BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

JUNE 2001

10

CONTENTS

COMING EVENTS

THEMATIC TERM ON SEMIGROUPS, ALGORITHMS, AUTOMATA AND LANGUAGES

ORGANIZERS

Gracinda M. S. Gomes (University of Lisbon, Portugal), Jean-Eric Pin (University of Paris VII, France) and Pedro V. Silva (University of Porto, Portugal).

DATE

May - July, 2001.

The Term is designed to make Coimbra the gathering point of researchers in the subjects of semigroup theory and automata theory during the months of May, June and July 2001. Besides providing a basepoint for the development of joint research projects, the Term includes multiple activities such as specialized schools and workshops on relevant specific subjects. Postgraduate students are particularly welcome.

Each school consists of several 5 hour courses held by prominent researchers. The workshops include 50 minute invited lectures and a limited number of 20 minute talks on the specific topics of the workshop, proposed by the participants.

The programme of events is the following:

2-11 May: School on Algorithmic Aspects of the Theory of Semigroups and its Applications

Invited lecturers: J. Almeida (Porto), C. Choffrut (Paris VII), J. Fountain (York), S. Margolis (Bar-Ilan), L. Ribes (Carleton), M. Sapir (Vanderbilt), M. Volkov (Ekaterinburg), T. Wilke (Kiel).

4-8 June: School on Automata and Languages

Invited lecturers: M. Branco (Lisbon), V. Bruyère (Mons), O. Carton (Marne-la-Vallée), A. Restivo (Palermo).

11-13 June: Workshop on Model Theory, Profinite Topology and Semigroups

Invited lecturers: J. Almeida (Porto), T. Coulbois (Paris VII), H. Straubing (Boston College), P. Trotter (Tasmania), P. Weil (Bordeaux).

2-6 July: School on Semigroups and Applications

Invited lecturers: K. Auinger (Vienna), M. Lawson (Bangor), W. D. Munn (Glasgow), A. Pereira do Lago (São Paulo).

9-11 July: Workshop on Presentations and Geometry

Invited lecturers: R. Gilman (Stevens Inst. of Tech.), D. McAlister (DeKalb), J. Meakin (Lincoln), S. Pride (Glasgow), N. Ruskuc (St. Andrews), B. Steinberg (Porto).

The venue for all events is the Observatório da Universidade de Coimbra, in the peaceful setting of Mount Santa Clara.

Sponsors

Fundação Calouste Gulbenkian

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Fundação Luso Americana para o Desenvolvimento

Universidade do Porto

Câmara Municipal de Coimbra

Faculdade de Ciências da Universidade de Lisboa

Registration fees

May school

Euro 120

June events

Euro 120

July events

Euro 120

Full Term

Euro 240

In return for their fees, the participants are entitled to receive school/workshop documentation and to participate freely in the social activities, including the corresponding Term dinners, to be held on May 10, June 12 and July 10.

Accompanying persons wishing to join the social programme will pay 75% of the normal fee. Early payments can be made by international cheque addressed to "Centro Internacional de Matemática" (CIM). The cheques should be sent to:

Patrícia Paraíba,
C.A.U.L., Av. Prof. Gama Pinto 2,
1649-003 Lisboa, Portugal.

For more information on these events and registration forms, please visit the site http://alf1.cii.fc.ul.pt/term2001/

Summer School
Analytical and Numerical Methods in Non-Newtonian Fluid Mechanics

Organizers

Estelita Vaz (University of Minho, Portugal), J. Maia (University of Minho, Portugal) and K. Walters (University of Wales Aberystwyth, United Kingdom).

Date

25-29 June, 2001.

Aims

The aim of the school is to interest young researchers into the field of Rheology and Non-Newtonian Fluid Mechanics by helping to bridge the gap between the available theoretical tools and the existing problems of a mathematical nature in industry and academia.

The School will be held in the Guimarães Campus of the University of Minho, Portugal. Guimarães is located at 50 km north-east of Porto, in the Minho province. Porto International Airport is served by most major airline companies.

Lecturers

K. Walters, University of Wales

A. R. Davies, University of Wales

M. H. Wagner, Technical University of Berlin

G. Marrucci, University of Naples

R. Keunings, Catholic University of Louvain

F. P. T. Baaijens, University of Eindhoven

Summer School Secretariat

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e-mail: s.school@ecum.uminho.pt

For more information on this event, please visit the site http://www.ecum.uminho.pt/SummerSchool/

Advanced School on Recent Developments in Large-Scale Scientific Computing

Organizers

Filomena Dias d'Almeida (Engineering Faculty, Univ. of Porto, Portugal) and Paulo Beleza de Vasconcelos (Economics Faculty, Univ. of Porto, Portugal).

