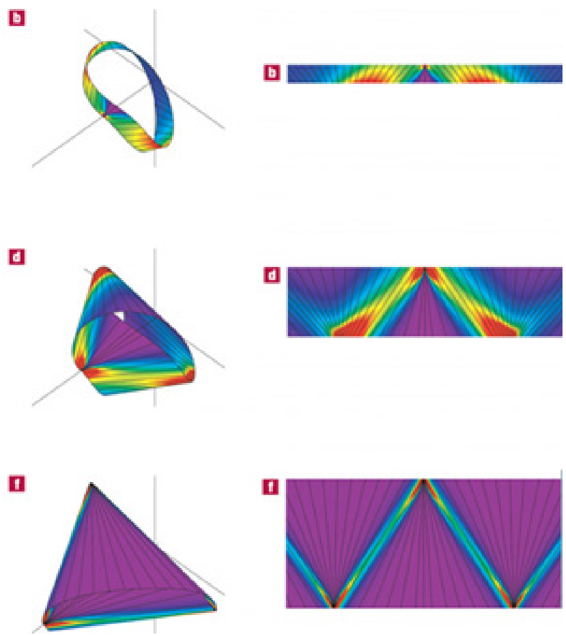
"A Twist on the Möbius Band".

That's the title that Julie J. Rehnmayer used for her *Science News Online* report (www.sciencenews.org/articles/20070728/mathtrek.asp) on recent answers to the question: when an inelastic rectangle (for example, a strip of paper) is twisted into a Möbius band in 3-dimensional space, what exactly is the resulting shape?



Any rectangle with side ratio $\sqrt{3}$ to 1 can be folded into a Möbius Band by reassembling it as a trapezoid (a), folding along the blue dotted line (b), and then folding along the green. The last fold (c) brings into congruence, with proper orientation, the sides that are to be identified. This configuration is the limiting case of the embeddings studied by Starostin and Van der Heijden.

When the side ratio is $\sqrt{3}$ to 1, the strip can be folded into a configuration that respects the edge identification. For narrower strips the band assumes a "characteristic shape" minimizing the total bending energy; the exact determination of this shape has been an outstanding problem at least since 1930. Evgueni Starostin and Gert van der Heijden (University College London) recently nailed down the solution using "the invariant variational bicomplex formalism" and numerical methods. (The variational bicomplex is, according to Ian Anderson — see www.math.usu.edu/~fg_mp/Publications/VB/vb.pdf—, a double complex of differential forms defined on the infinite jet bundle of any fibered manifold $n : E \to M$.) They report: "Solutions for increasing width show the formation of creases bounding nearly flat triangular regions ...".



Three of six characteristic shapes for length $= 2\pi$ and various widths shown in Srarostin and Van der Heijden's article: width 0.2 (b), 0.8 (d) and 1.5 (f). "The colouring changes according to the local bending energy density, from violet for regions of low bending to red for regions of high bending." Image reprinted by permission from MacMillan Publishers Ltd: *Nature Materials* (Vol. 6, 15 July 2007, p. 563), copyright (2008).

The Myth. The Math. The Sex.

"Everyone knows men are more promiscuous by nature." That's how Gina Kolata starts her piece "The Myth. The Math. The Sex." in the New York Times for August 12, 2007. We even have darwinian explanations for the phenomenon, with woman being "genetically programmed to want just one man who will stick with her and help raise their children." Surveys bear this out: Kolata mentions a British study which "stated that men averaged 12.7 heterosexual partners in their lifetimes and women, 6.5." Whoa! It turns out this is mathematically impossible. Kolata refers to David Gale, who sanitizes the context and gives us the

High School Prom Theorem: *We suppose that on the day after the prom, each girl is asked to give the number of boys she danced with. These numbers are then added up, giving a number G. The same information is then obtained from the boys, giving a number B.*

Theorem: $G = B$.

Proof: Both G and B are equal to C, the number of couples who danced together at the prom. Q.E.D.

If the numbers of men and women in the active heterosexual population are the same, as they approximately seem to be, the HSP Theorem does indeed imply that the average number of partners must be the same for both sexes. This should settle the matter. But Kolata makes the error of mentioning one study which reported an (almost identical) difference in the *medians* of the two distributions; this earns her a rebuke from Jordan Ellenberg, *Slate*'s math guru: "Mean Girls: The *New York Times* slips up on sexual math" (August 13, 2007, slate.com/id/2172186). "It's not every day I get to read a mathematical theorem in the *New York Times*, so I hate to complain. But Kolata isn't quite right here." Ellenberg goes on to give obvious examples of different medians with the same mean. Towards the end of the piece he acknowledges that some of Kolata's examples did in fact involve *means*; he changes tack and quotes serious studies of the problem of inaccurate self-reporting (unreliable memory plays a part). Kolata's essay is available online, thanks to the *Dallas Morning News*.

Geometry and the Imagination.

The 5-day conference with this title, held at Princeton on June 7-11 in honor of Bill Thurston's 60th birthday, was surveyed by Barry Cipra in the July 6, 2007 *Science*.

Bill Thurston (Princeton, March 1990).

Cipra's 2-page spread covers four of the presentations:

• **The smallest hyperbolic manifold.** In hyperbolic geometry, similar triangles must have the same area, and each hyperbolic manifold has its own specific volume. In the 1970s, Cipra tells us, "Thurston ... proved a surprising property of hyperbolic manifolds. Given any infinite collection of such manifolds, one member of the collection will be of smallest volume." In particular, one hyperbolic 3-manifold must have the smallest volume of all. A candidate was discovered shortly thereafter, by Jeff Weeks. The "Weeks manifold" remained for a long time the smallest hyperbolic 3-manifold known; only this year did David Gabai, Robert Meyerhoff and Peter Milley prove that there can be no smaller. Their work was posted on arXiv May 30, 2007 (www.arxiv.org/abs/0705.4325).

