BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

June 2005

18

Contents

Coming Events

Thematic Term on Optimization

Coordinator

Luís Nunes Vicente (University of Coimbra)

Dates

July 2005

Optimization (mathematical programming) is a well established discipline of mathematics which has been remarkably capable of finding new applications to science, engineering, and economics.

Two areas where optimization is playing an increasingly important role are finance and medicine. The 2005 CIM Thematic Term includes a workshop and a short-course on Optimization in Finance and a workshop on Optimization in Medicine.

One of the main events of this Thematic Term is the Workshop on PDE Constrained Optimization, where a short course is also planned. Optimization problems governed by PDEs is at the core of simulation-based optimization, an area of high demand and pivotal importance in multi-disciplinary engineering.

A Summer School on Integer Programming is also scheduled, focused on the newest recent developments obtained by geometric and algebraic approaches to combinatorial optimization problems.

The programme of events is the following:

July 5-8: Workshop on Optimization in Finance

Organizers

A. M. Monteiro (University of Coimbra), R. H. Tütüncü (Carnegie Mellon University, Pittsburgh, USA) and L. N. Vicente (University of Coimbra).

Aims

Optimization models and methods play an increasingly important role in financial decision making. Many problems in quantitative finance, originated from asset allocation, risk management, derivative pricing, and model fitting, are now routinely and efficiently solved using modern optimization techniques. This workshop will bring together researchers in the rapidly growing field of financial optimization and intends to provide a forum for innovative models and methods on new topics, novel approaches to well-known problems, success stories, and computational studies in this exciting field. Participants are encouraged to present and discuss their recent work and new, possibly controversial, approaches are particularly welcome.

The targeted audience for this workshop includes graduate students and faculty members working in applied mathematics, operations research, and economics, who have been interested in mathematical finance or plan to do so. The workshop will also be attractive for those doing quantitative modelling in the financial market.

A one-day short-course, intended for optimization researchers interested in quantitative finance as well as finance researchers and practitioners interested in optimization models and methods, will precede the scientific programme of the workshop. Invited and contributed presentations will be scheduled during the remaining three days.

The event will be held at the Faculty of Economics - University of Coimbra.

Short Course

It will be delivered by

R. H. Tütüncü (Carnegie Mellon University, USA)

S. Uryasev (University of Florida, USA)

Invited speakers

J. R. Birge (University of Chicago, USA)

T. F. Coleman (Cornell University, USA)

H. Konno (Chuo University, Japan)

J. M. Mulvey (Princeton University, USA)

R. T. Rockafellar (University of Washington, USA)

N. Touzi (CREST, France)

S. A. Zenios (University of Cyprus, Cyprus)

For more information about the event, see

`http://www.mat.uc.pt/tt2005/of/`

July 11-15: Summer School on Geometric and Algebraic Approaches for Integer Programming

Organizers

M. Constantino (University of Lisbon), L. Gouveia (University of Lisbon) and R. Weismantel (Otto-von-Guericke-University of Magdeburg, Germany).

Aims

The School is composed by five set of lectures, designed to introduce young researchers to the more recent advances on geometric and algebraic approaches for integer programming. Each set of lectures will be about six hours long. They will provide the background, introduce the theme, describe the state-of-the-art, and suggest practical exercises. The organizers will try to provide a relaxed atmosphere with enough time for discussion.

Integer programming is a field of optimization with recognized scientific and economical relevance. The usual approach to solve integer programming problems is to use linear programming within a branch-and-bound or branch-and-cut framework, using whenever possible polyhedral results about the set of feasible solutions. Alternative algebraic and geometric approaches have recently emerged that show great promise. In particular, polynomial algorithms for solving integer programs in fixed dimension have recently been developed. This is a hot topic of international research, and the School will be an opportunity to bring up-to-date knowledge to young researchers.

The school will be held at the Faculty of Sciences, Bloco C6 - located in the main campus of the University of Lisbon.

Lectures

Generating Functions for Lattice Points
A. Barvinok (University of Michigan, USA)

Geometric Approaches to Cutting Plane Theory
G. Cornuéjols (Carnegie Mellon University, USA)

Fast Algorithms for Integer Programming in Fixed Dimension
F. Eisenbrand (Max-Planck-Institut, Germany)

I. Experimenting and Applying the Rational Function Method: A LattE Tutorial
II. Transportation Polytopes: Structure, Algorithms, and Applications to Optimization and Statistics
J. de Loera (University of California, Davis, USA)

The Integral Basis Method and Extensions
R. Weismantel (Otto-von-Guericke Univ. Magdeburg, Germany)

For more information about the event, see

`http://www.mat.uc.pt/tt2005/ss/`

July 20-22: Workshop on Optimization in Medicine

Organizers

C. Alves (Technical University of Lisbon), P. M. Pardalos (University of Florida,Gainesville, USA) and L. N. Vicente (University of Coimbra).

Aims

The study of computing in medical applications has opened many challenging issues and problems for both the medical computing and mathematical communities. This workshop is intended to foster communication and collaboration between researchers in the medical computing community and researchers working in applied mathematics and optimization.

Mathematical techniques (continuous and discrete) are playing a key role with increasingly importance in understanding several fundamental problems in medicine.

For instance, mathematical theory of nonlinear dynamics and discrete optimization has been used to predict epileptic seizures. Next to stroke, epilepsy is among the most common disorders of the nervous system. Measures derived from the theory of nonlinear dynamics and discrete optimization techniques are used for prediction of impending epileptic seizures from analysis of multielectrode electroencephalographic (EEG) data.

Several examples of the use of mathematics in medicine can be found in recent cancer research. Sophisticated mathematical models and algorithms have been used for generating treatment plans for radionuclide implant and external beam radiation therapy. With Gamma Knife treatment, for example, optimization techniques have been used to automate the treatment planning process.

Optimization has been used to address a variety of medical image registration problems. In particular, specialized mathematical programming techniques have been used in a variety of domains including the rigid alignment of primate autoradiographs and the non-rigid registration of cortical anatomical structures as seen in MRI.

The invited presentations will be complemented by sessions of contributed talks.

The event will take place at the Institute of Biomedical Research in Light and Image (IBILI), Faculty of Medicine - University of Coimbra

Invited Presentations

Optimization of Gamma Knife Radiosurgery
M. C. Ferris (University of Wisconsin, USA)

Multicriteria Optimization in Radiation Therapy
H. W. Hamacher (Univ. of Kaiserslautern, Germany)

Optimization in Epilepsy
L. D. Iasemidis (Arizona State University, USA)

Optimal Reconstruction Kernels in Medical Imaging
A. K. Louis (University of Saarbrücken, Germany)

Optimization and Optimal Control in High Intensity Ultrasound Surgery
J. P. Kaipio (University of Kuopio, Finland)

Integer Programming in Radiation Therapy
E. K. Lee (Georgia Institute of Technology, USA)

Optimization in Medical Imaging Registration
A. Rangarajan (University of Florida, USA)

For more information about the event, see

http://www.mat.uc.pt/tt2005/om/

July 26-29: Workshop on PDE Constrained Optimization

ORGANIZERS

L. M. Fernandes (Polytechnical Institute of Tomar), M. Heinkenschloss (Rice University, Houston, USA) and L. N. Vicente (University of Coimbra).

AIMS

Optimization problems governed by partial differential equation (PDE) constraints arise in many important applications. Progress in computational and applied mathematics combined with the availability of rapidly increasing computer power steadily enlarges the range of applications that can be simulated numerically and for which optimization tasks, such as optimal design, parameter identification, and control are being considered. For most of these optimization problems, simple approaches combining off-the-shelf PDE solvers and optimization algorithms often lack robustness or can be very inefficient.

Successful solution approaches have to overcome challenges arising from, e.g., the increasing complexity of applications and their mathematical models, the influence of the underlying infinite dimensional problem structure on optimization algorithms, and the interaction of PDE discretization and optimization.

This workshop will combine a wide range of topics important to PDE constrained optimization in an integrated approach, fusing techniques from a number of mathematical disciplines including functional analysis, optimal control theory, numerical optimization, numerical PDEs, and numerical analysis and application specific structures.

A short course will be offered on the first day of the workshop.

Invited and contributed presentations will be scheduled during the remaining three days.

The event will take place at the Escola Superior de Tecnologia de Tomar and Hotel dos Templários, Tomar.

SHORT COURSE

Theoretical background on characterization and properties of solutions to PDE constrained optimization problems
F. Tröltzsch (Technical University of Berlin, Germany)

Numerical solution of PDE constrained optimization problems
M. Heinkenschloss (Rice University, USA)

INVITED PRESENTATIONS

Flow Control
M. D. Gunzburger (Florida State University, USA)

Multiphysics Problems
R. H. W. Hoppe (University of Augsburg, Germany)

State Constraints
K. Kunisch (University of Graz, Austria)

Time Dependent Problems
G. Leugering (Univ. Erlangen-Nürnb., Germany)

Model Reduction
A. T. Patera (MIT, USA)

Adaptive Solution of PDE Constrained Problems
R. Rannacher (University of Heidelberg, Germany)

Preconditioning of KKT Systems
E. W. Sachs (University of Trier, Germany)

For more information about the event, see

http://www.mat.uc.pt/tt2005/pde/

International Conference on Semigroups and Languages

University of Lisbon, July 12-15

Organizers:

Jorge M. André, New Univ. of Lisbon

Mário Branco, Univ. of Lisbon

Vitor Hugo Fernandes, New Univ. of Lisbon

John Fountain, Univ. of York, UK

Gracinda M. S. Gomes, Univ. of Lisbon

John Meakin, Univ. of Nebraska, USA

Confirmed Invited Lecturers

J. Almeida, Univ. of Porto, Portugal

R. Gilman, Stevens Institute of Technology, USA

M. Lawson, Heriot-Watt Univ., UK

S. Margolis, Bar-Ilan Univ., Israel

D. McAlister, Northern Illinois Univ., USA

D. Munn, Univ. of Glasgow, UK

F. Otto, Univ. of Kassel, Germany

J.-E. Pin, Univ. Paris 7, France

P. Silva, Univ. of Porto, Portugal

B. Steinberg, Carleton Univ., Canada

M. Szendrei, Univ. of Szeged, Hungary

D. Therien, McGill Univ., Canada

M. Volkov, Ural State Univ., Russia

P. Weil, Univ. of Bordeaux, France

For more information about this event, see

`http://caul.cii.fc.ul.pt/csl2005/`

Workshop on Statistics in Genomics and Proteomics

Hotel Estoril Eden, Monte Estoril, October 6-8

Organizers:

M. Antónia A. Turkman, Univ. of Lisbon

Kamil Feridun Turkman, Univ. of Lisbon

Lisete Sousa, Univ. of Lisbon

Luzia Gonçalves, New Univ. of Lisbon

Aims

The workshop will aim to bring together the leading researchers in the areas of statistics in genomics and proteomics, to describe the state of the art and also to present problems that will change the next generation of Biostatistics and Bioinformatics researchers.

The workshop will have 7 Keynote speakers and 5 Invited speakers (from Portugal) on topics which are at the forefront of research. The main areas of the workshop are:

- Analysis of Gene Expression Data
- Regulatory Networks
- Statistical Proteomics
- Physical Mapping
- Phylogenetics and Evolutionary Genomics

Preliminary List of Keynote Speakers

Terry Speed, Department of Statistics, University of California, USA

Dirk Husmeier, Biomathematics & Statistics Scotland SCRI, UK

Ruedi Aebersold, Institute for Systems Biology, Seatle, USA

Sophie Schbath, Institut National de la Reserche Agronomique, Unité Mathématique, Informatique & Génome, France

Korbinian Strimmer, Department of Statistics, University of Munique, Germany

Chris Cannings, Division of Genomic Medicine, University of Sheffield, UK

Simon Tavaré, Department of Biological Sciences, University of South California, USA

Preliminary List of Invited Speakers

Margarida Amaral, Department of Chemistry and Biochemistry, University of Lisbon and National Institute of Health, Lisbon

Líbia Zé-Zé, Sequencing Unit, ICAT, University of Lisbon

Pedro Fernandes, Gulbenkian Institute of Science, Lisbon

Rogério Tenreiro, Department of Plant Biology , University of Lisbon

Mário Silva, Department of Informatics, University of Lisbon

For more information about this event, see

`http://wsgp.deio.fc.ul.pt/`

Working Afternoons SPM/CIM

CIM, Coimbra

A joint initiative of the Portuguese Mathematical Society (SPM) and the International Center for Mathematics (CIM).