Date

3-6 July, 2001.

Aim of the School

The aims of this advanced school are: to present the state-of-the-art methods and tools to solve large scale linear problems, namely large linear systems and large eigenvalue problems, to bring together specialist researchers on computational mathematics and to encourage the interchange of new ideas, to create a suitable environment for the participants to get acquainted and involved in today's computational mathematics research problems.

The School will be held in the Faculty of Engineering, University of Porto.

Topics

Parallel architectures

Performance measures

Parallel programming paradigms

Nonstationary iterative methods for large linear systems

Direct methods for large sparse linear systems and preconditioners

Large scale eigenvalue problem

Linear algebra libraries for large scientific computations

Lectures:

Jean-Marie Chesneaux, Univ. of Pierre et Marie Curie, France

Jack Dongarra, Univ. of Tennessee and Oak Ridge Nat. Lab., USA

Iain Duff, CERFACS, France and Rutherford Appleton Lab., UK

Osni Marques, Lawrence Berkeley Nat. Lab., USA

Francisco Moura, Computer Science Dep., Univ. of Minho, Portugal

Orlando Oliveira, Physics Dep., Univ. of Coimbra, Portugal

Rui Ralha, Mathematics Dep., Univ. of Minho, Portugal

Application Lectures:

Mário Ahues, Mathematics Dep., Univ. of St. Etienne, France

Álvaro Azevedo, Civil Dep., Engineering Faculty, Univ. of Porto, Portugal

Joaquim Júdice, Mathematics Dep., Univ. of Coimbra, Portugal

Orlando Oliveira, Physics Dep., Univ. of Coimbra, Portugal

José Palma, Mechanics Dep., Engineering Faculty, Univ. of Porto, Portugal

School fee

The registration fee is 200 Euros (1 Euro = 200.482 PTE). It includes the school documentation and coffee. The social program will include a small guided tour through Porto by bus, a walk in Ribeira (old part of the city), a visit to a Porto Wine Cellar and a School Dinner that will take place in the same Porto Wine Cellar.

Scientific sponsors

CIM - Centro Internacional de Matemática

IDMEC - CENUME: Unidade de Métodos Numéricos em Mecânica e Engenharia Estrutural

CMAUP - Centro de Matemática Aplicada da Universidade do Porto

FCT - Fundação para a Ciência e Tecnologia (Programa Operacional Ciência, Tecnologia, Inovação do III QCA)

FEP - Faculdade Economia do Porto

FEUP - Faculdade Engenharia da Univ. Porto

FLAD - Fundação Luso-Americana

INESC PORTO - Instituto de Engenharia de Sistemas e Computadores do Porto

UP - Reitoria da Universidade do Porto

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DanCake - Dan Cake Portugal SA

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O!PORTO! - Porto Convention Bureau

UNICER - União Cervejeira SA

For the registration form and more information on this event (including travel information and accommodation), please write to LSC@fep.up.pt or visit the site

http://www.fep.up.pt/docentes/pjv/LSC.html

Workshop on Electronic Media in Mathematics

Organizers

F. Miguel Dionísio (IST, Technical University of Lisbon, Portugal), José Carlos Teixeira (University of Coimbra, Portugal) and Bernd Wegner (Technische Universität Berlin, Germany).

Date

13-15 September, 2001.

Aims

The workshop will provide an open forum for the exchange of information and presentations on electronic media in Mathematics for mathematicians, teachers of mathematics and people using mathematics in applications. Four main subject areas are to be covered: a) Computational algebra and computational tools. b) Visualization and animation software. c) Electronic information and communication. d) Electronic publishing and mathematical libraries.

The event will take place in Coimbra.

