What's New in Mathematics

Geometric Quantum Computation is the topic of a "Perspectives" article in the June 1, 2001 issue of Science. The author, Seth Lloyd, of the MIT Mechanical Engineering Department, explains some recent work in quantum computation. The new research shows how holonomy, in particular the phase changes undergone by a particle moving through a tailored electromagnetic landscape, might be harnessed as the operations of quantum computation. Lloyd describes holonomy "...imagine yourself walking over a gently curving landscape ... you wind up back where you started ... to your surprise you are now facing the opposite direction." Don't try this at home, unless you live on a very small asteroid.

"Surprisingly Square" is the title of a piece by Ivars Peterson in the June 16, 2001 Science News. Peterson is reporting on recent developments in algebra that bear on the problem: how many ways can you express a number as a sum of n squares? Carl Jacobi in 1829 found simple formulas giving the number of different ways of doing it with two, four, six or eight squares, using elliptic function theory. And there the theory stood until 1996, when Stephen C. Milne of OSU came up with "powerful new formulas" to extend Jacobi's calculations to higher n. Powerful, but "hard to fathom and use," according to Peterson, Milne's formulas spurred a search for alternate routes to the same information. Recently modular forms, the same tools that helped prove Fermat's Theorem, have been brought to bear on this problem, and with success. Don Zagier (Max Planck, Bonn) used them to re-do Milne's proof of a similar formula for triangular numbers, and Ken Ono (Wisconsin-Madison) extended Zagier's work to duplicate and clarify Milne's results on squares. Elliptic functions and modular forms are two different areas of mathematics, so their convergence on the sums-of-squares problem suggests hidden connections. As Milne puts it, "Why do the two seemingly unrelated approaches give the same results?"

Is π normal? Which means, do all digit sequences of the same length appear with the same frequency in its decimal expansion? Statistical evidence favors normality. For example, in the first 200 of the 206 billion digits recently computed by Yasumasa Kanada et al. at the University of Tokyo, 7 occurred 19,999,967,594 times. This information is from a piece by Ivars Peterson in the September 1, 2001 Science News. It seems sort of obvious that there should be no incestuous relationship between π and 10, but establishing a proof is another matter. Recent progress has been made, however. It builds on a 1995 discovery by David Bailey (Lawrence Berkeley National Lab), Peter Borwein (Simon Fraser) and Simon Plouffe (University of Quebec at Montreal), who "unexpectedly found a simple formula that enables one to calculate isolated digits of π –say, the trillionth digit– without computing and keeping track of all the preceding digits." This formula only works for the base 2 and base 16 expansions, not the decimal, but it seems like a step towards determining the normality of pi in those bases. Now Bailey and Richard Crandall (Reed) have proved the equivalence between the base-2 normality of π (and $\log 2$) and the equidistribution property for the orbit of certain self-maps of the interval. Peterson tells us which map works for $\log 2$: $x_n = 2\,x_{n-1} + \frac{1}{n} \pmod 1$, and relates the pessimistic opinion of Jeff Lagarias (AT&T labs), that the new problem may be as intractable as the old. As usual, π brings out the puns: Peterson called his piece "Pi à la mode," while the Nature comment was titled "Pi shared fairly."

Drunk on fractals. A 40-year old conjecture on random walks ("drunkard's walks") has recently been solved by "an important and rigorous application of fractals to probability theory and mathematical physics." This from Ian Stewart's News and Views piece "Where drunkards hang out" in the October 18 2001 Nature. The conjecture, due to Paul Erdös and S. J. Taylor, was proved this year by Amir Dembo, Yuval Peres, Jay Rosen and Ofer Zaitouni (preprint available online) in Acta Mathematica. The conjecture involves the number of times a planar random-walking particle can be expected to revisit its most frequently visited site in the first n steps. The answer is $\frac{(\log n)^2}{\pi}$. Fractals? According to Stewart, the particle "makes frequent excusions away from the most frequently occupied disc, but keeps returning to it. These excursions occur on all length scales, which is where fractal geometry comes in."