Programme for 2005/06:

- 3 September 2005 - Geometry - Organizer: Rui Loja Fernandes (IST, Lisbon)
- 5 November 2005 - Partial Differential Equations - Organizer: José Miguel Urbano (Univ. Coimbra)
- 7 January 2006 - Dynamical Systems - Organizer: José Ferreira Alves (Univ. Porto)
- 4 March 2006 - Statistics - Organizer: Paulo Teles (Univ. Porto)
- 6 May 2006 - Optimization - Organizer: Domingos Cardoso (Univ. Aveiro)

For more information, see

`http://www.spm.pt/investigacao/spmcim/spmcim.phtml`

Summer School on Mathematics "Escola Diagonal"

IST, Lisbon, September 5-9

An organization of the *Fundação Calouste Gulbenkian*, in the framework of the *Programa Gulbenkian Novos Talentos em Matemática*.

Also supported by *Fundação para a Ciência e a Tecnologia* (FCT), International Center for Mathematics (CIM), Portuguese Mathematical Society (SPM) and Center for Mathematical Analysis, Geometry, and Dynamical Systems of IST.

For more information, see

`http://www.math.ist.utl.pt/escola/`

CIM Events for 2006

The CIM Scientific Committee, in a meeting held in Coimbra on February 12, approved the CIM scientific programme for 2006.

The list of events is the following:

Graph Spectra

10-12 April 2006

Chairman of Organizing Committee:

D. Cardoso, Univ. Aveiro

Chairman of Scientific Committee:

D. Cvetkovic, Univ. Belgrado, Serbia-Montenegro

Lie Algebras and Quantic Groups

28-30 June 2006

Organizing Committee:

H. Albuquerque, Univ. Coimbra

S. Lopes, Univ. Porto

J. Teles, Univ. Coimbra

Mathematics in Chemistry

19-21 July 2006

Chairman of Organizing Committee:

J. C. Zambrini, Univ. Lisbon

Chairman of Scientific Committee:

J. A. Martinho Simões, Univ. Lisbon

Summer School on Statistical Tools in Knowledge Building

23-29 July 2006

Scientific Coordinators:

D. Pestana, Univ. Lisbon

C. W. Andersen, Univ. Sheffield, England

Mathematical Problems in Telecommunications

4-8 September 2006

Organizing Committee:

C. Salema, Telecommunications Institute and IST, Lisbon

C. Rocha, Mathematics Department, IST, Lisbon

A. Navarro, Telecommunications Institute and Univ. Aveiro

Mathematics in Biology and Medicine

11-15 September 2006

Organizing Committee:

J. Carneiro, F. Dionísio, G. Gomes and I. Gordo, Gulbenkian Institute of Science, Oeiras

For updated information on these events, see

`http://www.cim.pt/?q=events`.

CIM News

Seminar of the 2005 Annual Scientific Council Meeting

Held in Coimbra in February 12, 2005, the files of the talks

Geodesic flows on flat surfaces

(by Marcelo Viana, IMPA, Brasil)

A market-induced mechanism for stock pinning

(by Marco Avellaneda, Courant Institute of Mathematical Sciences, USA)

are available at http://www.cim.pt/?q=cscam

Meeting of the General Assembly of CIM

During the morning of May 21, 2005, the General Assembly of CIM met in the CIM premises at the Astronomical Observatory of the University of Coimbra. In the afternoon of the same day, the members of the General Assembly had the opportunity to attend a talk by Eng. António Segadães Tavares (one of the recipients of the 2004 IABSE Outstanding Structures Award) on the extension of the Funchal Airport, and another one by Prof. Manuel Castellet (the Chairman of ERCOM - European Research Centres on Mathematics) about ERCOM and the *Centre de Recerca Matemàtica* (Barcelona, Spain).

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the Univ. of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form in

`http://www.cim.pt/?q=research`

CIM on the Web

For updated information about CIM and its activities, see `http://www.cim.pt`

News from our Associates

(from UIMA)

OTFUSA2005

7-9 July 2005, Aveiro, Portugal

Conference on Operator Theory, Function Spaces and Applications – Dedicated to the 60th birthday of Professor F.-O. Speck.

It aims to bring together those enrolled in the research activities related with operator theory, function spaces and related applications. Therefore, promoting the exchange of ideas and knowledge, and reinforcing scientific contacts.

The main topics of OTFUSA2005 include:

1. Convolution type operators and related classes of singular operators, Bessel potential and pseudo-differential operators, factorisation theory, operator relations and normalisation problems;

2. The theory of function spaces and distributions around Lebesgue, Lorentz, Sobolev, Besov and Triebel-Lizorkin spaces, embeddings, interpolation, traces and extensions, representation formulas, oscillation;

3. Applications to mathematical physics, wave diffraction and scattering theory, elliptic boundary value problems, mixed problems in canonical domains, localisation, interface problems, boundary integral methods, boundary-domain methods, explicit solutions, regularity, singularities, fractal analysis and asymptotic behaviour.

For more information, see

`http://www.mat.ua.pt/otfusa2005/`

(from UNL)

Euro-Par 2005

30 August-2 September 2005, Lisbon, Portugal

Euro-Par is an annual series of international conferences dedicated to the promotion and advancement of all aspects of parallel computing. The major themes can be divided into the broad categories of hardware, software, algorithms and applications for parallel computing. The objective of Euro-Par is to provide a forum within which to promote the development of parallel computing both as an industrial technique and an academic discipline, extending the frontier of both the state of the art and the state of the practice. The Euro-Par conference series is traditionally organized in cooperation with ACM (SIGPLAN, SIGACT, and SIGARCH), in cooperation with IFIP (WG 10.3), and in technical cooperation with IEEE Computer Society TCPP.

For more information, see

`http://europar05.di.fct.unl.pt/`

(from UMinho)

CEIO 2005

26-28 October 2005, Guimarães, Portugal

The *Sociedade Galega para a Promoción da Estatística e da Investigación de Operacións*, in collaboration with the Department of Mathematics for Science and Technology, of Minho University, organizes the *I Congresso de Estatística e Investigação Operacional da Galiza e Norte de Portugal* and the *VII Congreso Galego de Estatística e Investigación de Operacións*, from the 26th to 28th of October 2005, in Azurém Campus of Minho University.

The main goals of this congress are to promote the dissemination of recent developments in Statistics and Operational Research and their applications in Galicia region and north of Portugal in a scientific, business and educational contexts. The program of the congress includes oral communications and posters, round tables, a tutorial and some social activities.

For more information, see

`http://www.mct.uminho.pt/ceio2005/`

CIM Summer School on Mathematics in Biology and Medicine 2004

Report

The school was held on 20-24 September 2004 at *Instituto Gulbenkian de Ciência* (IGC), organized by Gabriela Gomes, Jorge Carneiro, Pedro Coutinho, Isabel Gordo, Jose Faro and Francisco Dionisio, endorsed by Centro Internacional de Matemática (CIM), and sponsored by Fundação para a Ciência e a Tecnologia (FCT) and Lusolab. The aim of the summer school was to promote the use of mathematical modeling in biology and medicine and this aim was fully accomplished. Several areas of research on mathematical biology (such as epidemiology, population biology and population genetics, immunology and developmental biology) were tackled with success. The school included a short course by each of the invited speakers, and sessions for participants to present and discuss their research. The invited speakers and the subject of their lectures in the summer school were:

- Jacob Koella (University of Paris, France), *An evolutionary view of the epidemiology and control of malaria.*
- Gil McVean (University of Oxford, UK), *Modeling genetic variation.*
- Graham Medley (University of Warwick, UK), *Infectious disease control: the usefulness of toy models.*
- Markus Owen (University of Nottingham), *Mathematical and computational modeling in developmental biology.*
- Alan S. Perelson (Los Alamos National Laboratory, USA), *Modeling Viral Infections.*
- Lee Segel (The Weizmann Institute of Science, Israel), *Choosing an appropriate immune response.*

The school had the participation of about 60 research students and post-docs, from Europe, Africa and America. Overall, participants were interested and interactive. The outcome was pleasing and motivating, and we are preparing a similar event on a biannual basis.

The Organizing Committee: Jorge Carneiro, Pedro Coutinho, Francisco Dionisio, José Faro, Gabriela Gomes, Isabel Gordo.

Quantales as geometric objects: symmetry beyond groupoids?

Pedro Resende*

Departamento de Matemática, Instituto Superior Técnico,

Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

1 Introduction

Modern mathematics has become pervaded by the idea that in order to cater for certain notions of *symmetry*, in particular of a local nature, one needs to go beyond group theory, replacing groups by *groupoids*. A nice survey of some implications of this idea in algebra, geometry, and analysis (as of 1996) can be found in [19]. The same idea can be expressed in terms of *pseudogroups*, which provide another generalization of groups, or, even more generally, by abstract *inverse semigroups*. See the first chapters of [7] for motivations and a good historical account.

How do these two generalizations relate to each other? Many differences and similarities are illustrated by various constructions back and forth between groupoids and inverse semigroups [7, 13], and by their relations to operator algebras [1, 13, 15]. One recurrent aspect is that inverse semigroups are closely related to a particular notion of *topological groupoid*, namely to *étale groupoids*. A fundamental reason for this can be singled out in the form of a precise correspondence between these two concepts, bearing a close resemblance to the equivalence between local homeomorphisms and sheaves: to each inverse semigroup S of a certain kind (the analogue of a sheaf) we associate the groupoid of *germs* of S, which is an étale groupoid $\mathrm{Germs}(S)$ (the analogue of a local homeomorphism); and to each étale groupoid G we associate an inverse semigroup of "sections" $\Gamma(G)$, such that $S \cong \Gamma(\mathrm{Germs}(S))$ and $G \cong \mathrm{Germs}(\Gamma(G))$. We provide a brief description of this correspondence in §4.

In order to extend the correspondence beyond étale groupoids we need a more general type of semigroup. One good candidate is the notion of *quantale* [10, 18], of which the most general definition is that of a *sup-lattice ordered semigroup* (§5). For instance, the topology of any étale groupoid is closed under pointwise multiplication of open sets, hence being a quantale. There is an algebraic characterization of the quantales obtained in this way, through which a correspondence between étale groupoids and quantales is obtained [16], matching that of inverse semigroups and étale groupoids (§7). But we can also go beyond étale groupoids because slightly more general classes of quantales provide characterizations of more general groupoids, such as open groupoids (§8).

The purpose of this paper, which is to be regarded partly as a research announcement, is to highlight some aspects of the interplay between quantales, inverse semigroups, and groupoids. Many of the results mentioned have not yet been presented (some are joint work) and will appear in detail elsewhere.

2 Groupoids

A *groupoid* G is a small category in which every morphism is invertible, or, equivalently, a pair of sets G_0 (of *units*) and G_1 (of *arrows*) equipped with structure maps satisfying appropriate axioms,

$$G_1 \times_{G_0} G_1 \xrightarrow{\ m\ } G_1 \overset{i}{\circlearrowleft} \;\; \underset{d}{\overset{r}{\rightrightarrows}} \;\; G_0 \quad (u : G_0 \to G_1)$$

where the multiplication map m is defined on the set $G_1 \times_{G_0} G_1$ of *composable pairs of arrows*:

$$G_1 \times_{G_0} G_1 = \{(x, y) \in G_1 \times G_1 \mid r(x) = d(y)\} \,.$$

*Supported in part by FEDER and FCT through CAMGSD.

The above diagram makes sense in any category with pullbacks, for then the "object of composable pairs of arrows" can be defined, and in particular it makes sense in the category of topological spaces, where it gives us topological groupoids. A topological groupoid is *open* if d (equivalently, all the structure maps) is open, and *étale* if d (equivalently, all the structure maps) is a local homeomorphism. A related notion is that of *r-discrete* groupoid [15], which in the applications considered in [15] (due to the presence of a suitable measure) is the same as an étale groupoid.

In the category of *locales* (see §6 below) there are pullbacks and thus we can define groupoids in it, obtaining the notion of *localic groupoid*. Many topological definitions can be easily transferred to locales, in particular open maps and local homeomorphisms. *Open* localic groupoids and *étale* localic groupoids are defined accordingly. From any localic groupoid G there is a canonically associated topological groupoid $\Sigma(G)$ (its *spectrum*), whose spaces of arrows and units are respectively the spectra of the locales G_1 and G_0.

