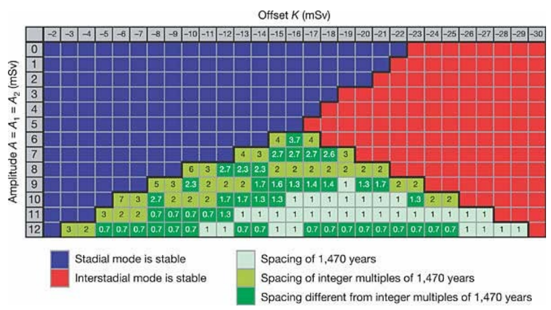
Glacial climate cycles and the least common multiple. In part of the late glacial period severe climate oscillations occurred with a period of almost exactly 1470 years; these are documented by ice-core samples from Greenland, and are called Dansgaard-Oescher (DO) events. The period of these oscillations has been mysterious, because there are no excitations of that frequency either in the solar record or in the variation of the Earth's inclination and orbit. Holger Braun and his colleagues in Heidelberg, Potsdam and Bremerhaven report in the November 10 2005 *Nature* on a possible explanation. There are two "pronounced and stable centennial-scale solar cycles," the DeVries-Suess (period 210 years) and the Gleissberg (86.5 years); the German group designed a model to test the hypothesis that the *sum* of these two excitations could be driving the DO oscillations. In general two different periodic astronomical phenomena will have irrationally related frequencies unless there is "phase locking," but there are *approximate* common periods: waiting long enough one can get the two back as close as one wants to their initial relative position. It turns out that for the DeVries-Suess and Gleissberg cycles, 1470 years is a very good approximation to a common period (it equals 7×210 and almost exactly 17×86.5). The team used CLIMBER-2 (`www-lsce.cea.fr/pmip/docs/climberdoc.html`), a global climate and biosphere simulation model that has been around since 1998, forcing it with

$$F(t) = -A_1 \cos(\omega_1 t + \varphi_1) - A_2 \cos(\omega_2 t + \varphi_2) + K$$

where ω_1 and ω_2 are the DeVries-Suess and Gleissberg frequencies, and K represents changes in the background climate compared with a baseline.



The response of the model for different combinations of periodic excitation amplitude $A = A_1 = A_2$ (vertical) and K — a baseline measurement of the general warmth of the climate (horizontal). The pale green squares represent the parameter ranges for which the model manifests a 1470-year periodicity. Image from *Nature* **438** 208, used with permission.

The results of the simulation show a region where the 1470-year period would be stable under perturbation. As the authors remark, the simulation also shows that similar oscillations could not happen today.

Math on the Millennium Bridge. The Millennium Bridge, a 325-meter footbridge spanning the Thames in London, opened on June 10, 2000. The November 3 2005 *Nature* ran a Brief Communication entitled "Crowd synchrony on the Millennium Bridge," describing what happened and giving a mathematical analysis. "Soon after the crowd streamed on ... , the bridge started to sway from side to side; many pedestrians fell spontaneously into step with the bridge's vibrations, inadvertently amplifying them." This is not the classical example of marchers across a bridge exciting a resonance of the structure. Rather there was a positive feedback loop in which the bridge invited the initially unorganized pedestrians into synchrony. The authors of the *Nature* communication — a five-man team led by Steven Strogatz of Cornell — modeled the phenomenon by "adapting ideas originally developed to describe the collective synchronization of biological oscillators such as neurons and fireflies." Their model starts with the differential equation for a forced, damped harmonic oscillator:

$$Md^2X/dt^2 + BdX/dt + KX = G(\sin\Theta_1 + ... + \sin\Theta_N)$$

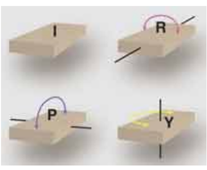
where $X(t)$ is the lateral displacement, and each pedestrian "imparts an alternating sideways force G $\sin\Theta_i$ to the bridge; ... $\Theta_i(t)$ increases by 2π during a full left/right walking cycle." What you wouldn't have seen in Introductory Differential Equations is *feedback*. Since feedback works through the phase difference between the natural oscillation of the bridge and the gait of the pedestrian, the authors make the pair $(X, dX/dt)$ into an angular variable by setting $X = A\sin\Psi$, $dX/dt = (K/M)A\cos\Psi$. Then the feedback is expressed in the set of equations

$$d\Theta_i/dt = \Omega_i + CA\sin(\Psi - \Theta_i + a);$$

here Ω_i is the natural walking rhythm of the i-th pedestrian and a is a phase lag. The model, once tuned by the adjustment of the parameter C, gives a close simulation of the actual event: as the number of pedestrians increases, nothing untoward happens until a critical number is reached, "when the bridge starts to sway and the crowd starts to synchronize, with each process pumping the other in a positive feedback loop."