Speakers:

Alberto Marini, Milan, Italy

Albrecht Gündel-vom Hofe, Berlim, Alemanha

Ana Ramalho Correia, Lisboa, Portugal

Bernd Wegner Berlim, Alemanha

Enrique Macias, Santiago de Compostela, Espanha

F. Miguel Dionísio, Lisboa, Portugal

Gertraud Griepke, Heidelberg, Alemanha

Hans Becker, Göttingen, Alemanha

Ken Brodlie, Leeds, UK

José Carlos Teixeira, Coimbra, Portugal

José F. Rodrigues, Lisboa, Portugal

Konrad Polthier, Berlim, Alemanha

Luis Borbinha, Lisboa, Portugal

Olga Caprotti, Linz, Áustria

Sponsors

CIM - Centro Internacional de Matemática

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CMUC - Centro de Matemática da Universidade de Coimbra

SPM - Sociedade Portuguesa de Matemática

FCT - Fundação para a Ciência e Tecnologia

APM - Associação de Professores de Matemática

Timberlake Consultants

Academia Global

For registration and other information on this event (including deadlines for abstracts), please write to emm@mat.uc.pt or visit the site

http://www.mat.uc.pt/EMM

Workshop - From Brownian Motion to Infinite Dimensional Analysis

Organizers

A. B. Cruzeiro (Grupo de Física Matemática - University of Lisbon, Portugal) and L. Streit (University of Bielefeld, Germany).

Date

18-22 September 2001.

Aims

The need for the development of infinite dimensional Analysis on spaces of continuous paths or of less regular objects such as distributions has become evident mainly by physical motivations (e.g. Quantum Mechanics and Quantum Field Theory).

These spaces are endowed with probability measures, one of the more regular cases being the law of Brownian motion. In this case Itô calculus provides the underlying techniques to manipulate irregular functionals of the paths and the corresponding infinite dimensional Analysis has developed intensively in the past recent decades giving rise to important results in Mathematics, but also applications outside the initial framework (e.g., Filtering and Control Theory, Financial Mathematics).

More recently, special attention has been given to the geometry of (curved) spaces. The goal of the workshop is to bring together various approaches to infinite dimensional Analysis.

The event will take place in Coimbra.

Speakers giving a series of lectures:

Bernt Øksendal (Univ. of Oslo, Norway)

Jurgen Potthoff (Univ. of Mannhein, Germany)

Other speakers:

Thomas Deck (Univ. of Mannhein, Germany)

Hermann Matthies (Technical University Braunschweig, Germany)

Marta Sanz-Solé (Univ. of Barcelona, Spain)

Ali Suleyman Ustunel (Ec. Nat. Sup. Telecommunications Paris, France)

Grants:

Students can apply for participation grants. Applications can be sent to cruzeiro@cii.fc.ul.pt.

For information on this event, please visit the site

http://gfm.cii.fc.ul.pt/Events/fbm2ida/

CIM News

CIM Events for 2002

The CIM Scientific Committee, in a meeting held in Coimbra on March 17, approved the CIM scientific program for 2002.

Thematic Term

The **Thematic Term** for 2002 will be dedicated to Mathematics and Biology. The application of mathematics to biology has had considerable effect on the development of new research areas at the interface of both sciences. The development of Mathematical Biology research requires interdisciplinary teams with great expertise on several scientific areas.

This Thematic Term has the objective of acting as a seed for the development and enlargement of mathematical research applied to biological systems centered on some expertise and areas that exist already within the teams working in Portugal.

The areas covered range from Ordinary Differential Equations; Dynamical Systems, Partial Differential Equations; Optimization; Numerical Analysis; Homegenization; Calculus of Variations; Nonlinear Continuum Mechanics; to Epidemiology; Population Dynamics; Molecular Geometry; Material Science; Bone Remodeling; Numerical Analysis and Design of Bone Prosthesis and Implants; Computer Simulation of the Mechanics of Soft Tissues and Muscles and Computer Simulation of the Heart and Circulatory System.

It is expected that a large number of graduate students and researchers not only from mathematics and biology, but also from engineering, physics and chemistry, may have the opportunity of exchanging their views and knowledge in order to establish a solid and fruitful collaboration in the near future.

School and Workshop on Mathematical and Computational Modeling of Biological Systems

17-21 June 2002

Organizers:

João A. C. Martins, I. Superior Técnico - Lisbon

E. B. Pires, I. Superior Técnico - Lisbon

Advanced School and Workshop on Bone Mechanics - Mathematical and Mechanical Models for Analysis and Synthesis

24-28 June 2002

Organizers:

Helder C. Rodrigues, I. Superior Técnico - Lisbon

José M. Guedes, I. Superior Técnico - Lisbon.

