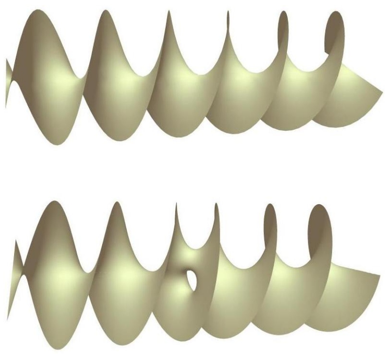
Originally published by the American Mathematical Society in MATH in the MEDIA, a section of the AMS Website, www.ams.org/mathmedia. Reprinted with permission.

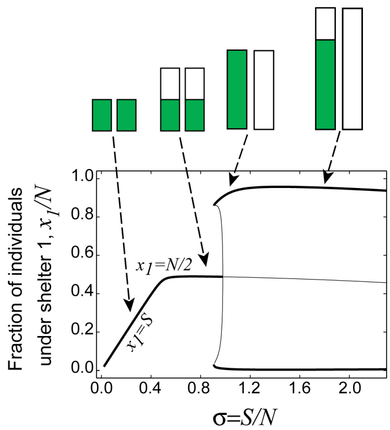
"A Helix with a Handle". That's the title of Fenella Saunders' piece in the May-June 2006 *American Scientist*. The subject is what its authors – Mathias Weber (Indiana), David Hoffman (Stanford) and Michael Wolf (Rice) – describe as "the first properly embedded minimal surface with infinite total curvature and finite topology to be found since 1776, when Meusnier showed that the helicoid was a minimal surface." (Their paper, which appeared in the November 15, 2005 *PNAS*, is available online – www.indiana.edu/~minimal/research/helicoid.pdf). In fact the new surface is closely related to Meusnier's; it is properly described as a helicoid with a handle, or a "genus-one helicoid", and is asymptotic to the helicoid at infinity.



The helicoid and the genus-one helicoid. This picture shows a segment of a cylindrical core through each of the surfaces, which actually extend to infinity in every direction. Image courtesy of Indiana University.

Saunders tries to start her readers off gently: "Dip a loop of wire into a soapy solution, and the film that covers the loop will be what mathematicians call a minimal surface." But soon we hear: "At any point, a minimal surface is maximally curved in one direction and minimally curved in the opposite direction, but the amount of curvature in each direction is exactly the same." The readers may have better luck with the project's interesting history. "Over a decade ago, Hoffman, with Fusheng Wei ... and Hermann Karcher ... , had created computer simulations of such handled helicoids, but an airtight demonstration of minimal surfacehood eluded them." They knew what it looked like, but they could not prove that it really was an embedded minimal surface. Saunders quotes Hoffman: "I think the information about how to solve this problem was lurking in the pictures all the time, but we just had to think about it for a long time and have the theory catch up with the evidence we had."

Dynamics of Roach Congregation. "Group-living animals are often faced with choosing between one or more alternative resource sites." Thus begins the abstract of a paper published April 11, 2006 in the *Proceedings of the National Academy of Sciences* (**105** 5835-5840). The authors, a French-Belgian team led by Jean-Marc Amé and José Halloy, report on "an experimental and theoretical study of groups of cockroaches (*Blattella germanica*) tested in a circular arena ... with identical shelters." When the number of shelters is two, the phenomenon can be described by the graph below, giving the occupancy of shelter 1 as a function of shelter size. Until the size of a shelter is enough for the whole population, the roaches split between the two shelters. But as soon as there is room for everyone in each of the shelters, then the roaches all occupy one and not the other.



Occupancy of two shelters as a function of shelter size S for a fixed number N of individuals. When S/N is less than .5, both shelters are filled; for S/N between .5 and 1, the animals split evenly between the shelters; if S/N is 1 or more, all the animals congregate in one of the shelters. Adapted from *PNAS* **105** 5835-5840, from which the equations below are taken. Image courtesy of José Halloy ULB.