A new topology for the internet. *Science News* for October 8, 2005 ran a short report by Katie Greene with the title "Untangling a Web. The Internet gets a new look." Greene is describing work to be published in the *PNAS*, in which John Doyle (Caltech) and his colleagues "offer a new mathematical model of the Internet." The conventional ("scale-free") model "indicates that a few well-connected master routers direct Internet traffic to numerous, less essential routers in the network's periphery." Doyle et al. prefer HOT models (the letters stand for Highly Optimized/Organized Tolerance/Tradeoffs), based on insights from biology and engineering. For the Internet, HOT modeling would predict "no ... central hubs and any highly-connected routers lie at the periphery." For security, a HOT model is clearly preferable to a scale-free one, since, as Greene puts it, "if one of those well-connected, outlying routers were taken out, Internet traffic would simply divert to another well-connected router." Whereas in the scale-free Internet, "a targeted attack on a central router could halt virtually all data flow." This is not completely hypothetical: as Greene reports, Doyle and his team have tested their model on Internet2 (an academic subnetwork whose map is known, and which according to Doyle is a "good representation" of the structure of the entire Internet). "The researchers report that their proposed model corresponds well to the structure of Internet2."

Math and the art of mattress flipping. Mattress flipping is one of those household chores that is bothersome because you never know if you are doing it right. Mattress manufacturers recommend periodic flipping for even wear: the four possible combinations of head and foot, top and bottom should receive equal exposure. Ideally there would be a maneuver you can execute each time you flip your mattress such that after four repetitions all four combinations will have been used. Brian Hayes calls such a maneuver a "golden rule" in his treatise on the subject in the September-October 2005 issue of *American Scientist*, and he gives us the bad news: no such golden rule exists.



Mathematically speaking, there are four ways to rotate a mattress so that it ends up aligned with the bed. Hayes uses the symbols I for the Identity rotation (wait until next week) and R, P, Y for the nautical terms Roll, Pitch and Yaw. Image courtesy Brian Hayes.

His argument runs as follows: no matter how creatively you manipulate your mattress, once it's back on the bed you will have performed one of the four operations I, R, P, Y shown in the figure. Each of these operations has the property that if you repeat it, you end up where you started. So you will have missed two of the configurations. Hayes goes on to define the mathematical concept of *group* and to give it content by comparing mattress flipping with another chore: "rotating" (interchanging) the tires on an automobile so that each tire is used, and undergoes wear, in the four different positions. Here there is a "golden rule:" repeating Q (counterclockwise substitution around the outside of the car) four times brings you back to where you started, and each tire will have seen all four positions.

first operation \ second operation	I	R	P	Y
I	I	R	P	Y
R	R	I	Y	P
P	P	Y	I	R
Y	Y	P	R	I

first operation \ second operation	I	Q	Q^2	Q^3
I	I	Q	Q^2	Q^3
Q	Q	Q^2	Q^3	I
Q^2	Q^2	Q^3	I	Q
Q^3	Q^3	I	Q	Q^2

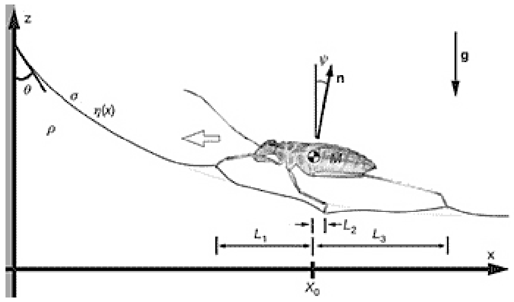
The multiplication tables for mattress flipping (left) and counter-clockwise tire rotation (right). For example, a P (flipping end over end) followed by an R (flipping right over left) has the same effect as a Y (planar rotation by 180 degrees). Each of these tables defines a group with four elements, but the two groups are intrinsically different. Images courtesy Brian Hayes.

The article, available online (www.americanscientist.org/template/AssetDetail/assetid/45938), ends with some fancier material: the complete group of permutations of four objects, and the group of rotations of a cubical mattress.