Workshop on Molecular Geometry Optimization

27-29 June 2002

Organizer:

Fernando Nogueira, Univ. Coimbra

Summer School on Mathematical Biology

15-19 July 2002

Organizers:

Alessandro Margheri, Univ. Lisbon

Carlota Rebelo, Univ. Lisbon

Fabio Zanolin, Univ. Udine

Furthermore, the 2002 program will contain the following event:

International Conference on Bounded Systems and Complexity Classes

28-29 June 2002

Aims: To draw together people interested in bounded formal systems related to computational complexity classes in order to discuss current work and assess directions of research.

Organizer: Fernando Ferreira, Univ. Lisbon

CIM Publications

Since 1996, CIM has published the following monographs and volumes of proceedings:

1. Pedro V. Silva, *Introdução à Teoria Combinatória de Semigrupos Inversos*, 1996.
2. João Tiago Mexia, *Introdução à Teoria Estatística do Risco*, 1996.
3. S. A. Robertson, *Three Talks on Convex Bodies*, 1997.
4. J. A. Green, *One Hundred Years of Group Representations*, 1997.
5. Paul A. Fuhrmann, *Linear Algebra and Control - Lecture Notes*, 1998.
6. Isabel N. Figueiredo (ed.), *Escola de Elementos Finitos e Aplicações*, 1998.
7. A. Ornelas, A. C. Barroso, J. Palhoto de Matos, J. Matias and P. Pedregal (ed.), *Mathematical Methods in Materials Science and Engineering - International Summer School*, 1999.
8. Grant Walker, *Some Aspects of the Action of Matrices over F_p on Polynomials*, 1998.
9. J. F. Queiró (ed.), *A Investigação Matemática em Portugal: Tendências, Organização e Perspectivas*, 1999.
10. Nazaré M. Lopes and E. Gonçalves (ed.), *On Nonparametric and Semiparametric Statistics*, 1999.

11. A. Sequeira (ed.), *International Summer School on Industrial Mathematics*, 1999.

12. A. Sequeira (ed.), *International Summer School on Computational Fluid Dynamics*, 1999.

13. A. Sequeira (ed.), *Navier-Stokes Equations and Related Topics (International Summer School)*, 1999.

14. L. Trabucho and J. F. Queiró (ed.), *O ensino da Matemática na universidade em Portugal e assuntos relacionados*, 2000.

15. M. Field, *Complex Dynamics in Symmetric Systems*, 2000.

16. M. Golubitsky and I. Stewart, *The Symmetry Perspective: From Equilibria to Chaos in Phase Space and Physical Space*, 2000.

17. L. N. Vicente (ed.), *Segundo Debate sobre a Investigação Matemática em Portugal*, 2001.

CIM Associates

The current CIM Associate institutions are:

- Sociedade Portuguesa de Matemática
- Universidade de Coimbra
- Universidade do Porto
- Faculdade de Ciências da Universidade de Lisboa
- Universidade do Minho
- Universidade Nova de Lisboa
- Universidade de Aveiro
- Universidade dos Açores
- Universidade da Beira Interior
- Universidade de Évora
- Universidade de Trás-os-Montes e Alto Douro
- Cooperativa de Ensino Universidade Lusíada
- Universidade da Madeira
- Universidade do Algarve
- Centro de Matemática Aplicada do IST
- Centro de Investigação em Matemática e Aplicações da Universidade de Évora
- Centro de Álgebra da Universidade de Lisboa
- Centro de Matemática da Universidade de Coimbra
- Universidade de Macau
- Centro de Matemática da Universidade do Porto
- Centro de Estruturas Lineares e Combinatórias
- Instituto Superior de Economia e Gestão

CIM Cooperation Activities

CIM has a Committee to deal with cooperation activities in Mathematics with Portuguese-speaking countries. The chairman of the Cooperation Committee is Prof. J. C. David Vieira (Aveiro).

CIM on the WWW

Complete information about CIM and its activities can be found at the site

http://www.cim.pt

This is mirrored at

http://at.yorku.ca/cim.www/

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the University of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form

(http://www.cim.pt/cim.www/cim_app/application.htm)

Feature Article

Linear Algebraic Methods in Additive Theory

by J. A. Dias da Silva

Departamento de Matemática - CELC
Universidade de Lisboa

Introduction

Additive Number Theory is the study of subsets of $\mathbb{Z}$ or $\mathbb{Z}_p$ (the set of the integers modulo p). Let $m \geq 2$ and $A_1, \ldots, A_m \subseteq \mathbb{Z}$ (or $\mathbb{Z}_p$). We denote by $A_1 + \cdots + A_m$ the subset of $\mathbb{Z}$ (or $\mathbb{Z}_p$)

$$A_1 + \cdots + A_m := \{a_1 + \cdots + a_m \mid a_i \in A_i,\ i = 1, \ldots, m\}.$$

The set $A_1 + \cdots + A_m$ is called the the *sumset of* $A_1, \ldots, A_m$.

