# An Interview with with Thomas J. Laffey

*Professor Laffey, please tell us about your formative years. How did you go into Mathematics, and what are your recollections of your university studies?*

I was born and raised on a small farm in the West of Ireland. Incomes in the area were low and, while neither of my parents had second level education themselves, they strongly encouraged me to study, as success at school was seen as the gateway to greater status and income. In the prevailing society, the medical doctor and schoolteacher had greatest status, even though their incomes would not match that of the bigger farmers and shopkeepers. I succeeded in winning a State scholarship to study at University College, Galway (UCG, now known as National university of Ireland, Galway (NUIG)). While this was, until recently, the smallest of the Irish universities, it was the one at which all State scholars were required to study. The State scholarships were very prestigious, since only about 20 per annum were awarded and they were also seen as the most lucrative available for open competition. The two programmes generally considered the most prestigious among those offered in UCG at that time were Mathematical Science and Classics (Latin and Greek) and the majority of the State scholars studied one of those. As a result, UCG produced a large proportion of Irish students who went on to do doctorates in Mathematics . (For other reasons, University College, Cork also produced a large number). The head of the Mathematics Department at UCG was Professor Sean Tobin, a group theorist, and, as a result, there was a strong algebra content in the courses offered. I was especially attracted towards research in group theory – in particular, I found the textbook of Walter Ledermann "Introduction to the theory of finite groups" inspiring and the tricky exercises and examples discussed by Tobin fascinating and challenging. I still remember a day in 1964 when, during a lecture on solvable groups, Tobin told us that a paper had just appeared by Feit and Thompson proving that all finite groups of odd order are solvable – it was clear from his description of it that we were not expected to read it for the examination!

*You went on to study with Ledermann for your doctorate.*

At that time, one had to choose between doing a PhD in the U.S. or the U.K. There was little tradition of research in Mathematics in Irish universities; the number of staff was very small and lecturers tended to have very large teaching loads. Also, there was no funding in place to support graduate students – this situation has only changed in recent times. I wrote to Walter Ledermann seeking support to do a doctorate at the then new University of Sussex (to which he had moved from Manchester) . After interview there, I was offered and accepted a tutorial studentship (the British equivalent of a teaching assistantship). The contrast between UCG and Sussex at that time was striking. Facilities at Sussex, such as the library, offices, restaurant etc., demonstrated wealth and style. The 1960s were a period of great economic development in Ireland , but the starting point for this was very low, and, despite the ongoing improvement in the country, university expenditure was very limited and the system functioned at a barely adequate level.

Thomas Laffey

I already had a Masters degree from UCG, so when I went to Sussex in September 1966, I was not required to do any further coursework, and I immediately began research. In the book by Curtis and Reiner entitled "Representation theory of finite groups and associative algebras", I was greatly impressed by Jordan's Theorem which states that there is a function $J$ defined on the natural numbers with the property that if $G$ is a finite subgroup of the group $GL(n, \mathbb{C})$ of invertible $n \times n$ matrices with complex entries, then $G$ has an abelian normal subgroup $A$ with $|G/A| \leq J(n)$. Several explicit functions $J$ were known, as a result of work of Jordan, Schur, Blichfeldt, Speiser , and I was interested in trying to get a better one. Ledermann (who, as an undergraduate in Berlin, was taught by Schur), encouraged me to work on this problem and, more generally, on the structure of finite subgroups of $GL(n, \mathbb{C})$ as a function of $n$. He was a most helpful and encouraging advisor and he was always available for discussion on group theory questions. He arranged for me to travel every week to Kings College, University of London, to participate in an advanced course on the representation theory of finite groups given by E.C. Dade, who was visiting the UK from Caltech in 1966-67 and I found this very stimulating. He also arranged for me to meet Walter Feit, for advice on problems concerning finite linear groups. There were a number of group theorists at Sussex at that time, but their interest was largely in theory of infinite groups, and I did not have much interaction with them. A paper which greatly influenced my work was "On a theorem of H.F. Blichfeldt" (Nagoya Math. J. 5 (1953) 75-77) by Noboru Itô, in which it is shown that if $p > n$ is a prime which divides the order of a finite solvable subgroup $G$ of $GL(n, \mathbb{C})$, and $G$ does not have a normal abelian Sylow $p$-subgroup, then $n$ is a power of 2 and $p = n + 1$ is a Fermat prime. I learnt several useful techniques from this paper – the proof is a wonderful demonstration of the minimal counterexample approach - and I was quickly able to build on them to get a best possible Jordan function for finite solvable groups and this formed the backbone of my thesis. During my second year at Sussex, I was more relaxed and started to read more widely in algebra and number theory and I attended all the colloquia and workshops taking place in the department. I shared an office with some graduate students of John Kingman in probability theory and David Edmunds in analysis and had many interesting and informative discussions with them on real and complex analysis. I submitted my thesis in June 1968. Roger Carter, whose name I was familiar with because of his discovery of Carter subgroups, was the external examiner and he arranged an invitation for me to attend the finite group theory meeting at the Mathematisches Forschungsinstitut in Oberwolfach that August and I first presented my thesis results there. That meeting was one of the high points of my research career; there I met several of the leading figures in the subject whose names were known to me through their papers and reputation. Among them were John Thompson, John Conway, Sandy Green, Helmut Wielandt, Bertram Huppert, Reinhold Baer, Michio Suzuki, Wolfgang Gaschütz. I still remember Thompson's lecture on Conway's group and its connections with unimodular lattices, modular forms and Ramanujan congruences There was a tremendous air of excitement and expectation about the future of the theory of finite simple groups.