The math of meniscus mountaineering. Walking on water is a way of life for many species of insects and spiders. But when they need to get onto dry land, they face a problem: surface tension, the same phenomenon that allows these creatures to exist, makes water curve upwards at the shore; the inclined surface marking the edge between wet and dry is called the *meniscus*.

Mesovelia approaches a meniscus. Image from *Nature* **437** 733-736, courtesy John W. M. Bush and David L. Hu.

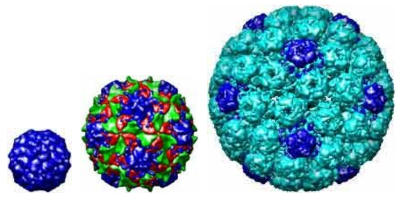
For small insects (say, millimeter-sized) the meniscus appears as a perfectly slippery slope. If they try to walk up, they slide back down. But some species have developed a method that seems like magic: they adopt a special posture and slide *up* the meniscus. David Hu and John Bush of the MIT Mathematics Department have recently worked out the math that makes this possible. Their article, "Meniscus-climbing insects," appears as a Letter in the September 29 2005 *Nature*.



A diagrammatic version of the photograph above, showing the positive and negative meniscus pockets created by *Mesovelia*'s three pairs of legs. Image from *Nature* **437** 733-736, courtesy John W. M. Bush and David L. Hu.

Their analysis is based on the well known observation that "lateral capillary forces exist between small floating objects, an effect responsible for the formation of bubble rafts in champagne and the clumping of breakfast cereal in a bowl of milk." More precisely, they calculate that a body of buoyancy T at distance x from the wall is attracted to the wall by a force $F = ATe^{-Bx}$, where A and B depend on properties of water and on the contact angle θ shown in the diagram. An insect like the water-walker *Mesovelia* faces the meniscus and exploits its three pairs of legs: it pulls up on the surface with the front and rear pairs as it pushes down with the middle pair. Even though the three sets of Ts must add up to something negative (the weight of the insect) the exponential advantage gained by the front legs being closer to the wall will propel the insect forward and up the hill. Where does the work come from? As the authors explain at the end, "by deforming the free surface, the insect converts muscular strain to the surface energy that powers its ascent." Many images and movies (recommended!) available at the project website (www-math.mit.edu/∼dhu/Climberweb/climberweb.html).

Virus geometry. A virus is essentially genetic material in a box. The box, or *capsid*, is assembled from specialized proteins called *capsomers*. Watson and Crick had observed in 1956, on topological grounds, that viral capsids could be expected to show the regularities of platonic solids. In fact, icosahedral-type symmetry is the most prevalent.



Satellite Tobacco Mosaic Virus (diameter = 168Å), Type 1 Poliovirus (304Å) and Simian Virus 40 (488Å) have different sizes and capsid structures, but all exhibit icosahedral symmetry. Images from Virus Particle Explorer (VIPER) (viperdb.scripps.edu), a website for virus capsid structures and their computational analysis.

Recent progress in understanding this bias towards icosahedra was reviewed ("Armor-plated Puzzle") by Peter Weiss in the September 3 2005 *Science News*. Weiss first describes research by the UCLA team of Roya Zandi, David Reguera, Robijn Bruinsma, William M. Gelbart and Joseph Rudnick (PNAS **101**, 15556-15560). This team used Monte-Carlo simulations to find locally energy-minimizing configurations of "pentamers" and "hexamers." As Weiss explains it, "They developed a computer model that treated capsomers as malleable disks. ... Then, by having the computer repeatedly shuffle those disks into arbitrary arrangements on a spherical surface, they simulated the formation of millions of hypothetical capsids. ... To explore all possible ratios of pentamers and hexamers, the researchers also programmed into the process random switching of disks between the two types." The lowest energies occurred with arrays of 12 pentamers, surrounded by 0, 20, 30 and 50 hexamers respectively. These corresponded exactly to the prediction, made in 1962 by Donald Caspar and Aaron Klug, of capsids made of 12 pentamers, or 12 pentameters with $20(T-1)$ hexamers, where T is one of the series 3, 7, 13, 19, ... of numbers of the form $h^2 + hk + k^2$; h and k are integers with $(h, k) = 1$.