Following Nathanson [16], in a *direct problem in Additive Theory* we establish properties on the sumset $A_1 + \cdots + A_m$ when properties of $A_1, \ldots, A_m$ are known. In an *inverse problem in Additive Theory* we study the structure of sets $A_1, \ldots, A_m$ whose sumset has prescribed properties, for example, the structure of sets whose sumset has small cardinality.

Some direct problems in Additive Theory have recently been approached by using tools of Linear Algebra. This happened after years of using additive results in Linear Algebra (sometimes reproved with this purpose)[12, 13, 14, 15, 18].

The linear algebraic approach of Additive Number Theory is based on the use of the degrees of invariant polynomials of (diagonal) linear operators as estimators for the cardinality of parts of their spectrum. To illustrate it we need to introduce some terminology and notation.

We denote by $\mathbb{N}_0$ the set of nonnegative integers. We use p to mean the characteristic of the field $\mathbb{F}$, in the case $\mathbb{F}$ has finite characteristic, and ∞ if $\mathbb{F}$ has characteristic zero (we assume the usual conventions on the symbol ∞). If A is a set, $|A|$ denotes the cardinality of A. If f is a linear operator on the finite dimensional vector space V over $\mathbb{F}$, we use $\sigma(f)$ for the spectrum of f (meaning either the family or the set of the roots of the characteristic polynomial of f, in the algebraic closure of $\mathbb{F}$). We use P_f to mean the minimal polynomial of f (that is, the monic polynomial of minimal degree satisfied by f). We say that f is *diagonal* or *of simple structure* if, for some basis of V, the matrix of f is diagonal.

Let $v \in V$. The subspace spanned by the images of v under the powers of f is called the f-*cyclic subspace* of v and denoted $\mathcal{C}_f(v)$, i.e.,

$$\mathcal{C}_f(v) = \langle f^j(v) \mid j \in \mathbb{N}_0 \rangle.$$

The identity operator on V is denoted by I_V.

The following theorems are basic tools for the next sections.

Theorem 1 *If f is a diagonal linear operator on V, the degree of the minimal polynomial of f is equal to the cardinality of its spectrum, i.e.*

$$\deg(P_f) = |\sigma(f)|.$$

Theorem 2 *The degree of the minimal polynomial of f is the maximum of the dimensions of the f-cyclic subspaces of the vectors of V, i.e.,*

$$\deg(P_f) = \max_{v \in V} \dim \mathcal{C}_f(v).$$

From the Cauchy-Davenport theorem to the Erdös-Heilbronn conjecture

Let p be a prime number. The following theorem was proved by Cauchy in 1813 [2], and reproved by Davenport in 1935 [5].

Theorem 1 *Let A and B be nonempty subsets of $\mathbb{Z}_p$. Then*

$$|A+B| \geq \min\{p, |A|+|B|-1\}.$$

A new proof for the Cauchy-Davenport theorem was obtained [7] using Linear Algebra. The first step needed to get this proof is to obtain the linear algebraic translation of the notion of sumset, i.e., given linear operators f and g to find a linear operator H such that

$$\sigma(H) = \sigma(f) + \sigma(g).$$

Basic Linear Algebra provides that operator, as we can see in the following theorem.

Theorem 2 *Let V and W be nonzero finite dimensional vector spaces over the field $\mathbb{F}$. Let f be a linear operator on V and g be a linear operator on W. The spectrum of the Kronecker sum of f and g,*

$$f \otimes I_W + I_V \otimes g,$$

is equal to the sumset of the spectra of f and g, i.e.,

$$\sigma(f \otimes I_W + I_V \otimes g) = \sigma(f) + \sigma(g).$$

We are now able to state the linear counterpart of the Cauchy-Davenport theorem.