This behavior is predicted by a mathematical model. First, the researchers determined from experiment that the probability Q_i of an individual leaving shelter i varies inversely with the crowdedness (the ratio of the number x_i of animals in the shelter to the shelter size S):

$$Q_i = \frac{\theta}{1 + \rho\left(\frac{x_i}{S}\right)^n}$$

where θ, ρ and n are experimentally derived parameters. On the other hand, the probability R_i for an exploring cockroach to join shelter i decreases linearly with the crowdedness:

$$R_i = \mu\Big(1 - \frac{x_i}{S}\Big)$$

where μ is experimentally derived. These two laws can be combined into a system of differential equations:

$$\frac{dx_i}{dt} = \mu x_e\Big(1 - \frac{x_i}{S}\Big) - \frac{\theta x_i}{1 + \rho\left(\frac{x_i}{S}\right)^n}$$

(here x_e is the number of unattached individuals) subject to the constraint

$$x_e + x_1 + x_2 + ... + x_p = N$$

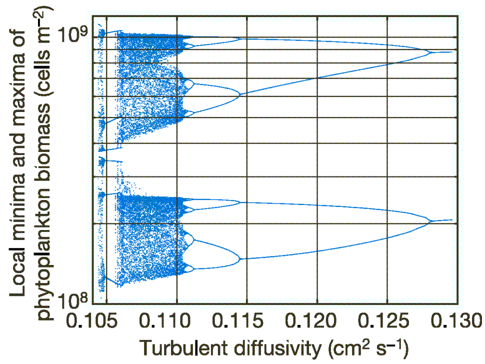
if there are p shelters and a total of N individuals. The authors solved numerically for the steady states, which for $p = 2$ appear schematically in the graph above. Larger numbers of shelters give more complex bifurcation. The authors explain why this collective behavior gives the optimal outcome for each individual roach, and speculate that "the collective decision-making process studied here should have its equivalent in many gregarious animals ... . " This work was picked up in the April 6 2006 *Nature* Research Highlights.

Mathematical Incompleteness in the Scientific American. The March 2006 *Scientific American* features a report by Gregory Chaitin, entitled "The Limits of Reason," describing his own work on the incompleteness of mathematics. "Unlike Gödel's approach, mine is based on measuring information and showing that some mathematical facts cannot be compressed into a theory because they are too complicated" and that therefore "... a theory of everything for all of mathematics cannot exist." Chaitin outlines his theory, including the irreducible number Omega: the first N digits of Omega cannot be computed using a program significantly shorter than N bits long. He sketches the argument that computing the first N binary digits of Omega would solve the halting problem for all programs of length up to N; so the uncomputability of Omega follows from Turing's proof of the unsolvability of the halting problem. It follows from its definition that "an infinite number of bits of Omega constitute mathematical facts ... that cannot be derived from any principles simpler than the string of bits itself. Mathematics therefore has infinite complexity, whereas any individual theory of everything would only have finite complexity and could not capture all the richness of the full world of mathematical truth." Chaitin then spends some time pondering the scientific and philosophical consequences of his work. "Irreducible principles – axioms – have always been part of mathematics. Omega just shows that a lot more of them are out there than we suspected." "If Hilbert had been right, ... there would be a static, closed theory of everything for all of mathematics, and this would be like a dictatorship. ... I much prefer an open system. I do not like rigid, authoritarian ways of thinking." "Extensive computer calculations can be extremely persuasive, but do they render proof unnecessary? Yes and no. In fact, they provide a different kind of evidence. In important situations, I would argue that both kinds of evidence are required, as proofs may be flawed, and conversely computer searches may have the bad luck to stop just before encountering a counterexample that disproves the conjectured result."

Fractals finger suspect Pollocks. Alison Abbott reports in the February 9 2006 *Nature* on a new mathematical development in the saga of the 32 small "possible Pollocks" recently discovered on Long Island. Large poured works by Jackson Pollock bring prices in the tens of millions of dollars; if these paintings are authentic they are very valuable. But their authenticity, accepted by some experts, has been challenged by others. Enter the physicist Richard Taylor. Taylor had published in 1999 (*Nature* **399** 422) his group's discovery that Pollock's poured works showed (as Abbott explains it) "two distinct sets of fractal patterns. One was on a scale larger than 5 cm; the other showed up on scales between 1 mm and 5 cm." and furthermore "that the fractal dimension of Pollock's works ... increased through the years as the artist refined his technique." In a later experiment, he analyzed "14 Pollock paintings, 37 imitations created by students at the University of Oregon, and 46 paintings of unknown origin." Abbott quotes Taylor: "The only shared thing in Pollock's very different poured paintings is a fractal composition that was systematic through the years." The non-Pollocks, when they had fractal structure, had different fractal characteristics. So it was natural for the Krasner-Pollock foundation to send six of the putative 32 for Taylor to examine. His diagnosis: "I found significant deviation from Pollock's characteristics." The foundation's final judgment has not yet been promulgated. The *Nature* piece has several echoes in the *New York Times*. It gets picked up as a news item by Randy Kennedy ("Computer Analysis Suggests Paintings Are Not Pollocks") on February 9. Their art critic Michael Kimmelman weighs in with "A Drip by Any Other Name" on February 12: "... the curious truth is that while a few drips and splashes can imitate Pollock's touch ... it is nearly impossible to replicate ... the full-scale complex rhythms and overlapping patterns, the all-over, depthless, balletic and irregular space he created." And then on the February 19 Op-Ed page, Professor Don Foster (English, Vassar, "Mind Over Splatter") brings us an academic perspective. He starts out serious: "At the heart of the controversy lie critical questions about artistic meaning and value that have vexed literary scholars no less than art historians." But he leaves us with: "Meanwhile, Jackson Pollock may be chuckling in his grave: if the object of Abstract Expressionist work is to embody the rebellious, the anarchic, the highly idiosyncratic – if we embrace Pollock's work for its anti-figurative aesthetic – may faux-Pollock not be quintessential Pollock? May not a Pollock forgery that passes for authentic be the best Pollock of all?"

