An Interview with John Horton Conway

Curia, 11th of September of 2004. Breakfast with John Horton Conway[1]

Professor Conway, can we start this interview with the genesis of On Numbers and Games *(ONAG)*[2] *and the Theory of Surreal Numbers?*

Of course. I was in Cambridge at the time, and used to play Go with a colleague, who was the English champion. I'm not good at Go, but became very curious about the game. Simultaneously, I'd been thinking about sums of partizan games for a long time. I already knew that such games formed a group. I've investigated the structure of that group. I've found a sequence of games of type A, $B = 2A$, $C = 3A$, etc., it was natural to associate them with the natural numbers; another sequence satisfied $2B = A$, $2C = B$, etc., it was natural to associate it with the dyadic rationals. I've realised in this way that the group of the games had interesting subgroups, isomorphic to other well-known ones. Later on I've convinced myself that I had obtained more than this, games were indeed numbers, they were not bound to contain subgroups isomorphic to the integers and the fractions alone, there were more general ones, like the irrationals and the infinite ordinals. It took me more than one year to obtain the definition in the final form. In 1970 I successfully presented my construction at the California Institute of Technology, suddenly we realised the stuff was important, a generalization of Dedekind's construction of the real numbers, and produced many other numbers. In the following year I went to Calgary's University in order to work with Richard Guy on this matter, and ended up writing down a paper, "All games bright and beautiful", where I presented this theory. Somewhat later, in a break during a conference, I mentioned this work of mine to Donald Knuth. Shortly after, on the pretext of having discussed with his wife, the latter spent one week in an hotel, in Norway, writing *Surreal Numbers*[3], which is the first book mentioning my construction. Actually, the term "surreal numbers" was coined by Knuth. Other authors have been writing some books and papers about the subject.

How was it possible to write ONAG in one week?

Well, I was involved, with Elwin Berlekamp and Richard Guy, in the project of writing a book about impartial games, I mean, games of Nim type. Nevertheless, I was uncovering so many things about the other class of games, that I preferred to take it out of my attention, by writing a book. I locked myself in the office and only stopped for eating and sleeping. In one week the book was finished. It remained the final chapter, which I have completed two years later, and some tables, but essentially the book was finished after one week.

John H. Conway, talking about tangles and knots in Curia, September 11, 2004

This book almost gave rise to a quarrel between friends ...

It's true. I sent a letter to my colleagues Berlekamp and Guy, which started "Dear colleagues, last week I wrote a book, which you will get by mail in a couple of days ...". Even before they have got the book, I already had a letter from Berlekamp, threatening me with a lawsuit. He thought I was stealing stuff from our joint project to publish under my name alone.

That lawsuit didn't go ahead, did it?

I was so distressed with Berlekamp's reaction, that I answered him by explaining what my really intention was, and offering to withdraw my name from the joint work we were doing. In the end he set down and we have

[1] with Nuno Crato (ISEG, Lisbon) at the same table.
[2] Shortly to be published in Portuguese by Gradiva house editor.
[3] Also published in Portuguese by Gradiva, with the title *Números Surreais*.

been three good friends until today. The collaboration continued and has culminated with the publication of *Winning Ways* (WW). Our typical method of work consisted in having me going to the blackboard and explain the theory, while Guy was taking notes. Later, Richard Guy would expose on the blackboard the result of his work on the notes he had taken. That was the point where the battle would begin, I would say "Richard, that was not what I have written!" and Richard Guy would answer "Of course it isn't. This is better!". The sessions were always very vibrant. The strong point of Berlekamp consisted in the analysis of some games, the most interesting of them is Dots and Boxes, which I know makes part of your National Championship of Mathematical Games[4].

But Berlekamp's style is different from yours, how does that not show up in "WW"?

The text suffered many stylistic changes, mainly done by Richard Guy. You may note, in certain chapters, mainly in the annexes, that the language of Berlekamp shows up more.

How did the possibility of working with Richard Guy came up, as he is much older than the two other co-authors?

My friendship started with his son, Michael Guy, who had been my colleague in Cambridge and is an excellent mathematician. It was through him that I got to know Richard, who was very interested in games, and that's it. Michael was my best friend for many years, but today it is his father which I know better ... In the beginning I had a strange feeling, I was in my twenties and he was almost fifty! But everything worked out well, we have had great fun together. Once we rented a house in New Jersey, where we were both working at Bell. Richard came in the house with me and said: the largest room is for me, as I'm the oldest. There were two books in that house: a novel and a children's book with magic tricks. Richard kept the novel with him and gave me the children's book (it was there that I learned the trick I have done some days ago, at Gulbenkian[5], though I have modified it a bit). The weeks we spent together in New Jersey were very amusing, and we worked a lot on mathematical games.

We know you were born in Liverpool. How were you as a student?

I was a good student at high school, which got me into Cambridge. Here I did my first degree and the PhD.

How influent was Cambridge in your career?

The system there, with lectures and tutorials worked out very well with my fellows and me. Some teachers were really good and the ones who dedicated themselves to the tutorial classes did that with high competence, the students could learn a lot.