3 Inverse semigroups

An *inverse semigroup* (see [7]) is a semigroup S equipped with an involution $s \mapsto s^{-1}$ that satisfies $ss^{-1}s = s$ and such that all the idempotents commute. The set of idempotents $E(S)$ is a semilattice. As an example, if X is a topological space with topology $\Omega(X)$ there is an inverse semigroup $\Gamma(X)$ (an example of a pseudogroup) whose elements are the homeomorphisms $h : U \to V$ with $U, V \in \Omega(X)$. The idempotents are the identities on open sets, and thus $E(S) \cong \Omega(X)$. By the Vagner-Preston representation theorem, every inverse semigroup is, up to isomorphism, contained in a pseudogroup.

Every inverse semigroup S has a *natural order* defined by $s \le t$ if $s = ft$ for some idempotent f, which in $\Gamma(X)$ coincides with the restriction order. Two elements $s, t \in S$ are *compatible* if $st^{-1}, s^{-1}t \in E(S)$, and S is *complete* if every set $X \subseteq S$ whose elements are pairwise compatible has a supremum, or join, $\bigvee X$ in S. The representation given by the Vagner-Preston theorem does not necessarily preserve joins. This follows immediately from the fact that every locale (see §6 below) is an inverse semigroup (all the elements are idempotents), but not every locale is spatial.

4 Étale groupoids and inverse semigroups

Let G be a topological étale groupoid. For simplification let us assume that G_0 is a sober topological space (for instance a Hausdorff space, as is often assumed in applications — see §6). A continuous local section $s : U \to G_1$ of d is a *local bisection* if $r \circ s$ is a homeomorphism onto its image. It is easy to see that the local bisections form a sheaf of sets on G_0. In addition, the set of all the local bisections has the structure of a complete inverse semigroup $S = \Gamma(G)$ for which $E(S) \cong \Omega(G_0)$, where the multiplication is defined in terms of the multiplication of G in a straightforward manner.

There is a converse to this construction. For each complete inverse semigroup S whose semilattice of idempotents $E(S)$ is (isomorphic to) the topology $\Omega(X)$ of a sober space X we can define a sheaf of sets $\mathcal{D}$ where for each "open" $U \in E(S)$ the set $\mathcal{D}(U)$ contains the elements $s \in S$ such that $ss^{-1} = U$. If $V \le U$ are idempotents, the restriction map $\mathcal{D}(U) \to \mathcal{D}(V)$ is given by multiplication: $s \mapsto Vs$. The fact that this is a sheaf rather than just a presheaf is precisely equivalent to the completeness of S. Now the standard construction of a local homeomorphism from a sheaf (see, e.g., [8, Ch. II.5]) gives us a space $\Lambda_{\mathcal{D}}$ of "germs", along with a local homeomorphism $d : \Lambda_{\mathcal{D}} \to X$. This is the domain map of an étale groupoid Germs(S) whose other structure maps are obtained from the inverse semigroup structure of S.

If $S = \Gamma(G)$ for an étale groupoid G, every local section of d is "locally a local bisection", and thus $\Lambda_{\mathcal{D}}$ is homeomorphic to the space of germs of local sections of $d : G_1 \to G_0$, which is homeomorphic to G_1. It follows that both G_1 and the domain map $d : G_1 \to G_0$ are recovered from S. It can be verified that the remaining structure maps of the groupoid Germs(S) agree with those of G, giving us an isomorphism $\mathrm{Germs}(\Gamma(G)) \cong G$. Also, we have $\Gamma(\mathrm{Germs}(S)) \cong S$.

Hence, a topological étale groupoid (with a sober space of units) is essentially "the same" as a complete inverse semigroup with a spatial locale of idempotents. These results can be generalized to unit spaces that are not sober.

5 Quantales

A *unital involutive quantale* is a *sup-lattice* (i.e., a partially ordered set in which every subset X has a join $\bigvee X$ — and therefore also an infimum[1], or meet, $\bigwedge X$)

[1] Sup-lattices are complete lattices. The name "sup-lattice" is motivated [4] by thinking of joins as the first class operations, with meets being just derived. Accordingly, the homomorphisms of the category of sup-lattices are required to preserve only joins.

equipped with an additional structure of involutive monoid (the involution is usually denoted by $a \mapsto a^*$, and the multiplicative unit by e), where the involution preserves joins, and so does the multiplication in each variable:

$$\begin{aligned} a(\bigvee_i b_i) &= \bigvee_i (ab_i) \\ (\bigvee_i a_i)b &= \bigvee_i (a_i b) \\ (\bigvee_i a_i)^* &= \bigvee a_i^* \ . \end{aligned}$$

A *homomorphism* $f : Q \to R$ of unital involutive quantales is a function that preserves joins, the multiplication, the multiplicative unit, and the involution:

$$\begin{aligned} f(\bigvee S) &= \bigvee f(S) \\ f(ab) &= f(a)f(b) \\ f(e_Q) &= e_R \\ f(a^*) &= f(a)^* \ . \end{aligned}$$

Quantales are ring-like structures, and there are corresponding notions of *module*. A *left module* over a unital quantale Q is a sup-lattice M equipped with a left action $Q \times M \to M$ that preserves joins in each variable. For details on modules over involutive quantales see for instance [17].

6 Locales

To a large extent, the results with which we shall be concerned can be conveniently expressed in the language of *locale theory*, of which we give a very basic outline (for details see [2]).

By a *locale* is meant a sup-lattice in which binary meets distribute over arbitrary joins:

$$x \wedge \bigvee Y = \bigvee_{y \in Y} (x \wedge y) \ .$$

(Hence, any locale is a unital involutive quantale with multiplication given by $\wedge$ and trivial involution $a^* = a$.) The motivating example of a locale is the topology $\Omega(X)$ of a topological space X, ordered by inclusion of open sets. By a *map* $f : A \to B$ of locales is meant a *homomorphism* $f^* : B \to A$, i.e., a function that preserves arbitrary joins and finite meets (including the empty meet $1_B = \bigwedge \emptyset = \bigvee B$):

$$\begin{aligned} f^*(\bigvee S) &= \bigvee f^*(S) \\ f^*(a \wedge b) &= f^*(a) \wedge f^*(b) \\ f^*(1_B) &= 1_A \ . \end{aligned}$$

Again, the motivating example is the map of locales $\Omega(X) \to \Omega(Y)$ which is defined by the inverse image homomorphism $f^{-1} : \Omega(Y) \to \Omega(X)$ of a continuous map $f : X \to Y$ of topological spaces.

A *point* of a locale A is defined to be a map of locales $p : \Omega \to A$ from the topology Ω of a discrete one point space to A. The *spectrum* of a locale A is the topological space $\Sigma(A)$ consisting of the points of A with open sets of the form

$$U_a = \{p : \Omega \to A \mid p^*(a) = 1\} \ .$$

(This defines a functor Σ from locales to topological spaces.) The assignment $a \mapsto U_a$ is a surjective homomorphism of locales. A is said to be *spatial* if this is an isomorphism.

If a space X is the spectrum of a locale then there is a homeomorphism $\Sigma(\Omega(X)) \cong X$. Spaces with this property are called *sober* (e.g., any Hausdorff space). The category of sober spaces with continuous maps is equivalent to the category of spatial locales and their maps.

Locales are often important as replacements for the notion of topological space in a *constructive* setting, by which is meant the ability to interpret definitions and theorems in an arbitrary topos. See [3]. For instance, the locale $\mathrm{RIdl}(R)$ of radical ideals of a commutative ring R can be regarded as the "constructive Zariski spectrum" of R because $\Sigma(\mathrm{RIdl}(R))$ is (classically) homeomorphic to the usual space of prime ideals with the Zariski topology.

7 Étale groupoids and quantales

Let G be a topological étale groupoid. The fact that all the structure maps are local homeomorphisms implies two immediate facts: the unit space G_0 (rather, its image $u(G_0)$) is open in G_1; the pointwise product of any two open sets $U, V \in \Omega(G_1)$ is an open set. This makes $\Omega(G_1)$ a unital (and involutive) quantale. A topological groupoid is étale precisely if its topology has this property [16, Th. 5.18].

The algebraic characterization of the unital involutive quantales that arise in this way has been done in [16], leading to a correspondence between localic étale groupoids and certain quantales. From a localic étale groupoid G one obtains a quantale $\mathcal{O}(G)$ through the localic analogue of the construction just described. For want of a better name, let us refer to such quantales as *étale groupoid quantales*. Among other things they are also locales. The converse construction yields, from an étale groupoid quantale Q, a localic groupoid $\mathcal{G}(Q)$ whose locale of arrows is Q, whose locale of units is $\downarrow e = \{a \in Q \mid a \leq e\}$, and such that [16]

$$G \cong \mathcal{G}(\mathcal{O}(G)) \qquad (7.1)$$

$$Q \cong \mathcal{O}(\mathcal{G}(Q)) \ . \qquad (7.2)$$

The correspondence between inverse semigroups and topological étale groupoids can now be recast in terms of these quantales. Let S be an *abstract pseudogroup*, by which will be meant an inverse semigroup S whose idempotents form a locale $E(S)$, and let us denote by $\mathcal{L}^\vee(S)$ the set of all the downwards closed sets of S which are closed under the formation of all the existing joins of S.

Theorem. *$\mathcal{L}^\vee(S)$ is an étale groupoid quantale. If in addition $E(S)$ is a spatial locale then $\mathcal{L}^\vee(S)$ is a spatial locale, and the (spectrum of) the groupoid of $\mathcal{L}^\vee(S)$ is the germ groupoid of S:*

$$\Sigma(\mathcal{G}(\mathcal{L}^\vee(S))) \cong \mathrm{Germs}(S) .$$

We are therefore provided with a generalization of the construction of germ groupoids to the localic setting.

The "duality" expressed by (7.1) and (7.2) does not extend to an equivalence of categories because the inverse image locale homomorphism h^* of a map of localic groupoids $h : G \to H$ is not the same as a homomorphism of étale groupoid quantales $\mathcal{O}(H) \to \mathcal{O}(G)$ [16][2]. A consequence of this is that we are provided with an alternative category (a subcategory of quantales) whose objects are the étale groupoids. This category may be the right one to consider in some situations. For instance, the assignment $S \mapsto \mathcal{L}^\vee(S)$ is part of a left adjoint functor from abstract pseudogroups to étale groupoid quantales, and thus the identification of $\mathcal{L}^\vee(S)$ with a groupoid allows us to think of $\mathcal{L}^\vee(S)$ as being the "universal", or "enveloping", groupoid of S, with the proviso that the universality should be understood in the category of quantales rather than groupoids (in other words, roughly, it is the topology of the groupoid that is "freely" generated, rather than the groupoid itself). Paterson's universal groupoid of an inverse semigroup S [13] coincides with the groupoid of germs of a larger inverse semigroup S', but the universality described in [13, Prop. 4.3.5] is different.

In fact the adjunction just mentioned takes place between abstract pseudogroups and a larger category of quantales (the category of *stable quantal frames* [16]). The latter deserves attention because it has good properties, but besides the étale groupoid quantales it contains other quantales. These can be identified with involutive graphs that are almost étale groupoids, with the exception that their multiplication is "fuzzy" because it assigns to each composable pair of arrows an open set of arrows rather than just an arrow [16, §4.4]. The usefulness of such a generalization in applications, in particular in terms of the idea of symmetry, is yet to be examined.

8 Open groupoids and quantales

The topology of a topological open groupoid G is, similarly to that of an étale groupoid, closed under pointwise multiplication of open sets, and thus it is a quantale. A similar situation exists for open localic groupoids. The axioms that provide the algebraic description of étale groupoid quantales in [16] can be weakened so as to provide a characterization of the quantales (no longer unital) associated to open groupoids. Such quantales are an algebraic counterpart of open groupoids that generalizes the role played by inverse semigroups.

A continuous representation of a topological open groupoid G consists of an *action* of G on a bundle $p : X \to G_0$ (with open p) i.e., a map

$$\alpha : X \times_{G_0} G_1 \to X$$

satisfying suitable conditions, where $X \times_{G_0} G_1$ is the pullback of p and the domain map d. It can be verified that α is necessarily an open map. This fact leads to an action of the open subsets of G_1 on those of X that makes $\Omega(X)$ a $\Omega(G_1)$-module, and an analogous situation exists for localic groupoids.