Theorem 3 (Linear Cauchy-Davenport [7]) *Let V and W be nonzero finite dimensional vector spaces over $\mathbb{F}$. Let f be a linear operator on V and g a linear operator on W. Then*

$$\deg P_{f\otimes I_W + I_V \otimes g} \geq \min\{p, \deg P_f + \deg P_g - 1\}. \quad (1)$$

The proof of this theorem was obtained by showing that for $v \in V$ and $w \in W$ the set

$$\{f \otimes I_W + I_V \otimes g)^k (v \otimes w) \mid \\ k = 0, \ldots, \min\{p, \dim \mathcal{C}_f(v) + \dim \mathcal{C}_g(w) - 1\} - 1\}$$

is linearly independent. From this fact we get the inequality

$$\dim \mathcal{C}_{f\otimes I_W + I_V \otimes g}(v \otimes w) \geq \\ \min\{p, \dim \mathcal{C}_f(v) + \dim \mathcal{C}_g(w) - 1\}. \quad (2)$$

Choosing $v \in V$ such that $\dim \mathcal{C}_f(v) = \deg P_f$ and $w \in W$ such that $\mathcal{C}_g(w) = \deg P_g$ (recall Theorem 2) we have

$$\deg P_{f\otimes I_W + I_V \otimes g} \geq \min\{p, \min \deg P_f + \deg P_g - 1\}.$$

The Cauchy-Davenport Theorem can now be easily derived. Let A and B be subsets of $\mathbb{Z}_p$ of cardinalities r and s respectively. Let f be a diagonal linear operator on an r-dimensional vector space, V, over $\mathbb{Z}_p$, whose spectrum is A. Let g be a diagonal linear operator on an s-dimensional vector space, W, over $\mathbb{Z}_p$, whose spectrum is B. Using Theorem 3 and replacing in (1) the degrees of the minimal polynomials of f, g and $f \otimes I_W + I_V \otimes g$ (recall Theorems 1 and 2) by the cardinality of their spectra we get the Cauchy-Davenport Theorem.

The Erdös-Heilbronn conjecture was another (direct) additive problem that has been successively fitted in the linear algebraic approach. In order to state this conjecture let us introduce some more terminology and notation. We say *m-set* to mean a set of cardinality m. Let A be a nonempty subset of $\mathbb{F}$. We denote by $\wedge^m A$ the set of the sums of the elements of the m-subsets of A (we refer to these sums as "sums of the m-subsets"or "m-restricted sums"). For instance, if $A = \{a_1, \ldots, a_n\} \subseteq \mathbb{F}$

$$\wedge^2 A = \{a_i + a_j \mid 1 \leq i < j \leq n\}.$$

In 1964 Erdös and Heilbronn [10] made the following conjecture:

Conjecture *Let p be a prime number and let A be a nonempty subset of $\mathbb{Z}_p$. The set of the sums of the 2-subsets of A has cardinality at least $\min\{p, 2|A|-3\}$, i.e.,*

$$|\wedge^2 A| \geq \min\{p, 2|A|-3\}.$$

In the linear algebraic approach to this conjecture the following more general problem was considered: "Let n be a positive integer. Find a lower bound for the set of cardinalities of $\wedge^m A$ when A runs over the set of finite subsets of $\mathbb{F}$ of cardinality n, i.e. find a lower bound for the set

$$\{|\wedge^m A| \quad | \quad A \subseteq \mathbb{F} \text{ and } |A| = n\}\text{"}.$$

Given a linear operator f we have, now, to find a linear operator H such that the spectrum of H is the set of the sums of the m-subsets of the spectrum of f. As before, this linear operator has already been considered in Linear Algebra. Let f be a linear operator on V. Consider the linear operator $D(f)$ on $\wedge^m V$, the mth exterior power of V, defined by the equalities [1, Ch. III, p. 129],

$$\begin{aligned} D(f)(v_1 \wedge \cdots \wedge v_m) &= f(v_1) \wedge v_2 \wedge \cdots \wedge v_m + \\ &\quad + v_1 \wedge f(v_2) \wedge \cdots \wedge v_m + \\ &\quad + \cdots + v_1 \wedge v_2 \wedge \cdots \wedge f(v_m), \\ &\quad v_1, \ldots, v_m \in V. \end{aligned}$$

The following theorem is a consequence of the definition of $D(f)$.