The Satellite Tobacco Mosaic Virus (12 pentamers), and the Poliovirus (12 pentamers plus 30 hexamers) fall into this classification, but the Simian Virus 40 does not: every one of its 72 capsomers is a pentamer. Weiss explains how Reidun Twarok (York University) read about this problem and saw how her previous work on quasi-crystals could be applied. "The technique employs some mind-bending concepts, such as a six-dimensional lattice based on a hypercube or other building block. Twarock considered lines and planes projecting from such a lattice onto a three-dimensional sphere representing a viral capsid. ... Exploring the expanded portfolio of possible capsid structures that her tiling method had revealed, Twarock found a tile arrangement for a capsid comprising 72 pentamers and no heptamers." This turned out to be exactly the SV-40 structure pictured above. Her work appeared in the *Journal of Theoretical Biology* last year (**226**, 477 - 482), with a more general classification of possible capsid structures available online (arxiv.org/pdf/q-bio.BM/0508015).

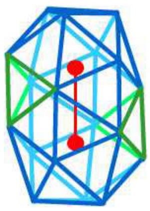
Math and narrative on Mykonos. "Can mathematicians learn from the narrative approaches of the writers who popularize and dramatize their work?" This is the sub-heading on a news feature piece by Sarah Tomlin in the August 4 2005 *Nature*. Tomlin is reporting on a conference held this summer on Mykonos, where a "select group of about 30 mathematicians, playwrights, historians, philosophers, novelists and artists" met to "find a common ground between story-telling and mathematics." The meeting was the brainchild of the poet and novelist Apostolos Doxiadis (*Uncle Petros and the Goldbach Conjecture*), who has formed a foundation (Thales & Friends) dedicated, according to its website (www.thalesandfriends.org), to "bridging the chasm between mathematics and human culture." Among the participants looking for that common ground from the mathematical side of the chasm, Tomlin quotes Timothy Gowers ("Most mathematics papers are incomprehensible to most mathematicians"), Perci Diaconis ("I can only work on problems if there is a story that is real for me") and Barry Mazur ("I don't think I personally understood the problem until I expressed it in narrative terms"). "Mazur," she tells us, "did not find a solution by using the narrative device of a cliff-hanger, but it helped him to frame the question — and that, he argues, may be as important." Mazur also is reported as suggesting "that similar narrative devices may be especially helpful to young mathematicians, who seem particularly poor at explaining their work to others." Tomlin also gives us a sobering quote from Diaconis: "To communicate we have to lie. If not, we're deadly boring."

The math behind "Intelligent Design". H. Allen Orr's "Devolution," subtitled "Why intelligent design isn't," ran in the May 30 2005 *New Yorker* under their Annals of Science rubric. Orr, Professor of Biology at the University of Rochester, examines the most recent instars of the Intelligent Design (I.D.) argument. In particular he mentions the claim that "recent mathematical findings cast doubt on the power of natural selection." This claim, Orr tells us, has been put forward by William A. Dembski, who "holds a Ph.D. in mathematics, another in philosophy, and a master of divinity in theology." Dembski, once on the faculty at Baylor University and now a member of the Center for Science and Theology at Southern Baptist Theological Seminary, uses the "so-called No Free Lunch, or N.F.L. theorems" to attack natural selection. These theorems analyze the efficiency of search algorithms. "Roughly, the N.F.L. theorems prove the surprising fact that, averaged over all possible terrains, no search algorithm is better than any other." Therefore the search algorithm posited by Darwinism (random mutation plus natural selection), looking for the best in the landscape of all possible adaptations, "is no better than blind search, a process of utterly random change unaided by any guiding force like natural selection." So runs the argument. "Since we don't expect blind change to build elaborate machines showing an exquisite coordination of parts, we have no right to expect Darwinism to do so, either." As Orr reports, "Dembski's arguments have been met with tremendous enthusiasm in the I.D. movement. In part, that's because an innumerate public is easily impressed by a bit of mathematics." But Orr mentions recent work showing that the N.F.L theorems "don't hold in the case of co-evolution, when two or more species evolve in response to one another. And most evolution is surely co-evolution. Organisms ... are perpetually challenged by, and adapting to, a rapidly changing suite of viruses, parasites, predators and prey. A theorem that doesn't apply to these situations is a theorem whose relevance to biology is unclear." He ends this discussion by quoting David Wolpert, one of the authors of the N.F.L. theorems, on Dembski's use of those theorems: "fatally informal and imprecise." Wolpert's paper, joint work with William Macready, is available online (www.santafe.edu/research/publications/wplist/1995), as is Wolpert's critique of Dembski's argument.