The differential geometry of quantum computation. "Quantum computers have the potential to solve efficiently some problems that are considered intractable on conventional classical computers." This is the start of "Quantum Computation as Geometry," a report in the February 4 2006 issue of *Science*. The authors are four members of the School of Physical Sciences, University of Queensland; a team led by Michael Nielsen. They continue: "Despite this great promise, as yet there is no general method for constructing good quantum algorithms, and very little is known about the potential power (or limitations) of quantum computers." What they propose in this report is an alternative approach to understanding the difficulty of an n-qubit computation, i.e. the complexity of the quantum algorithm that would be needed to carry it out. Such a computation corresponds to a unitary operator U (a $2^n \times 2^n$ matrix with complex entries). The authors' definition of difficulty is the length $d(I, U)$ of the shortest path from the identity matrix to U, where length is measured in a metric which penalizes all computational moves which require gates with more than two inputs. They show that this distance is "essentially equivalent to the number of gates required to synthesize U." "Our result allows the tools of Riemannian geometry to be applied to understand quantum computation. In particular we can use a powerful tool – the calculus of variations – to find the geodesics of the space." They remark that thinking of an algorithm as a geodesic "is in contrast with the usual case in circuit design, either classical or quantum, where being given part of an optimal circuit does not obviously assist in the design of the rest of the circuit." Finally they show how "to construct explicitly a quantum circuit containing a number of [one and two-cubit] gates that is polynomial in $d(I, U)$ and which approximates U closely."

Chaos in the deep. "Reduced mixing generates oscillations and chaos in the oceanic deep chlorophyll maximum" appeared in the January 19 2006 *Nature*. The authors, an Amsterdam-Honolulu collaboration led by Jef Huisman and Nga N. Pham Thi, investigated the stability of deep chlorophyll maxima (DCMs) – layers of high concentration of phytoplankton who flourish where there are sufficient nutrients welling up from the bottom and sufficient light filtering down from the top. The point of the article: "we extend recent phytoplankton models to show that the phytoplankton populations of DCMs can show sustained fluctuations." The authors set up a mathematical model, a reaction-advection-diffusion equation for the phytoplankton population density P coupled to a partial differential equation for the nutrient availability N. A common parameter in both equations is the "turbulent diffusivity" κ , the coefficient of the second-derivative terms. If κ is sufficiently large, "nutrients in the top layer are gradually depleted by the phytoplankton. The nutricline moves downwards, tracked by the phytoplankton population, until the population settles at a stable equilibrium at which the downward flux of consumed nutrients equals the upward flux of new nutrients." To investigate the behavior for lower κ, the authors ran "numerous simulations using a wide range of turbulent diffusivities." "The model simulations predict that the DCM becomes unstable when turbulent diffusivity is in the lower end of the realistic range. By a cascade of period doublings, reduced turbulent mixing can even generate chaos in the DCM."



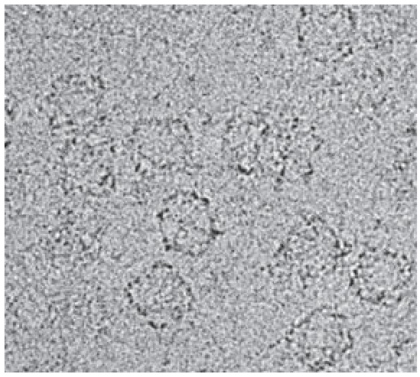
The numerical solution of the coupled $P - N$ differential equations shows bifurcation and eventually chaos as the mixing parameter is decreased. This is a close-up picture of the evolution of the local maxima and minima of the phytoplankton population as a function of turbulent diffusivity, near the low end of the realistic range $0.1 < \kappa < 1$. Image from Nature **439** 324, used with permission.

