What's New in Mathematics

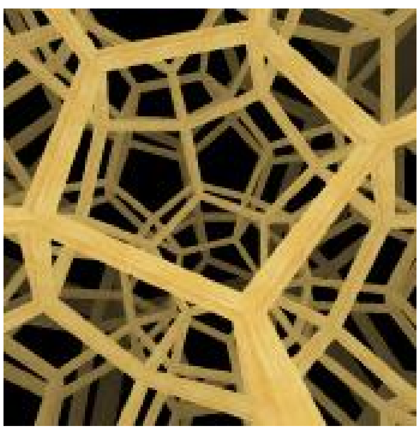
Largest prime yet. "MSU student's prime number largest one yet" is a story by Sharon Terlep in the December 4 2003 *Lansing State Journal*. "Michael Shafer, a 26-year old chemical engineering student, made math history by discovering the largest prime number known." Shafer did it by running a program that "hooked up... more than 200,000 computers worldwide." The program had been running for 19 days when "an alarm sounded letting him know his computers tagged a prime number." The number is a Mersenne prime (of the form $2^p - 1$, where p is prime); in Schafer's case $p = 20,966,011$ and the number itself has over six million digits. According to Shafer, "The number itself really isn't useful. What's more important is what's gone into developing the server and that the programs can get all these computers to work together for a common goal." And: "There may come a time when there's more important research that can harness this technology and use it for something more relevant." Terlep's story is available online; *Largest prime number ever* is found on the *New Scientist* News Service (http://www.newscientist.com/news/news.jsp?id=ns99994438).

Mathe-musical instrument. It's the tritare ("TREE-tar"), it's the invention of two Canadian mathematicians (Samuel Gaudet and Claude Gauthier, both at the University of Moncton), and it may revolutionize music. The story, by Karen Burchard, ran in the November 28, 2003 *Chronicle of Higher Education*. Gaudet and Gauthier are number theorists. In their research on "the odd-number portion of the 'p-series' problem" they came across a class of numbers with symmetries that seemed initially to have potential in engineering but ended up instantiated in a musical instrument "shaped like an inverted Y and equipped with six networks of strings that can produce a range of sounds, from guitarlike musical notes to percussive beats reminiscent of a church bell. If one string is plucked, it vibrates across all three of the fretboards". More details are available from various canadian sources. CBC - New Brunswick shows a photograph of the apparatus. Radio Canada has link to a 4-minute streaming video interview with the two inventors and their instrumentalist (en français. Gaudet: "Tout à coup on a pensé que ça pouvait faire un méchant instrument de musique."). Guitariste.com has some hints about the design (also en français). CBC Arts News has a link to a 1-minute broadcast where you can actually hear the tritare playing in the background.

Who invented the electronic digital computer? It was a professor of math and physics at Iowa State University, and it happened between 1937 and 1941. An article in the Ames, Iowa *Tribune* for November 1, 2003 reports on a meeting of "computer experts from around the world" to celebrate what would have been the 100th birthday of John Vincent Atanasoff who, together with his graduate student Clifford Berry, developed and built the first modern computer. What about ENIAC, you might ask. The Army Research Laboratory webpage is still running *The ENIAC Story* (Martin Weik, 1961) which states "The world's first electronic digital computer was developed by Army Ordnance to compute World War II ballistic firing tables," crediting Dr. John W. Mauchly and J. P. Eckert, Jr., of the University of Pennsylvania, for the original design and quoting from the patent (No 3,120,606) they filed on June 26, 1947. But 12 years after that story was written, a Federal Judge ruled that the ENIAC patents were invalid, and that "Eckert and Mauchly did not themselves first invent the automatic digital electronic computer, but instead derived that subject matter from one Dr. John Vincent Atanasoff". It's not a pretty story, and it's all told on the ISU Dept of Computer Science website.