Theorem 4 *Let f be a diagonal linear operator on the finite dimensional vector space V. Then $D(f)$ is diagonal and the spectrum of $D(f)$ is the set of the sums of the m-subsets of $\sigma(f)$, i.e.,*

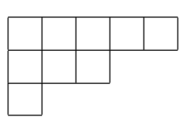
$$\sigma(D(f)) = \wedge^m \sigma(f).$$

To go on with the announced approach to the Erdös-Heilbronn conjecture, we need to express the image of the powers of $D(f)$, on certain decomposable exterior tensors, as linear combinations of a basis of $\wedge^m V$ (designed to fit in this problem). For this we introduce some combinatorial terminology and notation.

A *partition* of m is a decreasing sequence of nonnegative integers whose sum is equal to m. We say that a partition λ has *length* s (and write $s = \ell(\lambda)$) if the number of positive terms of λ is s. We denote by $\mathcal{P}_{m,s}$ the set of partitions of m of length at most s, and by $\mathcal{P}_s$ the set of partitions of length at most s, i.e.,

$$\mathcal{P}_s = \bigcup_{i \in \mathbb{N}} \mathcal{P}_{i,s}.$$

To each partition of m, $\lambda = (\lambda_1, \ldots, \lambda_t)$, we associate the Young tableau $[\lambda]$ which consists of m boxes placed in t rows, all starting in the same column, where the i-th row of $[\lambda]$ has λ_i boxes, $i = 1, \ldots, t$. For instance, the Young tableau associated with the partition $(5, 3, 1)$ is



Let λ be a partition of m. The (i, j)-*hook* of $[\lambda]$ is the subset of boxes of $[\lambda]$ consisting of the (i, j)-box of $[\lambda]$ (the box in the ith row and jth column of $[\lambda]$) together with the boxes in the same row to the right and the boxes in the same column under it. We denote by H^{λ}_{ij} the (i, j)-hook of $[\lambda]$ and by h^{λ}_{ij} the cardinality of H^{λ}_{ij}.

Let $v \in V$. The set

$$\begin{array}{c}\{f^{\lambda_m}(v) \wedge f^{\lambda_{m-1}+1}(v) \wedge \cdots \wedge f^{\lambda_1+m-1}(v) \mid \\ \lambda \in \mathcal{P}_m, \lambda_1 \leq \dim \mathcal{C}_f(v) - m\}\end{array} \tag{3}$$

is a basis for the mth exterior power of $\mathcal{C}_f(v)$. Then it is possible to express the image of powers of $D(f)$ on $v \wedge f(v) \wedge \cdots \wedge f^{m-1}(v)$ as a linear combination of this basis. The following theorem gives us that linear combination.

Theorem 5 ([8])

$$D(f)^t(v \wedge f(v) \wedge \cdots \wedge f^{m-1}(v)) =$$

$$= \sum_{\lambda \in \mathcal{P}_{t,m}} \frac{t!}{\prod_{i,j} h^{\lambda}_{ij}} f^{\lambda_m}(v) \wedge f^{\lambda_{m-1}+1}(v) \wedge \cdots \wedge f^{\lambda_1+m-1}(v).$$

With this expansion of $D(f)^t(v \wedge f(v) \wedge \cdots \wedge f^{m-1}(v))$ as a linear combination of the elements of the basis (3) it is possible to prove that, if $v \in V$,

$$\begin{array}{l}\{D(f)^t(v \wedge f(v) \wedge \cdots \wedge f^{m-1}(v)) \mid \\ \qquad t = 0, \ldots, \min\{p, m(\dim \mathcal{C}_f(v) - m) + 1\} - 1\}\end{array}$$

is a linearly independent set: Using arguments similar to the ones which have been used to prove the linear Cauchy-Davenport Theorem, we get what we can call the Linear Erdös-Heilbronn Theorem.

Theorem 6 ([8]) *Let V be a nonzero finite dimensional vector space over $\mathbb{F}$. Let f be a linear operator on V. Then*

$$\deg(P_{D(f)}) \geq \min\{p, m(\deg P_f - m) + 1\}.$$

Let A be a finite nonempty subset of $\mathbb{F}$. Taking f diagonal with spectrum A, and using the line of argument presented after the proof of the Linear Cauchy-Davenport Theorem, we obtain the following theorem :

Theorem 7 ([8]) *Let A be a finite nonempty subset of $\mathbb{F}$. Then*

$$| \wedge^m A| \geq \min\{p, m(|A| - m) + 1\}.$$

This theorem gave an affirmative answer to the Erdös-Heilbronn conjecture. In fact, taking $m = 2$ and $\mathbb{F}$ the field $\mathbb{Z}_p$ in the previous theorem, we conclude that the Erdös-Heilbronn conjecture is true.