Not surprisingly, it follows that the continuous representations of an open localic groupoid G can be identified with certain modules over $\mathcal{O}(G)$. Perhaps more surprisingly, the morphisms of groupoid representations can be identified with module homomorphisms (this is not true for groupoids themselves and their quantales, as we have remarked in §7), yielding a dual equivalence of categories.

9 Applications

There are many applications of groupoids in analysis, topology, geometry, and naturally also in algebra and category theory. For instance, Lie groupoids play an important role in differential geometry, and the interplay between such groupoids and operator algebras is a large part of what is meant by noncommutative geometry [1, 13, 15], where in general one constructs C*-algebras from locally compact groupoids that are equipped with Haar measures (such groupoids are necessarily open). Some instances of this interaction are particularly well behaved. For instance, any AF-algebra is a groupoid C*-algebra of an étale groupoid (an AF-groupoid), and the relation between the two is mediated by an inverse semigroup [15, Ch. III.1]. In another direction, in topos theory open groupoids are important due to the fundamental theorem of Joyal and Tierney [4] which states that every Grothendieck topos is equivalent to the category of continuous representations of an open localic

[2] A way around this would be to either expand or restrict the classes of morphisms under consideration. An analogous situation occurs with the equivalences of categories between inverse semigroups and inductive groupoids in [7, Ch. 4, p. 114].

groupoid. An immediate question is how useful a reformulation of this theorem in terms of quantale modules may be.

The original motivations behind the name "quantale" are also related to operator algebras [9], the idea being that the underlying "noncommutative space" of a noncommutative C*-algebra should be a quantale, generalizing the fact that the spectrum of a commutative C*-algebra is (the spectrum of) its locale of norm-closed ideals. In addition, it was suggested [9] that such a generalization of locales could provide the context for a constructive theory of noncommutative C*-algebras. This idea has led to several notions of *point* of a quantale [5, 11, 14] in the form of suitable "simple" modules, and to a representation theory in terms of which the "points of noncommutative spaces" can be classified. For instance, the equivalence classes of irreducible representations of a unital C*-algebra A can be identified (albeit nonconstructively) with the points of the quantale $\mathrm{Max}\, A$ of norm-closed linear subspaces of A [11], a somewhat surprising consequence of this being that the quantale valued functor Max is a complete invariant of unital C*-algebras [6]. Another example is the quantale of Penrose tilings of the plane [12], whose points (of a certain type) can be identified with the Penrose tilings.

Despite the progress achieved in this area, the interaction between quantales and C*-algebras is still not well understood, and attention should be given to the relations between groupoid C*-algebras, groupoid quantales, and quantales like $\mathrm{Max}\, A$.

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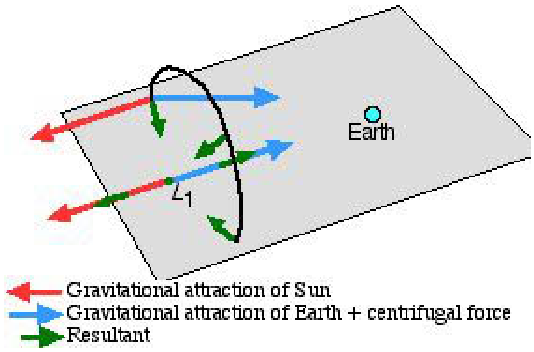
Royal Welcome to Abel Laureate Peter Lax. Norway's Crown Prince Regent awarded the 2005 Abel Prize to Peter D. Lax on May 24. The city of Oslo and the Norwegian Academy of Science and Letters prepared for several days of events that honored Lax, including the prize ceremony, lectures by and in honor of Lax, a banquet at the Akershus Castle, and some special events for local teachers and students.

HRH the Crown Prince Regent presented the Abel Prize 2005 to Peter D. Lax. Photo: Knut Falch/Scanpix (The Abel Prize/The Norwegian Academy of Science and Letters).

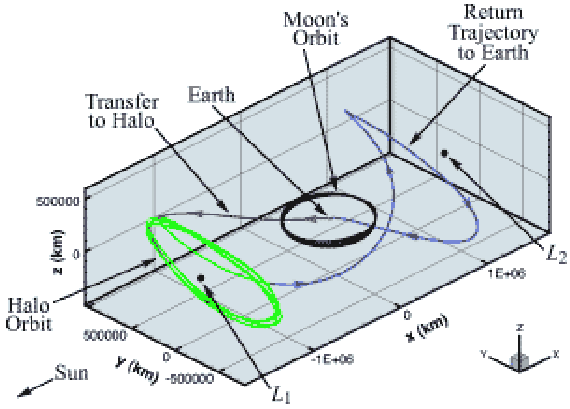
Lax received the US$980,000 Abel Prize "for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions." See the Abel Prize website (http://www.abelprisen.no/en/) for details about the international prize ceremony, all the festivities, and the 2005 Abel Symposium. In http://www.abelprisen.no/en/prisvinnere/2005/documents/popular2005eng9.pdf you may find the article "Peter D. Lax, Elements from his contributions to mathematics", by Professor Helge Holden, where the work of this year's Abel Laureate is described.

The math beneath the Interplanetary Superhighway. The cover story for the April 16 2005 *Science News* was Erica Klarreich's "Navigating Celestial Currents," with subtitle: "Math leads spacecraft on joy rides through the solar system." The spacecraft in question was NASA's *Genesis*. [The joy ride ended in what NASA terms a "hard landing" in the Utah desert. Fortunately much useful scientific information survived. More on the NASA website.] Klarreich's piece is about the way the mathematical analysis of the Solar System Gravitational Dynamical System (the sum of the gravitational fields of all the objects in the system) led to the discovery of extremely fuel-efficient orbits. Edward Belbruno, now at Princeton, pioneered this approach twenty years ago, and scored its first great success in 1991 when he rescued the Japanese *Hiten* spacecraft, stranded in Earth orbit without enough fuel, it seemed, to reach the moon. Belbruno showed how to exploit the chaotic nature of the SSGDS to calculate a long-duration, low-cost trajectory which would lead the spacecraft to its destination. Chaotic here does not mean disorderly, but refers to the enormous change in behavior that can be produced by a tiny change in initial conditions near an unstable critical point of the system. The *Hiten* rescue used the unstable critical points in the Earth-Moon system; there are three of them, known as the Lagrange points L_1, L_2 and L_3.



At L_1, the sum of Earth's gravity and the centrifugal force exactly balance the gravitational attraction of the Sun. At points on the (black) halo orbit the vector sum would pull a mass towards L_1; this can be balanced centrifugally by motion of the mass along the orbit.

The Genesis mission used the similarly defined and labelled points in the Earth-Sun system: with notation from the diagram above, at the L_1 point the Sun's gravity (red) is exactly balanced by the sum (blue) of the Earth's gravity and the centrifugal force produced by the yearly rotation of the Sun-L_1-Earth axis. (At L_2 and L_3 that centrifugal force balances the sum of the two gravities). The axis is crossed at L_1 by a two-dimensional surface at each point of which the (green) resultant of the red and blue vectors is tangent to that surface. A mass on that surface will fall towards L_1 unless it is orbiting rapidly enough, staying in the surface, to balance that attraction by centrifugal force. These are the "halo orbits" shown in Shane Ross's illustration below.



The *Genesis* mission parked in a halo orbit while it studied the solar wind, then looped around L_2 for a leisurely and inexpensive trip back to Earth. Image by Shane Ross (USC), used with permission.

But if the mass strays ever so slightly away from the surface, it will spiral either away from the Earth or away from the Sun. A tiny bit of fuel can send it on its way. We can think of "freeways" linking halo orbits around the three unstable equilibrium points. Want an inexpensive trip to Jupiter? Time your trajectory so that you're there when one of the Sun-Jupiter freeways crosses a Sun-Earth freeway; then a little nudge from your thrusters will do the trick. Klarreich's article is available online at `www.sciencenews.org/articles/20050416/bob9.asp`.

Saunders Mac Lane, 1909-2005. Saunders Mac Lane, AMS President from 1973 to 1974, died recently. Mac Lane helped develop category theory and co-authored *A Survey of Modern Algebra* with Garrett Birkhoff. The MacTutor History of Mathematics Archive (`www-groups.dcs.st-and.ac.uk/~history/Mathematicians/MacLane.html`) has Mac Lane's biography and list of publications. Read "Garrett Birkhoff and The *Survey of Modern Algebra*," (`www.ams.org/notices/199711/comm-maclane.pdf`) by Mac Lane in Notices of the AMS, December 1997. For the University of Chicago obituary for Saunders Mac Lane, which was released to the Associated Press, see `www-news.uchicago.edu/releases/05/050421.maclane.shtml`.

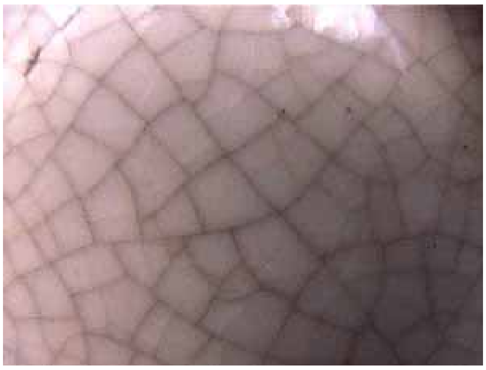
Peter Lax in the *New York Times*. Peter Lax won the Abel Prize this year.

Peter D. Lax, Abel Laureate 2005 (Photo: New York University).

On that occasion, he was interviewed by Claudia Dreifus of the *Times*; the interview appeared in the Science section on March 29, 2005, with a photograph showing Lax in his NYU office in front of a blackboard bearing the prominent and talismanic chalk inscription: $\delta = \log 4/\log 3$. Dreifus leads Lax through his early days: Budapest and Stuyvesant High School ("I didn't take any math courses at Stuyvesant. I knew more math than most of the teachers"). Lax was drafted in 1944 at the age of 18, and ended up at Los Alamos. "I arrived six weeks before the A-bomb test. ... Looking back, there were two issues: should we have dropped the A-bomb and should we have built a hydrogen bomb? Today the revisionist historians say that Japan was already beaten ... I disagree. ... I also think that Teller was right about the hydrogen bomb because the Russians were sure to develop it. And if they had been in possession of it, and the West not, they would have gone into Western Europe. What would have held them back? Teller was certainly wrong in the 1980's about Star Wars. ... The system doesn't work. It's a phantasmagoria." Dreifus asks what Von Neumann would think about the ubiquity of computers today. "I think he'd be surprised. ... But remember, he died in 1957 and did not live to see transistors replace vacuum tubes." Did he know John Nash? "I did, and had enormous respect for him. He solved three very difficult problems and then he turned to the Riemann hypothesis. ... By comparison, Fermat's is nothing." Does he believe high school and college math are poorly taught? "... In mathematics, nothing takes the place of real knowledge of the subject and enthusiasm for it."

2005 Wolf Prize. Gregory A. Margulis (Yale University) and Sergei P. Novikov (University of Maryland, College Park) have been named co-winners of the 2005 Wolf Prize in mathematics by the Wolf Foundation and will share $100,000. The selection committee cited Margulis for his "monumental contributions to algebra" and Novikov for his contributions to algebraic topology, differential topology and mathematical physics. The prizes were awarded on May 22 in Jerusalem.

The math of craquelure. "Four Sided Domains in Hierarchical Space Dividing Patterns" is the title of an item published on February 9, 2005 in *Physical Review Letters*, and picked up in the "Research Highlights" section of the February 24 2005 *Nature.* The authors, Steffen Bohn, Stephane Douady and Yves Coudert (Rockefeller University and ENS, Paris) begin with the observation that, in the tilings formed by the cracks in ceramic glazes, the average number of sides of a tile is four. This seems unnatural, at first glance: Generically the edges of a tiling meet three by three. Euler's characteristic for a convex domain gives Vertices - Edges + Faces = 1 or $V - E + F = 1$. Since every edge joins two vertices, generically $2E = 3V$; Euler's equation then gives $3V - 3E + 3F = 3$ and so $3F - E = 3$. When the number of faces is large we can write $3F = E$ and since each edge is shared by two faces, this means that the faces must be, on average, six-edged. How the six edges become four sides in crackle finishes is clear from the picture below.