Auxin and the Fibonacci Numbers. A European team led by Didier Reinhart and Eva-Rachele Pesce of the University of Bern has made great progress towards an understanding of the biochemical basis of phyllotaxis, the regular arrangement of leaves around a plant's stem that leads to spirals with charcteristic mathematical properties. Namely, the numbers of left and right-turning spirals are almost always two consecutive Fibonacci numbers. The team performed an ingenious set of experiments using recently developed mutant strains of Arabidopsis to show that the concentration of the plant hormone auxin and the distribution of primordia (leaf buds) participate in a positive-negative feedback system analogous to "the short-range activator and long-range inhibitor in reaction-diffusion mechanisms." They conclude: "Our model accounts for the reiterativity and the stability of organ positioning." But the way the precise divergence angles are determined (which is where the "golden angle" 137.5° and the Fibonacci numbers enter the picture) is only addressed speculatively. The research is reported in an article in *Nature*, November 20, 2003.

At home in dodecahedral space. The cover story in the October 9 2003 *Nature* is "Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background." Dodecahedral space was invented a hundred years ago by Henri Poincaré -he used it as a counterexample to an early version of his famous conjecture. You make a 3-dimensional space with no boundary by taking a solid dodecahedron and identifying opposite sides (after a rotation by $\pi/5$). If you are living in this space you don't feel any boundaries: as you cross one of the original faces you re-enter, slightly rotated, from the opposite side. This should feel perfectly natural, because the authors of the article (Jean-Pierre Luminet, Jeffrey Weeks, Alain Riazuelo, Roland Lehoucq and Jean-Philippe Uzan) give evidence to show that dodecahedral space may in fact be the shape of the universe we live in.



The view in dodecahedral space (if the framework of the docecahedron is visible). Adjacent cells are just the cell you're in, seen from different points. A spherical wavefront will intersect with itself in "circles in the sky." If detected, these would give an experimental confirmation of the theory. Three dodecahedra fit together evenly around an edge only if the space is positively curved. In physical terms, this means a value strictly greater than 1 for the mass-energy density parameter ?0, another point subject to experimental test. Image courtesy Jeff Weeks, used with permission.

The evidence comes from the spectrum of the temperature fluctuations on the microwave sky ("the waves from the Big Bang"). The data from the Wilkinson Microwave Anisotropy Probe reveal that the lowest-mode observable vibration (the quadrupole) is "only about one-seventh as strong as would be expected in an infinite flat space". The team calculated the spectrum of dodecahedral space, which "depends strongly on the assumed mass-energy density parameter Ω_0". They observe that for $1.012 < \Omega_0 < 1.014$ the values for both the quadrupole and the next-lowest mode (the octopole) give good matches to the experimental numbers from WMAP, while their range for Ω_0 falls "comfortably within WMAP's best-fit range of $\Omega_0 = 1.02 \pm 0.02$". Numbers from upcoming experiments including the Planck Surveyor should determine ?0 within 1"Finding $\Omega_0 < 1.01$ would refute the Poincaré space as a cosmological model, while $\Omega_0 > 1.01$ would provide strong evidence in its favour".

Infinite Wisdom, a piece by Erica Klarreich in the August 30 2003 *Science News*, surveys some recent work on the continuum hypothesis. Klarreich starts with a review of Cantor's proof that the set $\mathbf{R}$ of real numbers is strictly larger, in a precise sense, than the set $\mathbf{Z}$ of integers. In this connection she shows Helaman Ferguson's clever visualization of Cantor's diagonal argument:

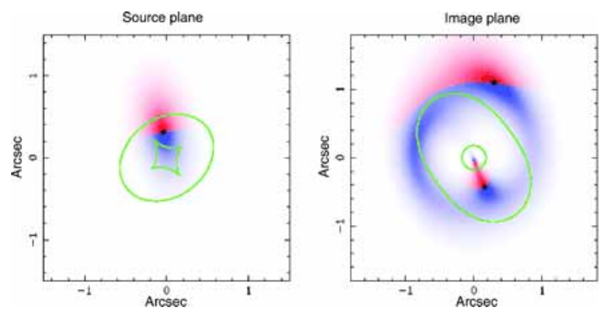
Cantor's Flickering Diagonal. The left half of this stereo pair shows the beginning of an enumeration of the real numbers between 0 and 1. The top line represents the start of the binary expansion of the first number on the list (white=0, black=1). The next line corresponds to the second number on the list, and so on. The right half is identical, except that each diagonal element (the first digit of the first number, the second digit of the second number, and so on) has been reversed: changed from white to black or from black to white. When the two images are fused, the reversed diagonal flickers in and out. The reversed diagonal is the binary expansion of a real number that cannot occur on the original list. Since this will happen for any list, the construction shows that there is no way of listing the real numbers between 0 and 1. Image courtesy Helaman Ferguson, used with permission.