*What did you do after you obtained your PhD?*

I applied for and obtained an advertised position as assistant lecturer in Mathematics at University College, Dublin (UCD) and took up duty there in September 1968. Though this was, and still is, by far the largest of the Irish universities, I did not know much about it before taking up employment there. Because of the very high teaching loads that had to be carried out by each lecturer in previous times, there was not a strong research tradition in Mathematics. However, as a result of the improving economy, more staff were then being appointed and teaching loads were being reduced to about ten lectures per week, so research was becoming more feasible. There was only one other algebraist in the department when I was appointed, the late Fergus Gaines. Fergus had been an undergraduate at UCD and then did his PhD at Caltech under the supervision of Olga Taussky Todd. In order to be able to discuss research with someone, I decided to read the literature related to Fergus's work. I learnt the number theory and ring theory required to read the papers of Olga Taussky Todd. I also, for the first time, became aware of the difference between the American style PhD and the one I did. I was impressed by the fact that Fergus had attended advanced courses and sat examinations on a wide variety of topics and had an altogether wider knowledge of Mathematics, before embarking on his research. Much later, when we came to set up our own PhD programme here, I was one of these who successfully argued for basing it on the American model.

*You lived in the US for a while. Was it there that you came to know Paul Erdös?*

I got the opportunity to spend a sabbatical year at Northern Illinois University (NIU) in DeKalb during the session 1972-1973. This place was the home of Henry Leonard, Harvey Blau and John Lindsey II, whose names were well-known to me in the context of finite linear groups. Suddenly, I was in an environment with several researchers in algebra. There I made my first contact with semigroup theorists – Bob McFadden and Don McAlister, both incidentally originally from Northern Ireland, headed a most active research team in the subject and I attended their twice-weekly seminars. There was a very active research group in Ring Theory headed by Bill Blair and John Beachy (though John was away in 1972-73) and I also participated in their seminars. The group theorists were very active also and I fully participated in their seminars. There was also a constant stream of colloquium speakers. I also found the analysts asking interesting question on matrix theory, some of which I could answer because of my earlier reading. This caused me to think seriously about doing research in matrix theory for the first time. Northern Illinois has a great reputation in the area of analytic number theory because of the presence of John

Selfridge (who was chairman of the department when I visited) and his research group. This group had a constant stream of research visitors – the first semester I was there, Paul Erdös, Derrick Lehmer and John Brillhart were there and I attended their seminars. I got to know Erdös and I often used to give him a ride to the department in my car on frosty mornings as he used to live in the same part of DeKalb as me, though I did not interact with him mathematically. Had I foreseen the concept of Erdös number, I might have tried to think of some question on which we might have collaborated. While some recent biographies of him suggest that he had no interests outside Mathematics, I found that he had considerable interest in current affairs. Many years later, in 1986, I met him at the ICM in Berkeley and he immediately started to give me a detailed account of a news report earlier that day on a fairly obscure political incident in Northern Ireland.