Archimedes palimpsest update. Capsule history: A 10th-century parchment containing several works of Archimedes (one of them known to us only by its title) was partially erased sometime between the 12th and 14th century and reused as a religious text. The new book, carefully preserved in monasteries, was found in 1906 by J. L. Heiberg, a scholar who recognized the subtext and was able to decipher and publish most of it. The book went out of sight and resurfaced in 1998 when it was sold at auction for $2 million. The collector who bought it has loaned it until 2008 to the Walters Art Museum in Baltimore. There modern techniques (X-ray fluorescence, optical character recognition and multi-spectral imaging) are being used to tease out the maximum possible of Archimedes' barely legible text. The update: Scholars in Baltimore were stymied by four pages which a 20th-century forger had overpainted with pseudo-medieval imagery, presumably to make the book more valuable. One of them, hearing that the ancient ink was iron-based, thought to take those pages to the Stanford Synchrotron Radiation Laboratory, where high-energy X-rays could make the hidden iron atoms fluoresce and give up their information. The result is four superimposed images (both texts, both sides of the parchment) but the message, which deals with floating bodies and the equilibrium of planes, is there for deciphering. This material is taken from a Stanford University news release posted online (www.sciencedaily.com/releases/2005/05/050521154449.htm). by *Science Daily* on May 22 2005.

The release was picked up in the May 19 2005 *Nature* ("Eureka moment as X-rays slice through forgery"). The *Nature* item shows one of the forged overpaintings and gives a glimpse of the SSRL radiograph.

World's largest nano-deltahedron. *Deltahedron* is chemists' name for a polyhedron with all faces triangular. These shapes occur as ions in cluster chemistry. Until recently, the largest one known had twelve lead atoms forming an icosahedral cage enclosing a platinum atom. Earlier this year, Jose Goicoechea and Slavi Sevov (Notre Dame) reported in the *Journal of the American Chemical Society* that they had assembled a deltahedral cage of eighteen germanium atoms around a palladium dimer. The structure is shown schematically below: two Pd-centered 9-atom Ge-clusters (blue) are joined with the interpolation of four (green) non-equilateral faces. The palladium dimer is shown in red, with the two palladium atoms approximately at the foci of the ellipsoidal cage. Goicoechea and Sevov report that the new cluster stays intact in solution. Their work was picked up under the Editor's Choice rubric in the May 20 2005 *Science*.



The structure of $[Pd_2@Ge_{18}]^{4-}$, after Goicoechea and Sevov. The Pd-Pd distance is about 3Å.

Math and the Unicorn Tapestries. Richard Preston's "Capturing the Unicorn," subtitled "How two mathematicians came to the aid of the Met" appeared under the Art and Science rubric in the *New Yorker* for April 11, 2005. It turns out that the Metropolitan Museum had a problem. The Unicorn Tapestries, the crown jewels of the Met's Medieval collection, were taken down for cleaning in 1998, and were photographed then as part of the Museum's high-resolution digital record project. The tapestries — there are six of them and a fragmentary seventh — are typically twelve feet high and somewhat wider. The digital Leica set up to do the job could only capture one 3 by 3-foot square at a time. But assembling the digital files into a coherent image was too large a job for the Museum's computers to handle. The data — more than two hundred CDs — were filed away, and the tapestries reinstalled on the museum walls. Fast forward to 2003, when David Chudnovsky meets a Metropolitan curator at a dinner party. He and his brother Gregory ("The Chudnovsky brothers claim they are one mathematician who happens to occupy two human bodies") soon take on the computing job, which should be a snap for their latest home-built supercomputer (called "the Home Depot thing" or just "It"). But there is a twist: even after geometric transformations have corrected for all possible perspective changes between adjacent frames, the images on the overlaps are hopelessly out of registration: it's as if the tapestry were a living being which had taken a breath between takes. Everything has slightly shifted. Coaxing the overlaps back into registration requires a new "warping" technique, as the Chudnovskys explain it, a 2-dimensional analogue of techniques used in DNA sequencing and speech recognition. The computation is huge: it takes the "Chudnovsky Mathematician" — Preston's coinage — and "It" three months to process just one tapestry, but "The Unicorn in Captivity" is digitally captured in seamless splendor.

The left-hand side shows the overlap between two adjacent frames of the photo mosaic, after all perspective corrections have been made. The right-hand side shows the two images brought into registration by the Chudnovskys' warping technique. Images courtesy Tom Morgan, IMAS.

End of story: One tapestry, apparently, was enough. The brothers have moved on to a bigger project, working on the design of what may be the world's most powerful supercomputer, for "a United States government agency."

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