Gallery

José Morgado*

José Morgado was born in Pegarinhos, a small village near Favaios, in the *Douro* region, north of Portugal, on February 17, 1921.

He enrolled Elementary School at Pegarinhos and later Secondary School in Favaios. Since his family could not afford his studies at the *Liceu* of Vila Real, he did not register for the third year. Some of his teachers knowing that he was "*an outstanding student, not only in one or two disciplines, but in all of them*", personally guaranteed the pursuit of his studies in Vila Real.

José Morgado made his university studies at the Faculty of Sciences of the University of Porto, where he completed his degree in Mathematical Sciences in 1944. Although it was wartime in Europe, the period he was a student at Porto was a singular moment in the history of Science and Mathematics in Portugal. By that time, a scientific revolution in Portugal was in process under the leadership of António Monteiro, who had recently returned to Portugal after his doctoral studies in Paris. Ruy Luís Gomes, Bento de Jesus Caraça, Aureliano Mira Fernandes, Manuel Zaluar Nunes, José da Silva Paulo, Manuel Valadares, Hugo Ribeiro and A. Pereira Gomes were among the participants of this scientific breakthrough. It was in the context of this truly outstanding generation in the history of Mathematics in Portugal that José Morgado studied and lived in Porto. Those were years full of activity and enthusiasm brought to an end by the repression of the *Estado Novo*. As José Morgado puts it [13]:

"*These activities were undertaken at the Physics Laboratory of the Faculty of Sciences of Lisbon, but in 1947 [...] fascism started a major offensive against the scientific workers and the universities.*"

Meanwhile, a long interim occurred in the life of José Morgado. According to his Curriculum Vitæ, "*between his removal from the official teaching in 1947 and his departure to Brasil in 1960, he survived by giving private lessons. First in Lisbon, he taught Calculus, Infinitesimal Analysis, Algebra, Descriptive Geometry, Projective Geometry and other mathematical disciplines, to students of the* Instituto Superior de Agronomia, Instituto Superior Técnico *and* Faculdade de Ciências de Lisboa. *After October 1958 he moved to Porto, and gave private lessons to students of the Faculty of Economics and the Faculty of Sciences of Porto.*" It was perhaps during this long experience of proximity with students that he developed his remarkable pedagogical qualities that made him a very popular tutor, and, later on, an exceptional teacher. Those 13 years of private teaching were interrupted by some periods in prison, due to his political activities. It was in prison that he wrote the second work in his list of publications [6], the first volume of a book about lattices, published by the *Junta de Investigação Matemática* in 1956.

José Morgado

Six years later, already in Recife, Brasil, he published the notes [11], from a series of lectures in Lattice Theory. These were presented at a seminar at the University of Ceará, and contain some interesting and original results.

As a teacher, José Morgado distinguished himself through his dedication, clarity and constant dialogue with his students. His high motivation and dynamism characterized his lectures.

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In Brasil, he played a preeminent role, together with Ruy Luís Gomes. They reinforced mathematical activities at the Federal University of Pernambuco, Recife, where he worked for 14 years. After the Revolution of April 25, 1974, he returned to Portugal. On October 4, 1974, he was reintegrated as an Assistant Professor at the *Instituto Superior de Agronomia*. On November 7, he was nominated for 2 years, as an invited full professor at the Pure Mathematics Group of the Faculty of Sciences of the University of Porto. On July 24, 1979, he got a permanent position there as a full professor. On February 1991, he retired and became a "Professor Jubilado" at the same university, where, in the following seven years, he continued to teach several optional courses.

José Morgado died in Porto on October 8, 2003.

José Morgado

José Morgado published at least 124 papers on research, history and popularization of Mathematics; 89 have been referred in the *Zentralblatt für Mathematik*.

The scientific work of José Morgado falls, essentially, in 3 areas: Lattice Theory, Group Theory and Number Theory. Next, we briefly mention his earliest work in Lattice Theory, and his later research in Number Theory.

The work of José Morgado in Lattice Theory includes 2 books, published in 1956 and 1962, and 25 papers from 1960 to 1966. We decided to choose his first articles because of the deep and ingenious way which he used to attack the problems. The first paper of José Morgado on Lattice Theory [7] gives a characterization of partially ordered sets P whose set $\Phi(P)$ of closure operators, ordered by $\varphi \leq \psi$ if $\varphi(x) \leq \psi(x)$ for all $x \in P$, is a complete lattice.[1] Ward [16] had proved that $\Phi(P)$ is a complete lattice whenever the same holds for P. Morgado showed that $\Phi(P)$ is complete if and only if, for every element $x \in P$, and every subset Σ of $\Phi(P)$, the set of the elements of P above x which are closed for some element of Σ admits a "quasi-infimum"[2] with respect to x and, moreover, there is some element above x which is essentially closed. Morgado was led to this study following a paper by Monteiro and Ribeiro [5] in which the authors aim to extend the notion of topological space to partially ordered sets, which play the role of the power set of a given set, using closure operators.