*How did your research interests evolve over the years?*

Having returned to Dublin in Autumn 1973, I worked on ring theory questions and gradually moved to ring theoretic questions about matrices such as simultaneous triangularizability, Research activity was increasing in Dublin and I also interacted with the functional analysis group at Trinity College, Dublin led by Trevor West. Olga Taussky Todd wrote to me in connection with some questions arising in a paper I wrote on simultaneous triangularization and thus began what was for me a long and very fruitful correspondence. Through Olga's influence, I worked on the $L$-property, simultaneous reducibility of matrices and on integral similarity and factorization results for integer matrices. Fergus Gaines was not so research oriented and I began to address questions which arose in work in his thesis or in his joint work with Bob Thompson or in the thesis (also supervised by Olga Taussky Todd) of Helene Shapiro. I also got to know two other former students of Olga – Charlie Johnson and Frank Uhlig – and also Hans Schneider and Richard Brualdi at Madison. Through Trevor West, I got to know Jaroslav Zemánek and the operator theorists connected to the Banach Center. As a result, by the late 1970s, I had moved entirely into linear algebra and functional analysis.

*You have been heavily involved in mathematical competition activities. What are your thoughts and experience concerning Mathematical Olympiads for the young?*

Every mathematician knows that one of the appealing aspects of Mathematics is the exhilaration and sense of achievement one gets by solving a challenging problem. People of my generation often experienced this first in the context of tricky exercises in Euclidean geometry, but this material is no longer taught at second level. Mainly through his initiative, Finbarr Holland of University College, Cork and I decided to try and run the American High School Mathematics Examination (AHSME) in Irish schools from 1978 onwards. The aim was to further interest in Mathematics among the more talented second level students. The competition organisers generously gave us permission to copy and distribute the papers to interested schools and the competition has been held annually here since 1978. The participating teachers manage the examination in their schools and the scores are coordinated by Finbarr and me and (small) prizes are awarded to the top performers. The reaction to this initiative was very positive, particularly from the more prestigious schools. The questions on the AHSME (and in the follow-up Invitational Competition) are elegant and tricky and a good score is indicative of mathematical potential. It served the purpose of increasing interest in the subject among mathematically gifted students well. However, Finbarr and I had the greater goal of trying to get funding in order to have an Irish team participate in the annual International Mathematical Olympiad (IMO). Unemployment was very high in Ireland in the 1980s and the tax take was low, so the State had difficulty paying its bills and there was little optimism that it would fund IMO participation. We planned to send a team to the IMO as soon as the host country was near, preferably the U.K,, as travel costs would be manageable within the small resources gained from limited sponsorship of the mathematics competition. Completely out of the blue, we got funding to send a team to the 1988 IMO in Canberra. Due to Irish being deported to the penal colony in Van Diemens Land (Tasmania) or voluntarily going to Australia to escape the Irish famine in the 19th century, over 50% of Australians claim some form of Irish heritage and, as the IMO there was nominated an official event of the 1988 Bicentennial Celebrations, the Irish Government wished to be represented at it. (It also helped that the chief organiser of the event was Peter O'Halloran, an Australian of Irish heritage, and that he lobbied the right people!). The Irish Department of Education has supported Irish participation in the IMO since 1988. Each year, they supply those involved in preparing students for the IMO with a list of the top performers in Mathematics in the Junior Certificate Examination (an examination taken by all students about age sixteen) and students on this list (through their school principals) are then invited to attend problem solving sessions and compete for a place on the Irish IMO team each year while they are still in secondary school. The training programme is offered in five universities, UCD, UCC, NUI Galway, NUI Maynooth and the University of Limerick, and usually takes place on Saturday mornings. While Irish performance at the IMO has not been impressive (like Portugal, we are definitely in the "amateurs" rather than "professionals" section), there have been considerable benefits in increasing interest in Mathematics among the very bright students. Several of our top postgraduate students in recent years have said that their first contact with UCD was the AHSME and the Saturday morning training sessions. One professor at Trinity College pointed out to me that the process also has an interesting sociological consequence – namely, most of the top students in the university system in all subjects now know one another, since these students tend to score very highly in Junior Certificate Mathematics and meet one another at the Saturday morning training sessions. I worry at times that the sheer difficulty of the IMO questions discourage some talented students from pursuing a career in the subject at university, but we explain very carefully the exceptional nature of the IMO, comparing it with running a 100 metre race against Marion Jones or using some other similar analogy, and the feedback from students and their teachers has been very positive.