The Gordian unknot. Alexander the Great cut the knot in 333 BC, and thereby destroyed important mathematical evidence. What was this knot that no one could untie? Keith Devlin reports in the September 13, 2001 Guardian that "A Polish physicist [Piotr Pieranski of Poznan] and a Swiss biologist [Andrzej Stasiak of Lausanne] have used computer simulation to recreate what might have been the Gordian knot." His piece is entitled "Unravelling the myth." Pieranski and Stasiak argue that the knot could not have had any free ends, so the cord was actually a circle. But if the circle had been topologically knotted, the problem would have been mathematically impossible, and therefore not a fair challenge. So the circle itself was tied into what had to be an unknot, and only the thickness of the cord made it impossible to loosen it. For example, the knot might have been tied in a wet cord which was then allowed to dry, and perhaps to shrink itself into an impossible configuration. Pieranski and Stasiak, motivated by interest in string theory and in the knotting of biological molecules, respectively, used a computer program to simulate the manipulation of such knots, and have found one so obdurate that maybe it has the structure of the original puzzle that Alexander "solved." Devlin's article is available online. Pieranski's home page has animations of the computer program in action.

The Abel prize is the name of a new "top maths prize," as Nature puts it in their September 13, 2001 "News in brief." The prize is being set up by the Norwegian government in honor of that country's greatest mathematician. The prize reportedly is aimed at bringing recognition of research achievements in mathematics up to the Nobel level. It will be given every year (starting in 2003) and the money is good: NKr 5 million (approx US$ 550,000).

Photo Solitons. Solitons are solutions to a non-linear wave equation. They have been observed in nature since 1844, when John Scott Russell chased a "solitary wave" as it sped down the Edinburgh to Glasgow canal without losing its shape. This phenomenon in another context turned out to be the key to understanding a strange phenomenon called "Fermi-Pasta-Ulam recurrence" (1953). In the computer simulation of the oscillations of a string consisting of 64 particles with non-linear interaction, the initial shape of the string dissolved as expected into a superposition of non-coherent modes, but after a certain time the modes magically reassembled into the original configuration. This was the "recurrence." In a News and Views piece ("Déjà vu in optics") in the September 20, 2001 Nature, Nail Akhmediev explains how the phenomenon was initially understood theoretically as a solitary wave in the solutions of the Korteweg-de Vries equation, the mathematical model for the original system, and how it is now understood that "essentially the Fermi-Pasta-Ulam recurrence is a periodic solution of the non-linear

Schrödinger equation." Now this phenomenon has been observed in a real physical system, using light beams in an optical fibre. The experiment was reported this year in Physical Review Letters by Van Simaeys, Emplit and Haelterman. "Because they took great care when setting up the initial conditions, the recurrence they saw was almost perfect."

Computing an Organism. The e-mail journal Science-Week for May 25 2001 picked up an item from the March 27 PNAS (98:3879): Stan Marée and Paulien Hogeweg, of the University of Utrecht, published an account of their simulation of the culmination behavior of the social amoeba ("slime mould") Dictyostelium dicsoideum. "Computing an organism" is the title of the accompanying commentary by Lee A. Segal. As the Science-Week editors note, "The D. discoideum morphogenesis cycle is one of the great puzzles of biology." Briefly, the "normal" stage of this organism is an amoeba, an independent unicellular organism. It eats bacteria and reproduces by binary fission. But when a population of these creatures is starving, they aggregate to form a slug 2 to 4 mm long which moves ("migration") as a single organism towards light. There ("culmination") the slug puts up a stalk approximately 1 cm high bearing at its tip a fruiting body containing spores, which eventually disperse over a wide area, each becoming a new "normal" amoeba. Marée and Hogeweg were able to construct a mathematical model of part of this amazing behavior, and to use it to run computer simulations of the process. Their model is a "a two-dimensional simulation using a hybrid stochastic cellular automata/partial differential equation schema" in which "individual cells are modeled as a group of connected automata: the basic scale of the model is subcellular." (...) The Science-Week editors conclude: "...viewing the simulation produced by the mathematical model of Maree and Hogeweg will no doubt startle many biologists. Perhaps the most important consideration is that this work provides evidence that computer modeling involving recognized subcellular dynamic entities may soon be used to predict (and explain) specific tissue development and tissue morphology. The implications for both basic and medical biology are profound."

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