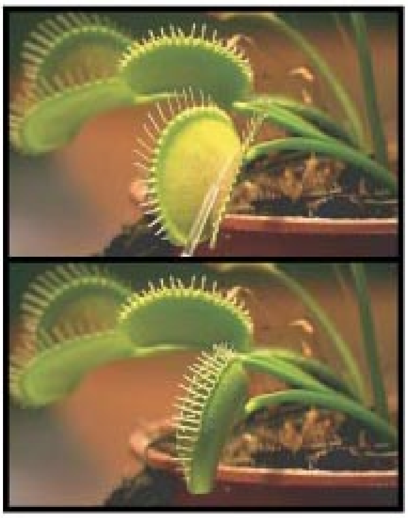
Craquelure in ceramics results from the differential shrinking of coats of glaze. The characteristic pattern is different from other naturally occurring tilings, which usually involve hexagons.

The authors explain the general mechanism at play: they define a *hierarchical space-dividing pattern* as one formed by "the successive divisions of domains and the absence of any further reorganization," and they show that "the average of four sides is the signature of this hierarchy." Another example is the organization of veins and sub-veins in the framework of a leaf (earlier work of theirs in this direction was referred to in the cover illustration of *Science* for February 6, 2004). Finally, they remark that the street network in a city where "growth resulted from self organization" is also of this type, and exhibit as evidence part of a 1760 map of Paris. Article available online at `http://asterion.rockefeller.edu/steffen/`.

Proof checking by computer assistants. Anyone who has ever been hoodwinked by a false proof of an intricate statement will be grateful to know that computers have been trained to take over the job of checking arguments. This is explained by Dana Mackenzie in the March 4 2004 *Science*, in an article with the title "What in the Name of Euclid Is Going On Here?" Mackenzie evokes the following problematic "scenario that has repeated itself, with variations, several times in recent years: A high-profile problem is solved with an extraordinarily long and difficult megaproof, sometimes relying heavily on computer calculation and often leaving a miasma of doubt behind it." The remedy is now at hand: software packages ("proof assistants") which "go through every step of a carefully written argument and check that it follows from the axioms of mathematics." The best-known examples are Coq, HOL and Isabelle. Recently Coq was used by Georges Gonthier to check the proof of the Four-Color Theorem, the archetype of Mackenzie's scenario. It passed; Gonthier's paper is available online. Isabelle was put through its paces by Jeremy Avigad to check the proof of the Prime Number Theorem. HOL-light has been used by Thomas Hales to check the Jordan Curve Theorem, a warm-up perhaps for a verification of his work on the Kepler Conjecture. Mackenzie muses on the philosophical implications of these new developments. "Ever since Euclid, mathematical proofs have served a dual purpose: certifying *that* a statement is true, and explaining *why* it is true. Now these two epistemological functions may be divorced. In the future, the computer assistant may take care of the certification and leave the mathematician to look for an explanation that humans can understand."

"Noether's Novelty," by John Derbyshire. In *National Review Online*, 21 April 2005, Derbyshire recalls that German mathematician Emmy Noether died 70 years ago in April. He describes her as "the greatest female mathematician of the 20th century, and quite possibly of all time." The article places Noether in the context of her time, place and culture—both societal and mathematical. Although she produced "a brilliant paper resolving one of the knottier issues in General Relativity" praised by Einstein, and although David Hilbert fought on her behalf to have her appointed to the faculty at Göttingen during World War I, she had many uphill battles. She won the admiration of colleagues and students but was "ill-paid and un-tenured," and when the Nazis came to power in 1933 she lost her job. While her mathematician brother Fritz emigrated to Siberia, Emmy came to Bryn Mawr in Pennsylvania, where she died two years later. Derbyshire concludes the piece with a quote from her obituary written by Albert Einstein, published as a letter to the editor in *The New York Times* on May 5, 1935, in which Einstein classifies Noether as one of the "genuine artists, investigators and thinkers" of the world.

Differential geometry and the Venus Flytrap. An international team led by Yoël Forterre has used "high-speed video imaging, non-invasive microscopy techniques and a simple theoretical model" to investigate how the Venus flytrap can snap shut rapidly enough to catch its prey. The authors, reporting in *Nature* for January 27, 2005, argue that "the macroscopic mechanism of closure is determined solely by leaf geometry." The image below shows clearly that both in the open state and the closed the leaf has positive gaussian curvature, and that it starts curved outward but ends curved inward.



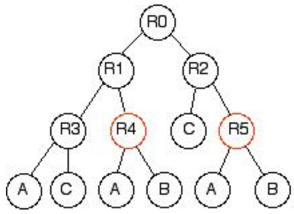
The rapid closure of the Venus flytrap (*Dionaea muscipula*) leaf in about 100ms is one of the fastest movements in the plant kingdom. Image from Nature (`www.nature.com`) **433** 422, used with permission.

Choosing curvilinear x and y coordinates on each half of the leaf, with x increasing in the direction of the spines, and y increasing perpendicularly to the right, they observe that the change in the principal curvature κ_x is the main actor in the phenomenon. "For a doubly-curved leaf ... bending and stretching modes of deformations are coupled, meaning that bending the leaf causes its mid-plane to be stretched. If the coupling is weak, the leaf can change its shape from open to closed by varying its gaussian curvature and stretch without a large energetic cost. In such a situation, the leaf deforms smoothly to accommodate the change in κ_x. If the coupling is strong, the leaf will not deform much (owing to the large energetic cost of stretching its mid-plane), until eventually the change in κ_x becomes so large that the leaf snaps shut rapidly." The authors derive ("poroelastic shell dynamics") a mathematical model which accurately mimics the detailed changes in leaf geometry. The Supplementary information posted on the Nature website (`www.nature.com/nature/journal/v433/n7024/suppinfo/nature03185.html`) includes a 1/4-speed video of the flytrap, tickled with a glass pipette as in the image above, snapping shut.

Wrong, Wrong and Wrong. Math Guides Are Recalled. That's the headline on an article by Susan Saulny in the March 25 2005 *New York Times*. The hapless New York math educators have done it again. This time, "City education officials were forced to recall test preparation materials for math exams late Wednesday after discovering that they were rife with errors, including basic arithmetic mistakes." Randi Weingarten, the head of the United Federation of Teachers, was reportedly outraged: "Tweed [the NYC Department of Education, located in the Tweed Courthouse] has no problem with excessively criticizing teachers for failing to meet its picayune mandates. But then it produces a test prep manual riddled with errors and misspellings. The hypocrisy is stunning." The *Times* printed two examples of questions with wrong answers and called on Alfred Posamentier, mathematician and dean of the City College School of Education, for the final word: "... in mathematics, where you have such an exact science, there is no room for error."

Amateur math in ancient Japan. *Science* magazine for March 18, 2005 ran a "News Focus" item by Dennis Normile, under the title 'Amateur' Proofs Blend Religion and Scholarship in Ancient Japan." Datelined Tokyo, the piece is prompted by an exhibition of Edo period *sangaku* (wooden tablets inscribed with geometric theorems) that opened at the Nagoya City Science Museum last May. During that period (1603-1868) "when Japan was isolated from the rest of the world, a unique brand of mathematics flourished in the country's shrines and temples. Amateur mathematicians crafted geometric theorems on elegant wooden tablets ... and offered them to the gods." The exhibition is due largely to the efforts of Hidetoshi Fukagawa, a high school math teacher who stumbled upon sangaku while "looking for material to enliven his classes," and has spent decades tracking them down and deciphering their contents. Some of the theorems stated (notably Soddy's Hexlet - see Bob Allanson's animation) were published on a sangaku many years (in this case, 114 years) before their discovery in the West. And this was all the work of "amateurs." As Fukagawa puts it: "There was no academia as we know it. So samurai, farmers and merchants all felt free to study mathematics." The tablets contain theorems but, in fact, no proofs. Fukagawa again: "Ostensibly, the tablets were left as gifts to the gods. In reality, people were showing off and challenging others to work out the proof."

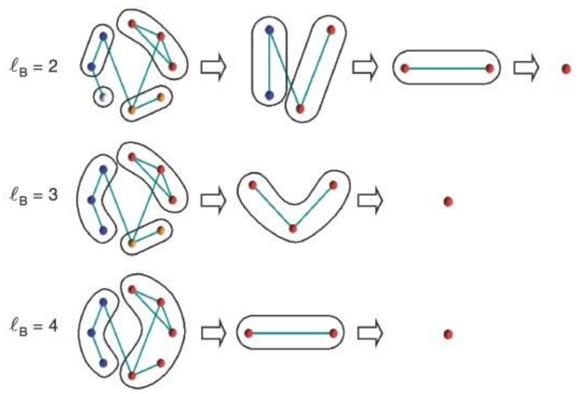
Algorithmic complexity in evolution. The idea of algorithmic complexity goes back, in some sense, to Leibniz (see Greg Chaitin's home page at `www.umcs.maine.edu/~chaitin/`). The general concept is suggested by Chaitin's definition: *The (algorithmic) complexity of a sequence of 0's and 1's is the length of the shortest computer program that will generate the sequence*. An international team, led by Ricardo Azevedo (University of Houston), has recently applied this concept to the study of the development of multicellular organisms. Their work appears as "The simplicity of metazoan cell lineages" (*Nature*, January 13, 2005). "Lineage" refers to the fact that all the cells in an organism descend from a single cell, the fertilized egg. But a typical metazoan has a large variety of different kinds of cells (brain, skin, bone, etc.). So in the family tree, traced back from a single cell in the complete organism, there must be one or more *nodes* where a mother cell divided into two dissimilar daughters. Algorithmically, each of these nodes corresponds to a division and differentiation rule. In the study, part of the tree (a *lineage*) is reduced by identifying functionally similar nodes. The number of reduced nodes divided by the original total is the algorithmic complexity of the lineage.



A schematic (non-biological) lineage illustrating the reduction process. Here nodes R4 and R5 are collapsed to RR4 in the reduced lineage: the algorithmic complexity of the original lineage is $4/5 = 80\%$.

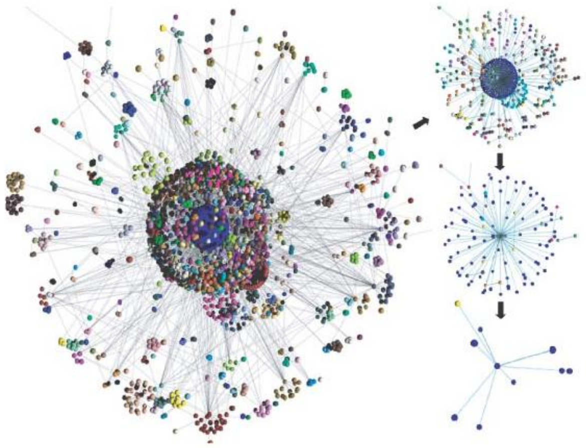
The team computed the complexity for lineages in four different multicellular organisms: three species of free-living nematodes (microscopic groundworms) and a sea squirt. The numbers worked out to 35%, 38%, 33% and 32% in the four cases. In a first analysis, the team "compared each real lineage to lineages with the same cell number and distribution of terminal cell fates but generated by random bifurcation." They found that real lineages were $26 - 45\%$ simpler than the corresponding random lineages. Conclusion: evolution selects for simpler lineages. (Tentative explanation: "the specification of simpler cell lineages might require less genetic information, and thus be more efficient.") In a second analysis, they "used evolutionary simulations to search for lineages that had the same terminal cell number and fate distribution as the actual lineages but were simpler." They found that after 20,000 to 50,000 generations they "could evolve lineages that were $10 - 18\%$ simpler than the ancestral, real lineages." One explanation is "developmental constraints imposed by the spatial organization of cells in the embryo." They added these constraints to their simulations and conclude that "the metazoan lineages studied here are almost as simple as the simplest evolvable under strong constraints on the spatial positions of cells."

Six degrees of self-similarity. The "six degrees of separation" phenomenon (so named when the network is acquaintance among people today) is often observed in complex networks. The "six" becomes the average diameter of the network. A Letter in the January 27 2005 *Nature* shows that many naturally occurring complex networks are also self-similar. The authors (Chaoming Sung and Hernán Makse (CCNY), Shlomo Havlin (Bar-Ilan)) focus on "connectivity between groups of interconnected nodes on different length scales," which they study by a renormalization procedure (see caption below).