*Can you give us a short overview of the Irish university system?*

Our system is modelled on the British one. Generally, courses to students were offered at two levels "general" and "honours", the honours courses being more challenging. Traditionally, the majority of students took the general courses, while the more talented took the honours ones. Entry to postgraduate programmes was open only to honours graduates and required a high level of performance in the honours courses. In recent times, both in the U.K. and Ireland, there has been a movement towards "honours only" programmes. Several factors have contributed to this: 1. Increased state funding has enabled students from wider socio-economic groups to go to university, while, at the same time, the number of places available did not increase proportionately, so the greater competition for places has led to an improvement in the academic calibre of the students gaining admission. 2. The status of the general degree also diminished in the view of employers. 3. New universities and technological institutes were created which, while getting weaker students than the established universities, produced large numbers of honours graduates. Here, we now offer traditional Honours Mathematics, with an emphasis on rigour and preparing students for graduate study later, and also an Honours programme called Mathematical Studies, more geared towards students going into employment in second level teaching or financial services immediately after obtaining a primary degree. The majority of Mathematics graduates come through the Mathematical Studies stream. We recently established a new programme called Mathematical Science in which a small select intake of students take a selection of Honours courses in Mathematics, Mathematical Physics and Statistics (with Computer Science also in their first year) and this has proved very popular with top students.

One concept that still survives here and in the U.K and some former members of the British Empire, is that of having an external examiner (or extern). In each subject, the university appoints an academic from another university to act in this position, usually on a three or four year contract. The duties of the extern are to vet the examinations, approve their standard and content and, through sampling, approve the marking and adjudicate on issues such as borderline cases of pass/fail or the grade of honours to be awarded. In particular, in most cases, the award of first class honours to candidates is approved on an individual basis by the extern. The extern has "total power" in that his recommendations are normally treated as sacrosanct by the university. While here, the extern visits the department about a week before the board meetings at which the results are approved and his recommendations are conveyed in written form to the meetings, in some universities the extern attends the final board meeting in person. During the mid-1990s, I acted as extern examiner in Honours Mathematics at the University of St. Andrews. There, the extern attends the final board meeting and signs the official record.

The system was originally established to ensure that the standard required for degrees in the various universities in Great Britain and its Empire were approximately the same (unlike, for example, the situation in the U.S., where the standard varies widely from university to university). However, as universities have increased in number, size and diversity of programmes, the influence of the system in uniformizing standards has diminished, but it is still valuable in guaranteeing certain minimal levels of knowledge and achievement, particularly in the case of first class honours graduates.

*Ireland is sometimes mentioned as a kind of 'miracle economy'. How has this influenced the university system?*

The past seven years has been a period of unprecedented economic growth in Ireland. Suddenly, the country changed from one with high unemployment to one in which anyone seeking employment could get it. Skill shortages in the computer industry arose and, as well as putting plans in place to increase the number of computer science graduates, the government started to actively encourage qualified people from inside the EU and also from the U.S., Australia, India and countries from the former Eastern Block to come to Ireland to satisfy the demand. Jobs were available immediately on graduation, so fewer students stayed on to do postgraduate studies. Incentives were put in place to encourage more students to go into graduate study and more foreign students availed of these and came to do their graduate studies here. Greater emphasis was put on the universities' mission of fostering research, vis a vis its teaching role. A policy of setting up a number of specialised research institutes in university campuses was put in place and also funding was made available, partly through the EU, to enable the appointment of postdoctoral researchers. As a result, universities now have some academic researchers with no teaching duties. While the number is small, this represents a major change in the structure. On the other hand, the increased wealth in society, combined with the increase in population due to the arrival of migrant workers, led to enormous increases in house prices and this made it more difficult for the university to attract foreign academics to fill senior positions.

At lower level, the government successfully sought to encourage a much higher percentage of secondary school-leavers to go on to third level through the remission of tuition fees at universities and institutes of technology, and other incentives. Companies such as Apple, Intel and Microsoft, which employ thousands of people here, hire mainly graduates, probably reflecting the fact that they have their roots in the American system where third level education is the norm. Another consequence of the economic changes has been that supermarkets, restaurants and other businesses in the services industry where salary levels are traditionally low, have difficulty hiring sufficient people on a full-time basis, and, as a result, senior second-level and all university students have unlimited opportunities to get part-time employment. Currently, the vast majority of students work about 20 hours per week (usually in the evenings or at weekends) during term-time in paid employment. This has led to a reduction in the time available for study and consequent downward pressure on the standards achievable in their courses. It also means that students are less interested in reforming the world and only a tiny minority take part in the traditional student activities of left-wing politics and anti-capitalist demonstration. The aftermath of September 11 and the general downturn in the computer industry is just starting to effect life seriously here and many of the recent developments, whose affordability was predicated on continuing high economic growth, are now being reviewed and it is likely that a period of financial stringency is on its way.