The Continuum Hypothesis is the statement that there is no intermediate size: there is no set with strictly more elements than $\mathbf{Z}$ and strictly fewer elements than $\mathbf{R}$. The truth or falsity of this hypothesis was number one on Hilbert's 1900 list of important unsolved problems. Klarreich continues with the history of the hypothesis, and of its relation to the standard axioms of set theory. She surveys the work of Kurt Gödel (1938) and Paul Cohen (1963): "Put together, those two results indicate that it's impossible either to prove or to disprove the continuum hypothesis using the standard axioms." Which brings us to Hugh Woodin

(U.C. Berkeley) and his recent work on the characterization of an axiom which could be added to the standard set and which would "answer all questions up to the level of the hierarchy that the continuum hypothesis concerns–the realm of the smallest uncountably infinite sets." Woodin calls such an axiom "elegant." Rather than try to construct such an axiom, "Woodin has proved –apart from one missing piece that must still be filled in– that elegant axioms do exist and, crucially, that every elegant axiom would make the continuum hypothesis false." Two survey papers by Woodin are available online: The continuum hypothesis, part I (http://www.ams.org/notices/200106/fea-woodin.pdf) and part II (http://www.ams.org/notices/200107/fea-woodin.pdf).

A heap of trouble. The July 3 2003 issue of *Nature* has a news feature by George Szpiro entitled "Does the proof stack up?" The topic is the fate of the research paper describing Thomas Hales' five year old proof of the Kepler Conjecture: the optimal way to pack equal spheres is the face-centered cubic arrangement used by grocers to stack oranges. The proof was unusual in that, after "reducing the infinite number of possible stacking arrangements to 5,000 contenders," it relied on a computer program to calculate the density of each arrangement; thereby verifying that face-centered cubic was the densest. Nevertheless Robert MacPherson (IAS, Princeton) asked Hales and his graduate student collaborator Sam Ferguson to submit their manuscript to the Annals of Mathematics. Understanding the complexity of the project, he named a team of twelve referees. But the referees have given up. Checking all three gigabytes of code, inputs and outputs turned out to be more than twelve humans could handle. So the paper is to be published with "a cautionary note ... stating that proofs of this type ... may be impossible to review in full." Hales is unhappy and has started a project to use computers to check every line of his proof; he estimates 20 person-years of work to carry it through.

Gravitational caustics. In the May 2 2003 *Science* a 7-member team led by Chris Carilli (National Radio Astronomy Observatory) published "A Molecular Einstein Ring: Imaging a Starburst Disk Surrounding a Quasi-Stellar Object." The QSO in question is PSS J2322+1944; images both in the Infrared (CO emission) and at 1.4 GHz show the "Einstein ring" diagnostic of "strong gravitational lensing by an intervening galaxy." In the absence of information about that particular lens, the team worked from a better known one and experimented with "various source configurations" until they could get a close match to the observed pattern. The model they derive is illustrated here.



"A gravitational lens model for the CO emission in PSS J2322+1944. ...The left panel presents the source plane distribution, corresponding to the true (i.e., undistorted by lensing) morphology of the system. The image plane distribution is presented in the right panel, corresponding to the observed morphology after being distorted by the gravitational lens. The pointlike QSO is represented by a black asterisk in the left panel and by two black asterisks in the right panel. The green solid lines are the caustics and critical lines in the source and image planes, respectively ... . The CO emission is modeled as an inclined disk ($i = 60^o$) around the QSO, and the north and south parts of the disk are color-coded red and blue, respectively, corresponding to different velocity regions on opposite sides of the QSO." (Image ©2003 Science, used with permission).

The paper argues from this reconstruction of the source that it must in fact be a star-forming galaxy.

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