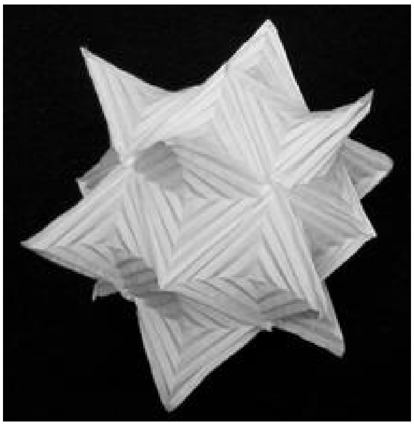
The authors' network renormalization, applied to a schematic network with 8 nodes. For each box length I_B the network is tiled with boxes in which all the nodes are $< I_B ub >$ steps away from each other. Then the boxes are replaced by nodes, which inherit connections, and the renormalization into boxes is repeated, with the same length criterion. The procedure terminates when the network has been collapsed to a single node. The total number of boxes required is $N_B(I_B)$. Finally $\log N_B(I_B)$ is plotted against $\log I_B$. If the points fall on a line, the network is said to be self-similar, with fractal (box) dimension d_B equal to minus the slope of that line. Image from *Nature* **433** 392, used with permission.

They apply this analysis to the following networks: hyperlinkages among the 325,729 web pages of a subset of the World Wide Web; 392,340 actors (linked if they have been cast together in at least one film); and various networks from molecular and cellular biology. All of these networks turn out to be self-similar: d_B for the WWW is 4.1, for the actors 6.3. Here is the author's illustration of renormalization carried out on their WWW sub-network, with $I_B = 3$:



Box size 3 renormalization of a 325,729-page subnetwork of the WWW. Image from *Nature* **433** 392, used with permission.

More Origami Mathematics. Each Tuesday the "Science Times" features a "Scientist at Work." For February 15, 2005 the scientist was Erik Demaine of MIT. Margaret Wertheim wrote the piece; she tells us that Demaine "is the leading theoretician in the emerging field of origami mathematics, the formal study of what can be done with a folded sheet of paper." Some pretty amazing things can be done. For example, Demaine, working in 1998 with his father Martin Demaine and with Anna Lubiw, showed that *any* polygonal shape can be made out of a single piece of paper by folding it flat appropriately and making one complete straight cut. The *Times* shows a swan; Demaine's Fold-and-Cut webpage (`theory.lcs.mit.edu/∼edemaine/foldcut/`) gives angelfish, butterfly, jack-o'lantern and more. Wertheim leads us through some of Demaine's other interests: linkages ("A linkage is a set of line elements hinged together like the classic carpenter's rule.") which are related to protein folding ("... molecular biologists would like to be able to predict from the chemical structure of a protein what shape it would fold into") and graph theory ("known to be fiendishly difficult, but Dr. Demaine is confident he can make headway once he immerses himself in its arcane lore.")



A "cube" constructed by Demaine, Demaine and Lubiw out of *hypads*, origami hyperbolic paraboloids. Image courtesy Erik Demaine, used with permission.

There is more in the article and much, much more on Demaine's website at `theory.csail.mit.edu/∼edemaine/`. For earlier media math origami see this bulletin for December, 2004.

Relativity. Incompleteness. Uncertainty. Thus runs the first paragraph of Eric Rothstein's February 14 2005 "Connections" column (every other Monday, in the *New York Times*). The piece is a meditation on Einstein, Gödel and Heisenberg, occasioned by the publication of Rebecca Goldstein's new book "Incompleteness: The Proof and Paradox of Kurt Gödel" (Atlas Books; Norton). Rothstein contrasts Heisenberg, whose "allegiance to an absolute state, Nazi Germany, remained unquestioned even as his belief in absolute knowledge was quashed," with Einstein and Gödel who "fled the politically absolute, but believed in its scientific possibility." Most of the column is saved for Gödel's Incompleteness Theorem. "Before ..., it was believed that not only was everything proven by mathematics true, but also that within its conceptual universe everything true could be proven. Gödel shattered that dream. He showed that there were true statements in certain mathematical system s that could not be proven. And he did this with astonishing sleight of hand, producing a mathematical assertion that was both true and unprovable." Rothstein, following Rebecca Goldstein, gives Gödel's result a positive twist: "But what if the theorem is interpreted to reveal something positive: not proving a limitation but disclosing a possibility? ... In this, Gödel was elevating the nature of the world, rather than celebrating powers of the mind. There were indeed timeless truths. The mind would discover them not by following the futile methodologies of formal systems, but by taking astonishing leaps, making unusual connections, revealing hidden meanings."

Crocheted Manifold. As Daniel Engber reported in the *Chronicle of Higher Education* for January 21, 2005, a team at the University of Bristol has used yarn and a crochet hook to build a model of the Lorenz manifold.

A close-up view of the crocheted Lorenz manifold. The origin, the center of the bulls-eye pattern on the right, is just hidden from sight. The wire looping through the origin is the strong stable manifold of the system. The manifold's vertical axis of symmetry can be seen as a diagonal across the upper half of this image. Photo: University of Bristol, used with permission.

This is the 2-dimensional stable manifold of the origin in the Lorenz system

$$\begin{cases} x' = \sigma(y - x) \\ y' = \rho x - y - xz \\ z' = xy - \beta z \end{cases}$$

with the classic choice of parameters $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$. According to Engber, Hinke Osinga and Bernd Krauskopf realized that the computer program they had devised for generating the Lorenz manifold could be adapted to produce a set of crocheting instructions.

Osinga and Krauskopf with their model of the Lorenz manifold. Photo: University of Bristol, used with permission.

"Each computed point on the manifold translates to a type of crochet stitch. A mere 85 hours and 25,511 stitches later, the project was finished." Osinga and Krauskopf's work appeared in the fall issue of *The Mathematical Intelligencer*; their preprint is available as a PDF file online (`http://www.enm.bris.ac.uk/anm/preprints/2004r03.html`). The crocheted Lorenz manifold struck the fancy of the international media, including the

BBC: *Mathematicians crochet chaos*
`news.bbc.co.uk/1/hi/education/4099615.stm`,

CBC Radio: *Crocheting Chaos*
`www.cbc.ca/aih/STEAM/2004/crocheted_chaos_20041216.html`,

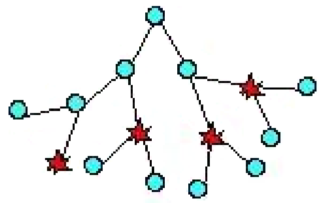
the Austrian ORF:
`science.orf.at/science/news/131349`,

Channel One in Russia:
`www.1tv.ru/owa/win/ort6_main.main?p_news_title_id=73197&p_news_razdel_id=9`.

"Blood, math and gore." It could work." That's the end of Alessandra Stanley's review of the new TV series "Numb3rs," in the January 21 2005 *New York Times*. The plot line involves "Don, a decent, workaholic F.B.I. agent who turns to his math genius younger brother, Charlie" for help in tracking down a serial rapist. As Stanley tells it, "Charlie looks at a water sprinkler and has an Archimedean moment: he realizes that the same principle that allows him to track the path of drops to determine their point of origin could be applied to the distribution of crime scenes on a map." (She quotes one character as saying: "If this works, we'll have a whole new system of investigating criminal cases.")

More about "Numb3rs". A more academic view was taken by NPR's "Math Guy" Keith Devlin, interviewed by Scott Simon on "Weekend Edition - Saturday" for January 22, 2004. Scott: "There's a scene where the mathematician brother is writing out a formula on the board. Firstly he seems to be listening to head-banging rock music and in addition to that he seems to be in the grip of a fever. Is that commonly what happens when mathematicians write out formulas?" Keith: "... Most people's impression of a mathematician, if that impression is of an elderly guy in a tweed suit and worn down shoes, they'd better walk around a university like Stanford or Cal Tech or MIT and just take a look. In fact when David Krumholtz was preparing for this role, he hung around Cal Tech for a while and just watched what he saw." Scott plays a clip in which Charlie consults a fellow mathematician who tells him: "Charlie, when you're working on human problems, there's going to be pain and disappointment." Keith: "This reflects one of the most interesting changes in the whole history of mathematics. ... Over the last few hundred years increasingly we've found that we can take this mathematics which was originally developed to study the physical world and apply it to the world of people, and by using computer graphics superimposed on action you can show people that mathematics, this abstract stuff, really applies to the real world and, in the case of a crime series, with positive outcomes for society." Scott: "Do you expect that this series could do for mathematics what 'The Simpsons' did for cartoons?" Keith: "I would hope it does succeed because the one thing they're trying to do is make mathematics look cool. I know it's cool, all my friends know it's cool. We do have an image problem, and I think a TV series like this can help get over it." The interview is available online at `www.npr.org/templates/story/story.php?storyId=4462642`.

Terror Network Theory. On December 11, 2004, Jonathan Farley was interviewed on Air America's "So What Else Is News" by the program host, resident whiz-kid Marty Kaplan. Farley, currently a Visiting Scholar at Harvard, turns out to be a mathematician with a mission. Inspired by the real and hypothetical mathematical derring-do evoked in "A Beautiful Mind," he has found an application of lattice theory to the war on terror. His problem is the structure of terror cells and what it takes to disrupt them. A current approach, he tells us, is to view a terrorist cell as a graph, "a picture where you've got a bunch of nodes or dots which represent the individuals, and then lines which connect individuals if they have some sort of communications link, or if they lived in the same flat in Hamburg at some time ..." Graph-theoretically, a cell is disrupted if the graph is disconnected. Farley noticed that with that kind of analysis "you're missing a key mathematical component of the terrorist network, namely its hierarchy. And that's where I come in, because my branch of mathematics, called lattice theory, deals with hierarchy and properties of order." Kaplan proposes a concrete example: suppose a cell has 15 people, "and the government has picked off 4 of them. To what degree can the government feel as though they have shut that cell down?" Farley explains that for a precise estimate you would need to know the structure of the cell, but he shows how, for a 15-node binary tree, hierarchically ranked from top to bottom, the graph-calculation and the lattice-calculation give very different answers. "If you've captured 4 guys you're pretty sure you've disrupted the cell, under the old way of thinking. But when you take the lattice-theoretic perspective, you see that actually you only have a 33% chance of disrupting the cell in that case."



Removal of four nodes at random has a 93% chance of disconnecting a 15-node binary tree, but only a 33% chance of breaking all top-to-bottom chains of command.

He elaborates: "If 4 people have been captured at random, it might still be possible for terrorist plans to be passed on from the leader down to one of the people at the bottom, one of the eight foot soldiers, in which case you might have another September 11, you might have a shoe-bombing ..." And finally: "Mathematics won't help you catch the terrorists, but it will help you analyze how good a job you've done in the past." Farley's work has also been covered by Ivars Peterson in *Science News Online* (`www.sciencenews.org/articles/20040110/mathtrek.asp`, January 10, 2004).

Ant Geometry. "Pheromone trails are used by many ants to guide foragers between nest and food. But how does a forager that has become displaced from a trail know which way to go on rejoining the trail?" Richard Feynman (in "Surely you're joking ...") considered this problem and speculated that a direction might be written into the pheromone trail (e.g. A-B-space-A-B-space). In fact, the ants use information encoded in the geometry of the (plentiful) bifurcations along the trail. This has been conclusively shown by Duncan Jackson, Mike Holcombe and Francis Ratnieks, a computer science-biology team at the University of Sheffield; work reported in the December 16 2004 *Nature.*

Exits on the Pheromone Highway branch on average 53° from the "away" direction, so each intersection reads like an arrow pointing home. Image © Duncan Jackson, used with permission.

The Sheffield team worked with colonies of Pharaoh's ant (*Monomorium pharaonis*). In one of their reported experiments, they allowed individual ants to walk along experimental, straight trails: "any reorientations occurring were as likely to be correct as incorrect in relation to the polarity of the trail when it was originally formed." In contrast, when meeting a trail bifurcation, "43% of fed ants [who would presumably be heading home] made U-turns upon meeting the bifurcation point when walking in the 'wrong' direction: that is, away from the nest. Conversely only 8% of fed ants walking the 'correct' way made ... corrections that led them to heading incorrectly away from the nest." On the other hand, of the unfed ants, who presumably would be heading out for food, "47% made course corrections at the bifurcation point when moving the 'wrong' way" while "only 8% walking the 'correct' way made incorrect course changes." The authors add: "Note that although only 45% of the ants corrected their orientation at a single bifurcation, real networks contain many bifurcations and many opportunities for course correction." [Interpreted as Markov chains, these numbers say that in the steady state 84% of the fed ants and 85.4% of the unfed will be moving in the appropriate direction.]