*How is the job market for mathematics graduates in Ireland nowadays?*

Over recent years, the job opportunities for Mathematics graduates in Ireland have become much more diverse. Up till then, the principal employment area for graduates with general degrees was second level teaching, while the smaller number of honours graduates either stayed in academic life or obtained positions in the central statistics office, the meteorological service or more generally, in the technical areas of the civil service. A smaller number obtained positions in operations research, principally in the international companies KPMG and Andersen Consulting, which have major facilities here. Nowadays, the majority of Mathematics graduates are employed in financial services – Dublin has a very successful international financial services centre (its success is enhanced through a low taxation policy) – and in the computer industry – this is a major component of Irish industry at present. While demographic changes, consequent of much lower average family size, are occurring here as elsewhere in Europe, lower class size has meant that there continue to be vacancies for second level teachers and, in fact, many schools are not able to find sufficient Mathematics graduates to teach their Mathematics courses and it is not uncommon for even the more advanced second level courses to be taught by graduates in commerce or agricultural or biological science who have had only one year of Mathematics at university.

As a subject to study at university, Mathematics has declined in popularity, and, while the total intake of university students has greatly increased each year, the gross total taking Mathematics has actually reduced. Two factors which contribute to this are (i) a continually reinforced philosophy in the media that technical knowledge is boring and that fuzzy thinking is to be encouraged (ii) a general perception that, as a subject, Mathematics is "hard". The same factors have led to decreased interest in physical science, computer science and engineering, despite the needs of the economy. Another noticeable change in students' outlook now is the emphasis they place on obtaining a qualification that should guarantee a large income for them in the future. Thus Medicine, Law, Actuarial and Financial Studies attract many of the brightest students. University entrance in Ireland is based on a points score compiled from a student's mark in six subjects at the State-run Leaving Certificate Examination, taken by all students at the completion of their second-level studies. There is a very good correlation between the total points scores and the Mathematics marks achieved by students – this correlation appears to be lower in the case of other subjects – and there is also an excellent correlation between the total points score and university performance in the traditionally challenging disciplines.

*What are your views about future trends in mathematics?*

Having being a graduate student in the late 1960s, I was influenced by the Bourbaki philosophy of formal definition based non-intuitive pure Mathematics "for its own sake." It is interesting to see the changes in this view over the years. Nowadays, one delights in pointing out applications of one's work outside the subject; the influence of physics in suggesting concepts to study, particularly in geometry, or that of computer science or electrical engineering in doing the same in relation to algebra, is stressed by leaders in the field. The subject has broadened and become more diverse because of these applications, while the status of Mathematics based on a new concept defined by a set of axioms "for its own sake" has reduced in my view. At the same time, progress on classical problems, particularly in number theory, has been achieved through the use of enormous technical machinery, such as arithmetic geometry in which number theory, algebraic and differential geometry, topology and analysis are combined to work effectively . This might be called "total Mathematics" by analogy with "total theatre". While one may regard the Bourbaki philosophy as passé, Grothendieck's ideas continue to inspire. It is hard to predict what areas will be seen as the most prestigious in, say, thirty years time. In the 1960s and 1970s, finite group theory, especially the classification of all finite simple groups, attracted universal acclaim, while nowadays, algebraic geometry and number theory and their connections, as in the Langlands programme, have a similar status. The fact that research in the area requires great background knowledge makes it very a demanding area for a student working towards a doctorate. The theory of finite simple groups reached that point in the mid 1970s and the subject became unpopular as an area in which to do a PhD. While Mathematics is often likened to a knowledge pyramid, it is important that a researcher need only know a relatively small part of the pyramid in order to make progress.

In Algebra, emphasis on commutative algebra has increased, and since this area has strong links with theoretical and algorithmic questions in computation, I expect this trend to continue.