• **Infinite trajectories in outer billiards.** Outer billiards was devised as a simple analogue of planetary motion. "An object starting at point x_0 outside some convex figure zips along a straight line just touching the figure to a new point x_1 at the same distance from the point of contact. It then repeats this over and over, thereby orbiting the figure in, say, a clockwise fashion." (Cipra). Are all such orbits bounded, or for some figure and some x_0 could the x_i wind up arbitrarily far away? The question had been open since the 1950s, but a set of unbounded examples was recently discovered by Richard Schwartz. The convex body he uses is the *kite* from Penrose tilings, and he exhibits "larger and larger clouds of smaller and smaller regions" converging to "a set of points from which the trajectories are unbounded." Details at Rich's website (www.math.brown.edu/~res).

• **Crossing number of the sum of two knots.** It is known that knots can't cancel. But how about partial simplification? "... if two knots are strung together to form one larger, more complicated knot, can the new knot be redrawn with fewer crossings than the original two knots combined?" Cipra quotes Colin Adams: "This problem has been out there forever." Some recent progress towards proving that the minimal crossing number $c(K_1\#K_2)$ of the knot sum is the sum $c(K_1) + c(K_2)$ of those of the addends was reported by Mark Lakenby, who showed that $c(K_1\#K_2)$ is at least $(1/281)[c(K_1) + c(K_2)]$. Cipra: "The basic idea is to think of each knot as enclosed in a spherical bubble and then carefully analyze what must happen to the bubbles if the knot sum is twisted into a new shape with fewer crossings." He remarks, "To prove the full conjecture, mathematicians will need to whittle this number [281] all the way down to one."

• **Update on the Poincaré conjecture.** "Pricey Proof Keeps Gaining Support" is Cipra's heading for his report on John Morgan's overview of Perelman's proof. "After poring over Perelman's papers for 4 years, topologists are confident of the result. ... Much of the confidence derives from alternative proofs researchers have devised in the wake of Perelman's work." Cipra quotes Thurston at the conference banquet: "I never doubted it would be proved. It's really wonderful to see the community ownership of this mathematics."

An Interview with F. William Lawvere - Part One

This is the first part of a conversation with F. W. Lawvere, that took place in Braga on the 28th of March 2007, during the Workshop "Applied and Computational Category Theory", a satellite event of the ETAPS 2007 Conference, and continued in June, in Carvoeiro (Algarve), during the Category Theory 2007 Conference — that celebrated the 70th birthday of F. W. Lawvere. The second part of this interview, conducted by Maria Manuel Clementino and Jorge Picado (University of Coimbra), will appear in the next issue of the Bulletin.

You have written a paper, published for the first time in 1986, entitled "Taking categories seriously"[1]. Why should we take categories seriously?

In all those areas where category theory is actively used the categorical concept of adjoint functor has come to play a key role. Such a universal instrument for guiding the learning, development, and use of advanced mathematics does not fail to have its indications also in areas of school and college mathematics, in the most basic relationships of space and quantity and the calculations based on those relationships. By saying "take categories seriously", I meant that one should seek, cultivate, and teach helpful examples of an elementary nature.

The relation between teaching and research is partly embodied in simple general concepts that can guide the elaboration of examples in both. Notions and constructions, such as the spectral analysis of dynamical systems, have important aspects that can be understood and pursued without the complications of limiting the models to specific classical categories.

The application of some simple general concepts from category theory can lead from a clarification of basic constructions on dynamical systems to a construction of the real number system with its structure as a closed category; applied to that particular closed category, the general enriched category theory leads inexorably to embedding theorems and to notions of Cauchy completeness, rotation, convex hull, radius, and geodesic distance for arbitrary metric spaces. In fact, the latter notions present themselves in such a form that the calculations in elementary analysis and geometry can be explicitly guided by the experience that is concentrated in adjointness. It seems certain that this approach, combined with a sober application of the historical origin of all notions, will apply to many more examples, thus unifying our efforts in the teaching, research, and application of mathematics.

I also believe that we should take seriously the historical precursors of category theory, such as Grassman, whose works contain much clarity, contrary to his reputation for obscurity.

Other than Grassman, and Emmy Noether and Heinz Hopf, whom Mac Lane used to mention often, could you name other historical precursors of category theory?

The axiomatic method involves concentrating key features of ongoing applications. For example, Cantor concentrated the concept of isomorphism, which he had extracted from the work of Jakob Steiner on algebraic geometry. The connection of Cantor with Steiner is not mentioned in most books; there is an unfortunate tendency for standard works on the history of science to perpetuate standard myths, rather than to discover and clarify conceptual analyses. The indispensable "universe of discourse" principle was refined into the idea of structure carried by an abstract set, thus making long chains of reasoning more reliable by approaching the ideal that "there is nothing in the conclusion that is not in the premise". That vision was developed by Dedekind, Hausdorff, Fréchet, and others into the 20th century mathematics.

F. William Lawvere (Braga, March 2007).

[1] *Revista Colombiana de Matematicas* 20 (1986) 147-178. Reprinted in *Repr. Theory Appl. Categ.* 8 (2005) 1-24 (electronic).