The Magic of Math, in Queens. On November 24, 2004 the *New York Times* ran "From Internet Arm Wrestling to the Magic of Math," Edward Rothstein's review of the new wing of the New York Hall of Science, in Queens. Rothstein glances at the high-tech baubles of the new installations, but saves most of his admiration for the *Mathematica* exhibition, which the Hall of Science recently acquired from the California Science Center. *Mathematica* was created for IBM in 1961 by the celebrated design team of Charles and Ray Eames. Rothstein remembers seeing it as a child: "I still recall wired structures rising out of soapy liquid, their swirling surfaces demonstrating solutions of mathematical problems; the cubic array of bulbs that translated simple multiplication into three-dimensional patterns of light; the suspended Möbius strip — a surface with only one side and one edge — on which a train continuously ran." And he ponders the difference between this exhibition, assembled at the apex of the post-sputnik wave of enthusiasm for science, and the flashier but shallower productions of today. "*Mathematica* samples varied branches of mathematics, not blanching from explaining functions or projective geometry; contemporary exhibitions set their sights lower, restricting each display's focus. *Mathematica* knows you won't fully understand it all ... . Contemporary displays are more concerned that you grasp a single concept. They are play stations in a science lesson."

Father of fractals. That's the title of Jim Giles' News Feature in the November 18 2004 *Nature*. Benoit Mandelbrot is the Father; the article is illustrated with a large and spooky image of part of "the set that bears his name." Giles gives us a capsule intellectual history of Mandelbrot, taking him from his 1963 paper on self-similarity in graphs of cotton prices, through his years as an "academic wanderer" and the 1982 publication of *The Fractal Geometry of Nature*, when "the worlds of math and physics took notice." After a brief and completely non-technical digression on fractals, we come to the main point of the paper: Mandelbrot's attitude. Apparently, he has not been very nice. "As so often happens in academia, questions of precedence were central." We hear reports of aggressivity and misbehavior at conferences. Then Giles focuses on Mandelbrot and Vilfredo Pareto (1848-1923), who had published "similar studies on power laws in economics" many years before. Giles claims that in the most recent reprinting of Mandelbrot's 1963 paper on cotton, "many references to Pareto have been removed." And that one paper by a third author, *Mandelbrot and the stable paretian hypothesis*, appears in the same collection with a new title: *Mandelbrot on price variation*. Are we supposed to be horrified? In fact Giles ends up fairly conciliatory: "Even researchers who have been the subject of his attacks praise his contributions to maths." [A deeper analysis would examine the divisions in post-war French society, politics and science (even mathematics!), and how they played themselves out in Mandelbrot's career.]

Prime Number Record Extended. The Great Internet Mersenne Prime Search (GIMPS) has discovered the largest known prime number. The number, $2^{25,964,951} - 1$, has almost eight million digits and is the 42nd known Mersenne prime. (Mersenne primes are prime numbers of the form $2^p - 1$.) The number was discovered on the computer of Dr. Martin Nowack, a German eye surgeon, through GIMPS, a distributed computing project (`www.mersenne.org/prime.htm`).

Originally published by the American Mathematical Society in What's New in Mathematics, a section of AMS Website, `http://www.ams.org/new-in-math`. Reprinted with permission.

An Interview with Marco Avellaneda

When and how did you start looking at mathematical problems arising in finance?

I became interested in Mathematical Finance and option pricing in the mid-80's when I was finishing my dissertation at the University of Minnesota. David Heath, who would later invent a famous model for interest rate derivatives, gave a very interesting talk in the Probability Seminar on the Black-Scholes model. I think that this was the first time that I heard about options. Later, in New York, I had friends that were using the models in the early stages of interest-rate derivatives. My own entry into theoretical finance was in 1994, after teaching courses at Courant Institute. The demand for looking at these problems then came from graduate students and the industry, not from the mathematical community.

Has it been a rewarding experience?

In many ways. First, it gave me a window into an application of mathematics that I did not know. As I learned more about the subject, I became involved in consulting and the more applied aspects of the field. Already in my first consulting project, I realized that I needed to learn how to program computers, in order to get numerical results from models involving PDEs. Doing research in Math Finance made me learn computing, finance and economics, which were new to me. It has also been, to some extent, financially rewarding, although I am not rich.

In your opinion what are the major differences between the type of models and analysis in areas like composite materials or porous media and those applied to finance? Do you look at all of them just as applications?

Before entering mathematical finance, I worked in other fields such as composite materials and porous media. A lot can be said with regards to the differences in methodologies between Physics and Quantitative Finance. Whereas Physics is based on understanding and describing natural phenomena, Finance is ultimately a social science. This means the way that we think is very different. What the two fields share in common is that they are based on data analysis. My strategy, as a mathematician is always to try to "improve the science", by selecting those problems where I think that I can contribute. However, I always try to learn as much as possible about the field, to avoid being a dilettante.

What have been for you the major contributions in the theory of option pricing?

Without a doubt, the Black-Scholes-Merton formula and its consequences is the major contribution. It takes some knowledge of finance to actually appreciate the scope of the BSM theory. In other words, one must look beyond the equations and the mathematics and understand the ideas behind it. The impact of Black-Scholes is tremendous, because it gives a "roadmap" for transferring financial risks across financial instruments of different kinds. The latest fashion in finance, credit derivatives, makes heavy use of Black-Scholes-Merton technology.

Marco Avellaneda

From a mathematical point of view what is still open to prove or conjecture?

I think that the scope of the question may be too narrow, in this context. Quantitative Finance is mostly modelling, followed by algorithms to extract numbers from models. To this, one has to add a heavy dose of statistics and econometrics. Proving theorems is essentially a 'quaint' endeavor, since the field is not based on axioms, but on data and market observation.

How do you measure the impact that the more recent and intense academic research in mathematical mod-

els for finance had in practice (from a trader point of view)?

As always, some research is better than others. Surprisingly though, Wall Street (in the generalized sense) is an avid consumer of academic research. I think that every honest attempt at contributing to the implementation of theoretical ideas in the practical world is taken very seriously... Let me give examples. In equity derivatives, the calibration of volatility surfaces, studied by many academics (too many to mention here), has been incorporated in the pricing of exotics and in managing volatility risk. The Heath-Jarrow-Morton (Cornell University) and Brace-Gatarek-Musiela (University of New South Wales) interest rate models, found their way from academia to the trading floors of the major houses. The Duffie-Singleton (Stanford University) model for pricing credit derivatives is also widely used. And the list goes on. In my opinion, the top investment houses, which have the best traders and make the most money, also have the best quantitative finance teams. In certain areas of trading, like correlation trading in equities and credits, traders are often former academics and researchers that use models to find opportunities and to manage their positions.

The Black-Scholes formula for pricing options is still heavily used by traders despite the amount of research since Black and Scholes. Is that surprising for you?

Not in the least. The reason is that the Black Scholes formula is used to convert prices into implied volatility and Greeks. So, in practice, Black Scholes establishes a correspondence between complex financial instruments, such as a callable-convertible bond, and a number called the implied volatility. Implied volatility gives you a common measure of the expected rate of change (in absolute values) of the underlying asset. For example, the implied volatility of Google should be higher than the implied volatility of GE (at least most of the time). Implied volatility allows, for instance, to compare the values of derivatives on different markets. Also, BSM provides traders with the Greeks (delta, gamma, vega, theta), which are used to set limits on exposure to market moves on a large, portfolio, scale. If there were no Black Scholes, derivatives markets could not exist. It would have to be invented quickly, since it is absolutely necessary for trading for the reasons explained above.

Financial derivatives have developed so much over the last 20 years. There are always new products coming up. How do you predict the evolution of derivatives? Will we ever reach a sort of "stationary point" in this field?

The evolution of derivatives will continue as markets become more interconnected, information flows more freely, and people find ways of "slicing-and-dicing" and trading financial risks. The current phenomenon is the Credit Derivatives markets, which have grown from zero in 1994 to 1 trillion dollars in 2005. Credit derivatives give ways in which the risk of defaults on bonds and loans are transferred from lenders to third-party investors, who "sell protection" to the lenders. This has interesting consequences in terms of credit risk, some of which remain to be fully explored. The credit derivatives markets are now in the process of producing radical innovations every few months. Based on this development, it is difficult to anticipate that the activity will diminish. It should be noted, however, that derivatives markets in different underlying assets, go through periods of growth and then reach a level of maturity characterized by strong liquidity and price-discovery and limited arbitrage opportunities. For example, options on interest rate swaps were exciting 10 years ago and are now "plain vanilla" instruments. The expansion of derivatives markets usually go in the direction of new products: derivatives on weather, the trading of carbon emissions, earthquake contracts, and so forth. I think that the 800-pound gorilla is now credit derivatives.

Do you think that Internet has improved the efficiency of financial markets? What is your opinion about the impact of the finance information spanned by the Internet?

The Internet has definitely improved the efficiency of financial markets. First of all, it renders possible to deliver almost real-time information at very low cost across geographical boundaries. Since finance is based on information, it was only natural that people would take advantage of the web. Low cost trading systems (not only for retail investors but also for professionals) are increasingly delivered as ASPs. This means more trading, hence more competition, hence more efficiency. Also, the amount of research and data that is available on the web distributes information more efficiently, a fact that obviously impacts markets.

Let us take a break from finance for a little while. You have been recently in Coimbra, for the CIM Scientific Council Meeting. What do you think about CIM? Could you share with us some of your impressions about CIM or about Portuguese mathematics in general?

I think CIM can serve two purposes: facilitate the access to mathematical research for graduate students and young faculty and attract foreign mathematical scientists to Portugal. I think that a good model might be in Institute for Mathematics and its Applications at the University of Minnesota. The dynamics of CIM will be to solicit and finance workshops and meetings which specialize in one or other aspect of mathematics, allowing communication between researchers and transfer of information between those who are experts and those who are interested in learning a new field. I have been exposed to Portuguese mathematics and engineering since the 1980's. I have interacted with good mathematicians, such as Irene Fonseca, Joaquim

Júdice, José Francisco Rodrigues, Pedro Girão, Marcelo Viana, Hugo Beirão da Veiga, Maria Grossinho, Paula de Oliveira, and you of course, just to name a few. Prior to coming to CIM, I visited the University of Coimbra during the Y2K celebrations, in a very nice meeting organized by Paula de Oliveira. I am fluent in Portuguese, with a *Carioca* accent, which helps...

Since we took the conversation to Portugal, we would like to ask you about the prospects of mathematical finance in a small country like Portugal. Do you think derivative markets in small countries will ever reach a desired liquidity?

Globalization works against local markets. This means that, in the current state of affairs, Portuguese financial companies would most likely add value by their knowledge of local companies and investment opportunities, attracting outside investments, that sort of thing. This is obviously far from mathematics. On the other hand, the perspectives are brighter if you think about the question from an European perspective. After all, Portugal is in Europe, so if it develops local talent in finance, this will probably make it attractive for financial firms to establish businesses in Portugal and hire local talent. The fact that a company is in Portugal, Spain or Germany is more a question of cost than anything else, because boundaries and regulations are falling rapidly. This creates opportunities for highly educated people. For example, Paris has small financial markets compared to the financial capital of Europe, which is London. Nevertheless, there is a lot of quantitative finance activity in France due to the strong mathematical education, and a lot of new financial products and ideas come from there.

Should Portuguese universities invest in Math Finance curricula? At what level?

Portuguese universities should invest in Mathematical Finance curricula taking into consideration the demand from students and from industry. This should be a gradual process, in which faculty can acquire a sense of which are the best areas to pursue, and then guide the students towards them. When I say "best", I mean that they combine intellectually challenging mathematics with useful material that young graduates can use in their first professional experience. The subtle thing about Math Finance programs is that you are not necessarily training individuals to be professors but, yet, you must give them a high-quality education. As far as funding is concerned, I also believe that Math Finance programs should satisfy the "law of the market": those who will use this education to go into business should not expect the government to finance their studies. For this reason, it is important to explore the possibilities of internships of fellowships sponsored by financial companies, a system that works very well in other countries. Funds from basic sciences should not be diverted into mathematical finance, but rather new funding opportunities should be identified. I am sure they exist.