*Finally, we would be grateful for some words on your connection to Portugal, as well as your views of mathematics in Portugal in light of your participation in the 1999 assessment committee.*

Since my involvement with linear algebra dates from the mid-1970s, it was only at that time that I became aware of the work of Graciano de Oliveira and his group at Coimbra. I think that I first learnt about it from Bryan Cain of Iowa State University at a conference in Santa Barbara. I was very honoured to be invited to speak at the first international linear algebra conference in Coimbra and this was also the occasion of my first visit to Portugal. Since then, I have maintained strong links with the algebraists in Coimbra and Lisbon. The research groups in linear and multilinear algebra have strong international visibility and contain a number of leaders in research in these subjects. I quickly also became familiar with the research in semigroups in Lisbon and the great strength of the subject there as well as in Coimbra and Porto. However, it was only as a member of the FCT triennial evaluation panel in 1999 that I got a global picture of mathematical research in Portugal and the structure of its research programmes. The traditional system whereby graduates are appointed to essentially permanent teaching positions at a young age and then do their PhDs while carrying out heavy teaching duties is quite unlike the systems I had previously encountered. The legal separation of the teaching functions of academics from their research functions was also a surprise. I was quite impressed by the level of research activity throughout Portugal and the number of conferences, workshops etc. organized there. The negative impact of large teaching loads and, especially, the necessity of offering examinations in the same course several times to cater for individual students' whims, made the high level of activity even more creditable. Given that my primary interest is in algebra, I found it interesting that, in Portugal, research in this subject is mainly concentrated in linear and multilinear algebra , semigroup theory, category theory and algebraic logic. The level of achievement in these areas is excellent and gains the country international recognition and status. However, and allowing for the fact that Portugal is a small country, I was surprised that there was not a greater variety of areas being investigated and, in particular, that there was very little research interest in number theory. I can understand the reason for the lack of diversity and believe that, without doubt, the situation will change as time goes on. The strong international links between leading research groups here and cognate centres abroad is to be lauded. The research centres produce a very impressive amount of material, such as textbooks in Portuguese, strategies for second-level Mathematics teaching etc., which, while not published internationally, makes a valuable contribution to the health of Mathematics in the country. Generally, the system of funding centres appears to work well and researchers are much more encouraged in their work than the corresponding people in Ireland.

Interview by J. F. Queiró

After completing a doctorate in Sussex University under the supervision of Walter Ledermann, Thomas Laffey joined the Mathematics Department of University College Dublin in 1968 and has remained there since. He served two terms as head of department (1986-90 and 1996-99). His principal research interests are in algebra, particularly in finite group theory and algebraic linear algebra.

He was the founding editor of the Newsletter (now Bulletin) of the Irish Mathematical Society and is currently one of the two editors of the Mathematical Proceedings of the Royal Irish Academy and a member of the editorial board of three other journals.

## Gallery

# José Joaquim Dionísio

Professor J. J. Dionísio was for years almost a legend to me. If my memory serves me right, I started hearing about him around 1957 or 58, and later I studied carefully some of his articles in the field of Linear Algebra. It was a time when mathematical research was almost unknown in Portugal, so to read papers by one of the few Portuguese active researchers was a stimulus to my imagination. I believe it was only in 1970 that I met him for the first time, at the School of Sciences, Lisbon University.

Prof. Dionísio was an assistant at Coimbra University, where he received his doctorate in 1954, and he moved to the University of Lisbon in 1956, where he stayed for the rest of his career.

Prof. Dionísio was one of the people who had a large influence on my own scientific career. Another one was Prof. Luís de Albuquerque, with whom I was in close contact since my student days. It was through him that I received the influence of J. Dionísio, then unknown to me. Luís de Albuquerque had been his friend for a long time, and he often talked to me about Dionísio's work, which I studied long before I met him for the first time.

As a professor, Dionísio taught courses in several areas. In research, he worked mainly in Linear Algebra, although he published articles in other fields in Portuguese journals. He was a cultivated man. He gave particular attention to the History of Mathematics, teaching a course on it and contributing to the Biographic Dictionary of Authors.

I had the opportunity to be in close contact with him in the periods 1972-75 and 1984-89, when I was his colleague at the School of Sciences, Lisbon University.

Perhaps the most remarkable aspect to which I can bear witness is his influence, unknown to many, on the Portuguese Linear Algebra group. Although indirect, this influence was important.

When I graduated, I knew two people interested in Linear Algebra: Dionísio and Albuquerque.

José Joaquim Dionísio

I never knew which of them was the first to have this interest, nor their mutual influence nor even why they chose the field. I asked Prof. Albuquerque, who, as far as I can remember, did not give a complete answer, and I believe his interest in the subject came about more or less by chance. He may have become curious about matrices during his undergraduate studies. A few years after his doctorate, Albuquerque spent a year in Germany, where he studied stochastic processes. I remember him talking a lot about stochastic matrices.

Again I recall that, at the time, the situation in Portugal concerning research was backwards, very different from today. Research was seldom mentioned, or it was described as something mysterious. It is impossible to understand what was done then without first recalling the university atmosphere.