Their explanation for the periodic behavior: if κ is low, the phytoplankton sink faster than the nutrients are welling up; without sufficient light their numbers decline. This lets more nutrients through up to more luminous layers, and "fuels the next peak in the DCM."

An ominous note: “Climate models predict that global warming will reduce vertical mixing in the oceans.”

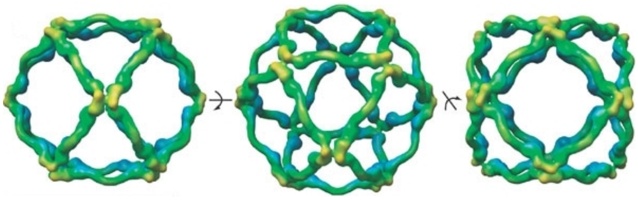
Plant growth and the Golden Ratio, re-evaluated. The *Botanical Journal of the Linnaean Society* ran in its January 2006 issue an article by Todd Cooke (Maryland) with the title: “Do Fibonacci numbers reveal the involvement of geometrical imperatives or biological interactions in phyllotaxis?” From the abstract: “This paper reviews the fundamental properties of number sequences, and discusses the under-appreciated limitations of the Fibonacci sequence for describing phyllotactic patterns.” Apparently golden-ratio giddiness has spread to botany, and this paper aims to be a corrective. Prof. Cooke’s main point is that although “it is inescapable that the spiral phyllotaxes of vegetative shoots are overwhelmingly characterized by low Fibonacci numbers,” the common belief that “such spiral arrangements are attributable to the leaf primordia being positioned in optimal packing” must be questioned and ultimately rejected. The argument, as I understand it, runs as follows. Suppose consecutive primordia are arranged at an exactly constant angular difference. If that difference is *exactly* the golden angle, here given as 137.5°, then one does indeed achieve optimal packing. But “even slight variation from the Fibonacci angle disrupt[s] optimal packing.” E.g. constant angle 137.45° or constant angle 137.92° don’t work. “It is difficult, if not impossible, to imagine any biological system being capable of organizing itself with such discriminating accuracy as a direct response to a hypothetical geometrical imperative for optimal packing. It seems more likely that the spiral phyllotaxes observed ... are the outcome of some biological process, the consequence of which is that such structures tend to approach optimal packing.” There are two points here. The mathematical one is shaky. The golden ratio is (supremely) irrational, and the evidence for its occurrence in the likeliest interval between consecutive primordia (*viz.*, the appearance of numbers of spirals corresponding to its rational approximators 2/3, 3/5, 5/8, etc.) is excellent. On the other hand the question whether optimal packing is an “imperative” or a “consequence” does not seem to me to be one that science can answer. The end of this article addresses the identification of the biological process governing phyllotaxis. Cooke refers to the 1992 *Physics Review Letters* paper (**68**, 2089-2010) by Stéphane Douady and Yves Couder, where they “managed to create spiral phyllotaxis on a lab bench” working with mutually repelling ferrofluid drops floating on silicon oil in a varying magnetic field. Presumably something analogous is happening at the growing tip of a plant. “The ... mechanism ... appears to involve the interaction of mathematical rules, generating process, and overall geometry. In particular, it seems quite plausible that the mathematical rules for phyllotaxis arise from local inhibitory interactions among existing primordia. These interactions are apparently mediated by the expression of specific genes whose products regulate growth hormones ... .” This work was picked up in the “Research Highlights” of the February 9 *Nature*.

Cuboctahedral vesicles in eukaryotic cells. A eukaryotic cell is a complex, three-dimensional organism. Just as our food is ingested in one place and moved to another for processing, with the nutrients then ferried about the body by the bloodstream, so in a cell’s internal economy a critical role is played by transportation. The agents of intracellular transport are vesicles: molecular cages that enclose their cargo and move it from A to B. A paper in the January 12 2006 *Nature* explores the structure of one type of vesicle: those whose skin is made from the coat protein complex II, or COPII. The authors (a Scripps Research Institute team of 8, led by Scott Stagg) explain that the structural part of COPII consists of a lattice formed by the protein complex Sec13/31.