What balance can you make of the Math Finance program at Courant?

It is a complete success, in terms of having an excellent placement ratio (which may be due to the current market conditions) and, to my delight, a group of students that are very gifted mathematically. A significant proportion of these come from Europe. By and large, we are keeping a high mathematical level as well as a high level in financial modelling and tracking what Wall Street is doing.

It is typical to end these interviews asking the interviewees about the favorite papers they have written, their plans for future work, and so on. What do you expect to do ten years from now?

Part of me wants to continue doing Finance and another wants to relax more and perhaps find a new area of research to dive into. I don't know what I will be doing in ten years. I find that research is a lot of fun and, in this sense, I have been well rewarded. I would love to continue to travel and to spend more time in Brazil and Argentina, where my parents and siblings live. Also, I love the "easy living" of Latin America when I was growing up, and one day I may just end up there.

Interview by Luís Nunes Vicente and Ana Margarida Monteiro (University of Coimbra)

Marco Avellaneda is Professor of Mathematics and Director of the Division of Financial Mathematics at New York University's Courant Institute of Mathematical Sciences, where he has been since 1985.

Since his PhD in Mathematics obtained at the University of Minnesota in 1985, Marco Avellaneda has published approximately one hundred research papers in applied mathematics. He is co-author of the book Quantitative

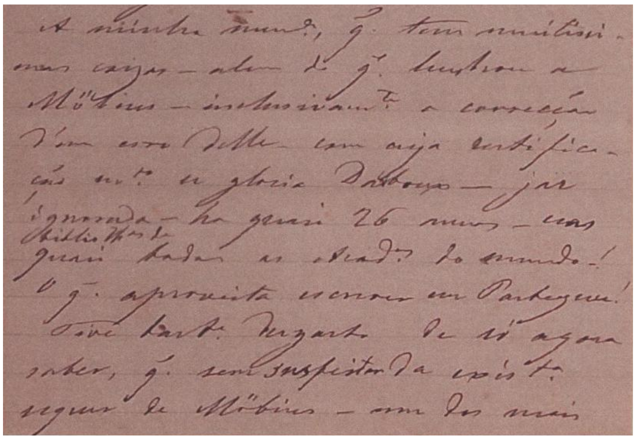
Modelling of Derivative Securities: From Theory to Practice, CRC Press, 1999. He is well known in Finance for his pioneer work on the Uncertain Volatility Model and the Weighted Monte Carlo pricing algorithm.

Marco Avellaneda has been on the board of a number of journals and consulted for several international financial companies. Professor Avellaneda is a founding partner of Finance Concepts, a financial consulting and training firm with offices in Paris and New York. He is currently the Managing Editor of the International Journal of Theoretical and Applied Finance. Professor Marco Avellaneda is also a member of the Scientific Council of CIM.

Daniel da Silva

Almost all the results obtained by Darboux were published in my memoir, which has many more things than those obtained by Möbius (...) lays down ignored for almost twenty–six years in the libraries of almost all the Academies of the world. The reward of writing in Portuguese!

This was written in 1877 by Daniel da Silva, a Portuguese mathematician of the nineteenth century with a wide range of interests, from Number Theory to Experimental Physics through Mechanics and Actuarial Sciences, in a letter addressed to da Silva young friend and disciple Gomes Teixeira.



A minha mem.a, q. tem muitissi-
mas cousas — alem do q. lembrou a
Möbius — inclusivam.te a correcção
d'um erro d'elle com cuja rectifica-
ção vai a gloria Darboux — jaz
ignorada — ha quasi 26 annos — nas
bibliothecas de
quasi todas as Acad.s do mundo —!
O q. aproveita escrever em Portuguez!
Tive [illegible] [illegible] de só agora
saber, q. sem suspeitar da exist.a
[illegible] de Möbius — um dos mais

Excerpt from the letter of da Silva to Gomes Teixeira

After a life dedicated to Science (and with a poor health) and despite the fact that he was recognized as a leader of the Portuguese mathematical community, with several honours and prizes, he sadly realized that, although having worked on some important problems of mathematics (so important, in fact, that they attracted the attention of such famous mathematicians as Darboux, as da Silva refers in the above letter) and obtained substantial results on those subjects, his name and his work would be ignored by the international mathematical community and the credits of his discoveries would be given to others.

In this note we will sketch the life and works of this mathematician.

Daniel da Silva

Daniel Augusto da Silva was born in Lisbon, on the 16th of May of 1814. In 1829 he was admitted in the Navy (*Companhia dos Guardas-Marinhas*) becoming a Navy officer in 1833. Still in 1829, he enrolled in the *Academia Real da Marinha* (which was to be transformed later, in 1837, into the Polytechnical School of Lisbon and, in 1911, into the Science Faculty of the University of Lisbon), finishing is course in 1832; that same year he entered the *Academia Real dos Guardas-Marinhas* — the *Escola Naval* (Navy School), after 1845 —, the school that at the time prepared the Navy officers. In 1835 he completed the three years course in that school and, in the following year, moved to the University of Coimbra, at the time the only university in Portugal, to pursue his studies in Mathematics. In 1839 he received a degree in Mathematics. He had been always a distinguished student and received high marks and prizes in both Navy Schools, in Lisbon, and in the University of Coimbra. When much later (in 1878) his friend, disciple and main biographer Gomes Teixeira became a professor in Coimbra, some old professors could still remember ([9, pag. 188]) the brightness of da Silva as a student.

In 1845, with the transformation of the *Academia Real dos Guardas-Marinhas* into the *Escola Naval*, Daniel da Silva was appointed as a professor there, teaching Mechanics, Astronomy and Optics; later on, in 1848, he became professor of Artillery, Fortification, Geography and Hydrography. In 1865 he retired from the *Escola Naval* and in 1868 he retired from the Navy as a high rank officer (*Capitão de Fragata*).

In 1851, da Silva published his first scientific work, which is also one of his two major works: *Memória sobre a rotação das forças em tôrno dos pontos de aplicação* (Memoir on the rotation of forces around the applications points), published in *Historia e Mem. Acad. Sci. Lisboa* 3 (2ª Ser.). In this paper he studied the action of several forces acting on a solid body, namely he searched for equilibrium conditions. This kind of problems in the field of Statics had been previously studied by, among others, Poinsot, in 1803 (whose work da Silva was aware of), Möbius in 1837 and Midding in 1841 (these two authors were unknown to da Silva), and, later on (1877), by Darboux. Da Silva rediscovered, in his work, most of the results of Möbius and Midding and found the results later published by Darboux. An analysis and comparison of these works was done in [10]. It was Darboux's paper, of which da Silva became aware through a review by Moigno in the *Jornal des Mondes*, that prompted his letter to Gomes Teixeira referred to above. Although, during the da Silva lifetime, this work of his had been ignored, due to [10] and Teixeira efforts, the name of da Silva appears in some publications on the history of Statics.

With that work Daniel da Silva was elected a corresponding member of the *Academia Real das Sciencias de Lisboa* in 1851, becoming a full member in 1852 and later on, in 1859, a honorary member.

He published some other works on Mechanics, but soon a different subject called his attention: Number Theory. In 1854 he published his second major work, *Propriedades Gerais e Resolução das Congruências binomiais* (General properties and resolution of binomial congruences). This long paper (163 pages) had not the same destiny of his work in Statics. A detailed analysis of it was published by C. Alasia de Quesada [1, 2]; due to Alasia's papers the name of da Silva appears several times in the classical book of Dickson, [4]. Curiously, Alasia came across this paper by someone completely unknown to him, in an antique bookshop, where he found a complimentary copy of it sent by the author to J. Liouville.

It is in this work that da Silva presents (in Chap. 1), without proof, the sieve formula for the cardinal of set unions, to which is sometimes attached his name (see e.g. [7, page 19]), although the formula had been used by Nikolas Bernoulli in the eighteenth century, and may be even earlier. The novelty with da Silva treatment, as pointed out in [3] (see also [5, 6]), is that da Silva developed, long before Cantor and even Boole, some kind of (naive) set notions and notations. In fact, he considers a collection S of numbers and he denotes by S_a (respectively ${}^aS$) the collection of elements of S that have (resp. do not have) property a. Similarly, $S_{ab\dots}$ (respectively ${}^{\dots ba}S$) denotes the collection of elements of S that have (resp. do note have) properties $a, b, \dots$. Then he writes symbolically

$${}^aS = S - S_a = S[1-_a], \quad {}^{\dots cba}S = S[1-_a][1-_b][1-_c]\dots .$$

He denotes by φS the number of elements of a collection S and then he writes:

$$\varphi^{\dots cba}S = \varphi S[1-_a][1-_b][1-_c]\dots ,$$

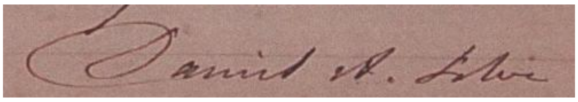
where it is understood that the multiplications in the second member should be done formally, obtaining, in his words, *an additive series and a subtractive series in the second member*. This means that the second member is to be equal to

$$\varphi S - (\varphi S_a + \varphi S_b + \varphi S_c \cdots) + (\varphi S_{ab} + \varphi S_{ac} + \varphi S_{bc} + \cdots) - \cdots ,$$

which is precisely the sieve formula.

He applies this formula to the study of certain functions of Number Theory, namely the Euler function φ; he also presents a generalization of the classical Theorem of Euler on the function φ. Chapter 2 is devoted to linear congruences in one and several variables. Some authors, following Gomes Teixeira [9, page 171], refer to this study of systems of linear congruences, thus anticipating the classical results of H. J. Smith of 1861. In fact, although da Silva carefully studies linear congruences of one and several unknowns, for systems with less equations than unknowns, he just mentions that, by elimination procedures, the system can be reduced to one equation. The chapters 3 to 9 are devoted to a detailed analysis of the congruences $x^n \equiv 1 \bmod n$ (primitive roots) and $ax^n \equiv b \bmod n$. There, da Silva studies the existence of solutions, as well as their number, and formulæ and techniques to obtain these solutions. Several of those results, now classic, were new at the time. The last chapter, entitled *applications*, seems to be just a draft of what da Silva intended to do; in fact, in some sections (e.g. *Continued Fractions*) there is nothing but the title! This was due to the severe illness that affected the author in 1854 (probably some kind of mental breakdown) from which da Silva never completely recovered. According to Gomes Teixeira [9], da Silva could not finish the last chapter and the proofreading of the book.

Due to his illness, da Silva published nothing between 1854 and 1866. After that, he published 2 more papers on mechanics (in 1866 and 1873) and several papers and newspaper articles of some importance on quite different subjects: actuarial science, statistics and demographical studies (see [9, page 174-176]. In 1873 he published his last paper, once again changing his field of interest, now turning to experimental physics; in this paper he studied properties of flames describing several experiences about the most bright part of a flame. Daniel da Silva died in Lisbon, on the 6th of October of 1878.



An interesting aspect of da Silva personality is shown in the letter he wrote to a young 20 year student of the 3rd year of the mathematical course in Coimbra. After hearing, in the mechanics course, the high references the teacher made to da Silva, the student decided to send da Silva a small paper (on continued fractions) he had just published. Daniel da Silva acknowledged the reception of the paper with words of high praise; he further refers his love, passion and hard work in mathematics, his illness, and how difficult was to him, since then, to concentrate in mathematics; he adds that he had a great admiration and interest for everyone who works on mathematics and ends the letter by saying:

To say that I esteem the author of the Memoir I received is much more than a mere compliment; it is just the statement of a necessary condition of my mental organization.

That student was Gomes Teixeira, and this letter was the beginning of a close friendship between the old master and the young mathematician; da Silva had been always very supportive of Gomes Teixeira, a well-known Portuguese mathematician (see [8]), at the beginning of his career.

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The CIM Bulletin is published twice a year. Material intended for publication should be sent to one of the editors. The bulletin is available at http://www.cim.pt.

The CIM acknowledges the support of: Fundação para a Ciência e a Tecnologia, Apoio do Programa Operacional Ciência, Tecnologia, Inovação do Quadro Comunitário de Apoio III; Departamento de Matemática da Universidade de Coimbra; Departamento de Matemática da Universidade de Aveiro; Centro de Matemática da Universidade de Coimbra.