What's New in Mathematics

What color is my hat?

This is the crux of "the hat problem," as presented to readers of the Science section of the New York Times on April 10, 2001. The article, dispatched from Berkeley by Sara Robinson, describes the puzzle as follows:

"Three players enter a room and a red or blue hat is placed on each person's head. The color of each hat is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each person can see the other players' hats but not his own. No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats, the players must simultaneously guess the color of their own hats or pass. The group shares a hypothetical 3 million prize if at least one player guesses correctly and no players guess incorrectly. The same game can be played with any number of players. The general problem is to find a strategy for the group that maximizes its chances of winning the prize."

There is a strategy, and the surprising thing is how well it works. In fact, when the number n of players is 1, 3, 7, 15, etc. (one less than a power of 2) the strategy promises a win $n/(n+1)$ of the time. That strategy is given in terms of Hamming codes, a special class of error-correcting binary codes that sit at the intersection of electrical engineering and abstract algebra. Robinson interviewed Elwyn Berlekamp, the Berkeley math professor who worked out this strategy. She quotes him giving the following life lessons to be deduced from the problem: "The first is that it's O.K. to be wrong as long as you contrive not to be wrong alone. The other, more important lesson is a need for teamwork that goes against the grain of most mathematicians. If the evidence suggests someone on your team knows more than you, you should keep your mouth shut. Most of us assume that each player's strategy is oriented toward him getting it right, and it's not. It's the whole team."

Solitons in matter.

Solitons, or solitary waves, were first discovered as surface waves in canals. They manifest solutions of the nonlinear wave equation which have the remarkable property of maintaining their form unchanged as they propagate. Eran Sharon, Gil Cohen and Jay Fineberg, three members of the Racah Institute of Physics in Jerusalem, have a "Letter to Nature" in the March 1, 2001 issue where they show how perturbations to a crack front in a brittle material result in long-lived and highly localized waves ('front waves') with many of the properties of solitons. They conclude that (presumably novel) nonlinear focussing processes, "perhaps analogous to processes that occur in classical soliton formation, are at play."

God, Stephen Wolfram, etc.

What has Stephen Wolfram, alumnus of Eton and Oxford, veteran of Argonne, CalTech, and the Institute for Advanced Study, MacArthur Fellow at age 21, been doing since his release of Mathematica ("the most popular scientific software ever made") in 1988? He has been planning the complete mathematization of science, and the overhaul of mathematics itself, through his work on cellular automata. This from a long essay by Michael S. Malone, in the online Forbes ASAP for November 27, 2000, entitled "God, Stephen Wolfram, and Everything Else." Cellular automata go back to Von Neumann, but gained wide fame through John Conway's game "Life". How will Wolfram bring about his revolution? To a mathematician the article does not offer any useful clues. The one specific example given, the pattern of markings on a Textile Cone Shell, fits into perfectly conventional science, but it is not clear whether this example is to be taken literally or not, i.e. whether this remark is relevant. A piece appearing in Forbes, and containing statements like "Everything from cars to cartoons, from farms to pharmaceuticals, may reflect the richness of the natural world as seen through Wolfram's cellular automata" and "Within 50 years, more pieces of technology will be created on the basis of my science than on the basis of traditional science," inevitably sounds more like the publicity for an IPO than the presentation of news about current scientific research. The beautiful and moving initial image (the 2-billion-tile rose generated from black and white squares laid according to "half a dozen ... arbitrary rules") typifies the essay. We do not know if the rose is fact or metaphor. We have no way of judging if the tremendous technical developments hinted at are fact or science fiction. "A New Kind of Science," Wolfram's magnum opus on the topic, is promised for sometime this year.

Unknotting the unknot.

The February 9, 2001 issue of Science has a nice piece by Charles Seife entitled "Loopy Solution Brings Infinite Relief." The subject is the recent discovery, by Jeff Lagarias (AT&T) and Joel Haas (U.C. Davis), of an upper bound on the number of Reidemeister moves required to remove all the crossings from the projection of a topologically unknotted curve. The three Reidemeister moves are elementary local changes in the projection; they correspond to moves you might actually make trying to unsnarl a tangle. So there is finally an upper bound on just how long it might take to do it. Seife: "Finite numbers, however, can still be ridiculously large. All Lagarias and Hass guarantee is that if a knot crosses itself n times, you can untangle it in no more than $2^{100,000,000,000\ n}$ Reidemeister moves. In other words, if every atom in the universe were performing a googol googol googol Reidemeister moves a second from the beginning of the universe to the end of the universe, that wouldn't even approach the number you need to guarantee unknotting a single twist in a rubber band. ... Still, [Lagarias] says, just showing that a limit exists may inspire future researchers to whittle it down to a reasonable size. (Macedonian swordsmen need not apply.)"

How do fish swim?

"The dynamics of swimming fish and flapping flags involves a complicated interaction of their deformable shapes with the surrounding fluid flow."This is the beginning of a "letter to Nature"(14 December 2000) from a Courant Institute/Rockefeller University team headed by Jun Zhang. Their research used flexible filaments in a flowing soap film 3-4 microns thick. In particular they report that beyond a certain critical length the system becomes bi-stable, with both a "stretched-straight state"and a "stable flapping state"possible. The stable flapping state has an especially simple mathematical form: "Unlike a simple pendulum, the undulation is well fitted by a travelling harmonic wave with a spatially varying envelope: $y(x,t) = f(x)\ \sin(2\pi\nu t + 2\pi x/\lambda)$. Here, $y(x,t)$ is the horizontal displacement of the filament from the centre-line, measured at a vertical distance x from the fixed point for time t. $f(x)$ is a spatial envelope function (increasing monotonically from the fixed point), ν is the flapping frequency and λ the wavelength."An interesting final point: "Swimming offers alternatives comparable to the bistability of our filament. The stretched-straight state is the analogue of a glide, whereas the flapping state is analogous to swimming. Efficient propulsion uses the natural oscillations of the swimmer, which in the filament is a property mediated by stiffness."A web presentation of this research is available.

Jock Math.

You may wonder, halfway through the season, if your team has a mathematical chance of winning the league. If your sport is soccer ("football"), then well may you wonder. It turns out that this is an NP-complete problem, equivalent to the notorious traveling salesman problem, and therefore computationally as hard as a problem can get. This information comes from a piece by Justin Mullins in the January 27, 2001 New Scientist, entitled "Impossible Goal" and explaining this recent discovery, due to Walter Kern and Daniël Paulusma of the University of Twente, and also, independently, to Thorsten Bernholt, Alexander Gülich, Thomas Hofmeister and Niels Schmitt of the Dortmund University Computer Science department. "Fans had a much easier time in the days when teams got 2 points for a win and 1 for a draw. Kern and Paulusma have shown that this is mathematically simpler than a travelling salesman problem, and the time to solve it increases more slowly as it gets bigger. The switch a few years ago to 3 points for a win turned it into an NP-hard problem. "

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