Part of a micrograph of Sec13/31 cages preserved in vitreous ice. The cages are approximately 600Å(0.06 microns) in diameter; their images show the planar projection of their cuboctahedral structure. Image from *Nature* **439** 235, used with permission.

Using electron cryo-microscopy, they determined that the most elementary cages formed by Sec13/31 have the structure of a cuboctahedron, but they suggest that in order to enclose larger cargoes, the same units could organize into the small rhombicuboctahedron, the icosidodecahedron or the small rhombicosidodecahedron. These semi-regular solids all share with the cuboctahedron (and the octahedron) the property that four edges meet at each vertex. The condition corresponds to the assymmetry in the molecular realization of the Sec13/31 complex: the two ends are different, so it cannot assemble into a network with odd-ordered vertices.



The three axes of symmetry of the Sec13/31 cage. Each edge is a Sec13/31 protein complex. The color (blue-green-yellow) encodes distance from the cage center. Note the assymmetry in the edges. Cage diameter approximately 600Å. Image from *Nature* **439** 236, used with permission.

Deal or No Deal? The *New York Times* ran a "Critic's Notebook" column by Virginia Heffernan on December 24, 2005. The subject was the popular game show "Deal or No Deal," and the title was "A Game Show for the Probability Theorist in Us All." Here's how the game works (you can test it out on the NBC website (www.nbc.com/Deal_or_No_Deal/game) – click on "Start game!" once it has uploaded).

- Twenty-six known amounts of money, ranging from one cent to one million dollars, are (symbolically) randomly placed in 26 numbered, sealed briefcases. The contestant chooses a briefcase. The unknown sum in the briefcase is the contestant's.
- In the first round of play, the contestant chooses 6 of the remaining 25 briefcases to open. Then the "banker" offers to buy the contestant's briefcase for a sum based on its expected value, given the information now at hand, but tweaked sometimes to make the game more interesting. The contestant can accept ("Deal") or opt to continue play ("No Deal").
- If the game continues, 5 more briefcases are opened in the second round, another offer is made, and accepted or refused. If the contestant continues to refuse the banker's offers, subsequent rounds open 4, 3, 2, 1, 1, 1, 1 briefcases until only two are left.
- The banker makes one last offer; the contestant accepts that offer or takes whatever money is in the initially chosen briefcase.

The psychology is what makes the game fun. As Heffernan explains: "So far, no game theorist from the Institute for Advanced Study has appeared to try his hand at 'Deal or No Deal' and play as a cool-headed rationalist. Instead the players on the American show are, like most game-show contestants, hysterics." In fact the three scientists at Erasmus University who conducted an exhaustive analysis of the decisions made by contestants in the Dutch version of the game (jackpot 5 million Euros) remark that "For analyzing risky choice, 'Deal or No Deal' has a number of favorable design features. The stakes are very high: ... the game show can send contestants home multimillionaires – or practically empty-handed. Unlike other game shows, 'Deal or No Deal' involves only simple stop-go decisions that require minimal skill or strategy. Also, the probability distribution is simple and known with near-certainty. Because of these features, 'Deal or No Deal' seems well-suited for analyzing real-life decisions involving real and large risky stakes." Their report is available online (papers.ssrn.com/sol3/papers.cfm?abstract_id=636508).

Mathematical patterns in asthma attacks. A mathematically powered breakthrough in the study of the incidence of asthma attacks, with potentially important therapeutic implications, was reported in the December 1 2005 *Nature*. Urs Frey (University Hospital of Berne) works in pediatric respiratory medicine; Béla Suki (Boston University) is a physicist who "analyses complex nonlinear systems, such as the factors that contribute to avalanches" (quote from an "Authors" sketch at the beginning of the issue). With their collaborators, they analyzed the records of a "previously published, randomised, placebo-controlled, double-blind crossover study" following 80 asthmatic subjects for 3 six-month treatment periods. In that study, the PEF (peak expiratory flow) of each subject was measured twice daily; the subject was also assigned a daily asthma symptom score. The team's strategy was to "examine whether the statistical and correlation properties of the time series of PEF recordings can be used to predict the risk of subsequent exaggeration of airway instability." They can. To disentangle the effects the authors created a "nonlinear stochastic model of the PEF fluctuations" ("a cascade connection of a linear dynamic system followed by a second order nonlinear system with no memory. ...") They were able to tune this model to match the statistical characteristics of the experimental data, and then use it to measure the impact of the characteristics separately. One startling conclusion from their analysis is that short-acting bronchodilators, such as the popular drug albuterol, can actually aggravate medium-term risk of an asthmatic attack.