You did your PhD under Harold Davenport, an expert in Number Theory, but your thesis was in Logic. How did that happen?

Well, I have always been very fond of Number Theory. Davenport's lectures were excellent, I even liked his accent from the North! It was natural to choose him for supervisor. Davenport gave me a problem to think about (Waring's problem). We met every Thursday, so that I could show him my progress. There was none along the first year, and I started feeling guilty. At the end of the academic year I spent some weeks thinking on the problem and could solve it. When the classes resumed I showed him my work. He took it for one week. In the following meeting he told me: Conway, what we have here is a poor PhD thesis ... Davenport never congratulated anybody, so this was the best we could expect to hear from him. In this way the message he wanted to get through was the following: if you don't do anything more, this work will give you the PhD, but you should work more. Actually, the average time for a PhD is three years, and this happened right after the first year.

After this our Thursday's meetings were always open to discussions on nearly any subject in mathematics, philosophy, etc.

Why not presenting the very same work after the three years?

A Chinese mathematician solved the same problem and published his work meanwhile. Therefore, after this, my work was no longer worth a PhD. Another reason is that I also got interested on some problems of Logic and Set Theory which, fortunately, gave me enough material to finish my degree. Davenport also had interest in these subjects, so I could keep the same supervisor.

Your interest in Logic didn't last for long ...

In the conferences I used to go I could see many important problems in the theory being solved, but all proofs extended over hundreds of pages ... it was difficult to work. At the same time, some colleagues introduced me to some hot questions in Group Theory, and my dormant old interest for this theory waked up again. In the following fifteen years I dedicated myself professionally to this field. After this period I published, together with

[4]Pavilhão do Conhecimento, Lisbon, 26th of November of 2004. `http://ludicum.org`

[5]John Conway visited Portugal last September to present a series of lectures, at *Fundação Gulbenkian* in Lisbon, in a Summer School of the "New Talents in Mathematics" programme.

some colleagues, the Atlas of Finite Groups, in 1984. I simultaneously moved to Princeton.

This is when the packing of spheres enters the scene, right?

Right, with Sloane, I devoted myself to the sphere-packing problem and Kepler's conjecture. So, I became a geometer! All my work after this point has to do with geometry.

What is the mathematical discovery you are most proud of?

Well, the answer must be the surreal numbers. This is, nevertheless, a surprising answer, even for me, due to the short mathematical content involved. In this theory, after introducing the definitions, everything is constructed in a few pages. The simplicity of the process is amazing. The amount of work I invested in the Atlas was enormous, along many years. The reconstruction of the Monster was also a nontrivial task. If I had to prove someone I am a competent mathematician, I'd show him my production in Group Theory. The book *Packing of Spheres*, which I have written with Sloane, got very positive criticisms. One of them in particular, from Gian-Carlo Rota, was so enthusiastic, that I copied it and hanged it on the wall. It helped me during times of depression. The *Book of Numbers*[6] was also very well accepted, being translated in nine languages, I believe.

And what do you think was the most important result of last century?

Well, we have Gödel's theorems, for example, which are extremely important. We have also the recently announced proof of Poincaré's conjecture, if it proves correct. But maybe the work of Wyles, in the proof of Fermat's last theorem, is the most remarkable result. It is hard to say, because, as we all know, the relevance of a result depends more on its future than on its past ...

Interview by Jorge Nuno Silva (University of Lisbon)

John H. Conway was born in 1937 in Liverpool, and received both his BA (1959) and his PhD (1964) from Cambridge. He is one of the preeminent theorists in the study of finite groups and the mathematical study of knots, and has written over 10 books and more than 140 journal articles on a wide variety of mathematical subjects. He has also done pathbreaking work in number theory, combinatorial game theory, coding theory, the sphere-packing problem, tiling and quadratic forms.

Before joining Princeton University in 1986 as the John von Neumann Distinguished Professor of Mathematics, Conway served as professor of mathematics at Cambridge University. There, from 1962 until 1986 he was Lecturer, Reader, and Professor in Mathematics. He remains an honorary fellow of Caius College.

Among the general public he is best known for his work on combinatorial game theory, including the classic game of Nim and many others, and for the invention of the Game of Life, popularized by Martin Gardner's columns in *Scientific American* in the early 1970s. He is also one of the inventors of sprouts, as well as philosopher's football, and he developed detailed analyses of many other games and puzzles, such as the Soma cube. He also created a new system of numbers, the surreal numbers, the subject of a mathematical novel by Donald Knuth. Conway may well have the distinction of having more books, articles and Web pages devoted to his creations than any other living mathematician.

He was elected a Fellow of the Royal Society in 1981, is a Member of the American Association for the Advancement of Science, and recipient of the Berwick Prize of the London Mathematical Society (1971), Pólya Prize of the London Mathematical Society (1987), Frederic Esser Nemmers Prize (1999), Leroy P. Steele Prize of the American Mathematical Society (2000), and Joseph Priestley Award (2001). He was also awarded an Honorary DSc by the University of Liverpool in 2001.

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[6] In Portuguese it was published by Gradiva under the title *Livro dos Números*.