In the papers [9, 8], Morgado introduced the notion of *quasi-isomorphism* between complete lattices: this is a bijection $q : L \to M$ that takes the maximum of L to the maximum of M and such that, for all non-empty subsets $S \subseteq L$ and $T \subseteq M$, there exist nonempty sets $S' \subseteq S$ and $T' \subseteq T$ such that $q(\bigwedge S) = \bigwedge q(S')$ and $q^{-1}(\bigwedge T) = \bigwedge q^{-1}(T')$. In [8], he established that the group of automorphisms of $\Phi(L)$ is isomorphic to the group of quasi-automorphisms of L. In [9], he proved that $\Phi(L)$ and $\Phi(M)$ are isomorphic if and only if L and M are quasi-isomorphic. In [10], he showed, among other results, that, if L is a finite lattice, then a permutation q of L is a quasi-isomorphism if and only if q fixes the maximum of L and

$$q(x \wedge y) = q(x) \wedge q(y) \tag{1}$$

whenever x and y are not comparable;

he also observed that, in the finite case, a bijection $q : L \to M$ taking the maximum of L to the maximum of M and satisfying condition (1) is not necessarily a quasi-isomorphism.

The work of José Morgado in Number Theory can be naturally divided in 3 periods:

- a first period, from 1962 to 1964, when he published papers about arithmetic functions, where he studies unitary analogs of several arithmetic functions, these analogs being defined on the basis of the unitary divisors of a number;[3]

[1] Given a partially ordered set P, a function $\varphi : P \to P$ is said to be a *closure operator* if, for all $x, y \in P$, $x \leq \varphi(x)$, $\varphi(x) \leq \varphi(y)$ whenever $x \leq y$, and $\varphi(\varphi(x)) = \varphi(x)$. An element x is said to be *closed for* φ if $\varphi(x) = x$ and it is said to be *essentially closed* if it is closed for all $\varphi \in \Phi(P)$.

[2] Given a non-empty set X of elements of P above x, a *quasi-infimum of X with respect to x* is the largest element y such that $x \leq y \leq X$ and, for every non-empty subset S of X and every s such that $x \leq s \leq S$, there exists z such that $\{y, s\} \leq z \leq S$. Note that, given subsets $T, U \subseteq P$, we write $T \leq U$, if $t \leq u$ for all $t \in T$ and $u \in U$.

[3] We recall that d is said to be an unitary divisor of n if $gcd(d, n/d) = 1$.

- a second period, from 1972 to 1977, when he introduced some results about generalizations of Euler's theorem[4], starting from the study of the regular integers (in the sense of von Neumann) mod n;
- a third period, from 1982 to 1995, when he published a series of articles about Fibonacci numbers and its generalizations, as well as about finite sets of integers in arithmetic progression and satisfying certain properties; all of this motivated by a problem going back to an observation of Fermat, suggested by a problem from the *Arithmetica* of Diophantus, and subsequently studied by L. Euler.

The subject of the papers about Fibonacci numbers and its generalizations, related to the third period, can be described in a somewhat more concise way if we introduce the following notation (see [1, 2]): a finite set $S \subseteq \mathbb{N}$ is said to be a *P_t-set with m elements*, or, when $t = 1$, a *Diophantine m-tuple*, if for every pair x, y of distinct elements of S, $xy + t$ is a perfect square. In the question of Diophantus studied by Fermat, one is asked to construct what is nowadays known as a *Diophantine quadruple*. Starting from the numbers 1, 3 and 8, Fermat shows how to search for a fourth element, finding the number 120. In 1969, A. Baker[5] and H. Davenport proved that $\{1, 3, 8, 120\}$ is the unique Diophantine quadruple containing the set $\{1, 3, 8\}$. In 1977, Hoggatt and Bergum showed that, for all $n \in \mathbb{N}$, the set

$$\{F_{2n}, F_{2n+2}, F_{2n+4}, 4F_{2n+1}F_{2n+2}F_{2n+3}\},$$

where $(F_n)_n$ denotes the Fibonacci sequence, is a Diophantine quadruple. Morgado generalized this result, showing in [12] that the set

$$\{F_n, F_{n+2r}, F_{n+4r}, 4F_{n+r}F_{n+2r}F_{n+3r}\},$$

is a P_t-set for some t, which can itself be expressed by means of certain Fibonacci numbers. On the basis of this work of Morgado, A. Dujella gives in [3] more examples of P_t-sets with 4 elements consisting of Fibonacci and Lucas numbers. In [15], G. Udrea generalized the results of Morgado to sets formed by Chebyshev polynomials of the second kind, and in [14] Morgado generalized these results to polynomials including as particular cases Chebyshev polynomials of the second kind and of the first kind, thus proving a theorem that includes all previous results of the same type. Morgado also found some impressive identities between Fibonacci numbers, as for example:

$$F_nF_{n+1}F_{n+2}F_{n+4}F_{n+5}F_{n+6} + (F_{n+2} + F_{n+4})^2 = \left(\frac{F_{n+1}F_{n+2}F_{n+6} + F_nF_{n+4}F_{n+5}}{2}\right)^2,$$

which holds for all $n \in \mathbb{N}_0$.

It is worth noticing that the question of the existence of a Diophantine quintuple is still an open problem, although it was recently proved by A. Dujella, in [4], that there cannot be more than a finite number of such quintuples and that there are no Diophantine sextuples.

[4] namely that $a^{\varphi(n)} \equiv 1 \pmod n$ holds whenever $gcd(a, n) = 1$.

[5] who was awarded the Fields medal in 1970 for his work on Diophantine equations and transcendental numbers.

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