Multiplicities and generalized sums

Let $A = \{a_1, \ldots, a_n\}$ and B be finite nonempty subsets of $\mathbb{F}$. For $c \in A+B$ define $\nu_c(A,B)$, the *multiplicity* of c in $A+B$, as the cardinality

$$\nu_c(A,B) = |\{(a,b) \mid a \in A,\ b \in B,\ \text{and } a+b=c\}|.$$

We write $\mu_i(A,B)$ (or simply μ_i) to mean the cardinality of the set of the $c \in A+B$ that have multiplicity greater than or equal to i, i.e.,

$$\mu_i(A,B) = |\{c \in A+B \mid \nu_c(A,B) \geq i\}|.$$

Similarly, if $c \in \wedge^2 A$ we denote by $\nu_c^{(R)}(A)$, the *multiplicity* of c in $\wedge^2 A$, as the cardinality

$$\nu_c^{(R)}(A) = |\{(r,s) \mid 1 \leq r < s \leq n,\ \text{and } a_r + a_s = c\}|.$$

The symbol $\mu_i^{(R)}(A)$ (or simply $\mu_i^{(R)}$) indicates the set of the elements c of $\wedge^2 A$ whose multiplicity is greater than or equal to i, i.e.,

$$\mu_i^{(R)}(A) = |\{c \in A+B \mid \nu_c^{(R)}(A) \geq i\}|.$$

In 1974, J. M. Pollard [17] established an average theorem for the multiplicities in $A+B$ proving that, if $A, B \subseteq \mathbb{Z}_p$, then, for $t = 1, 2, , \ldots, \min\{|A|, |B|\}$, we have

$$\sum_{i=1}^{t} \mu_i \geq t \min\{p, |A|+|B|-t\}. \tag{1}$$

Extending the arguments used to prove the Cauchy-Davenport and Erdös-Heilbronn theorems, and using some recent results on Linear Algebraic Control Theory [19], it was possible to generalize Pollard's theorem in the following two different ways:

Theorem 1 *Let A and B be finite nonempty subsets of $\mathbb{F}$. Then, for $t = 1, 2, \ldots, \min\{|A|, |B|\}$, we have*

$$\sum_{i=1}^{t} \mu_i \geq t \min\{p, |A|+|B|-t\}.$$

Theorem 2 *Let $A \subseteq \mathbb{F}$ and $1 \leq t \leq \lfloor \frac{|A|}{2} \rfloor$. Assume that $|A| \geq 2$. Then we have*

$$\sum_{i=1}^{t} \mu_i^{(R)} \geq t \min\{p, 2(|A|-t)-1\}.$$

Consider, now, the elementary symmetric polynomial of degree k in the indeterminates $X_1, \ldots, X_m$,

$$s_k(X_1, \ldots, X_m) = \sum_{\alpha \in Q_{k,m}} X_{\alpha(1)} \cdots X_{\alpha(m)},$$

where $Q_{k,m}$ denotes the set of strictly increasing maps from $\{1, \ldots, k\}$ into $\{1, \ldots, m\}$. Let $A_1, \ldots, A_m$ be subsets of $\mathbb{F}$. We denote by $s_k(A_1, \ldots, A_m)$ the subset of $\mathbb{F}$

$$s_k(A_1, \ldots, A_m) = \{s_k(a_1, \ldots, a_m) \mid a_i \in A_i,\ i = 1, \ldots, m\}.$$

This concept generalizes the notion of sumset of $A_1, \ldots, A_m$. In fact, $s_1(A_1, \ldots, A_m)$ is the sumset of $A_1, \ldots, A_m$, i.e.

$$s_1(A_1, \ldots, A_m) = A_1 + \cdots + A_m.$$

It is natural to search additive results for these generalized sumsets. Again, the linear algebraic approach worked for this generalization.

Let $V_1, \ldots, V_m$ be nonzero finite dimensional vector spaces over $\mathbb{F}$. Let T_i be a linear operator of V_i, $i = 1, \ldots, m$. If $\alpha \in Q_{k,m}$ let

$$\delta_\alpha(T_1, \ldots, T_m) = S_1 \otimes \cdots \otimes S_m,$$

where $S_i = I_{V_i}$ if $i \notin \operatorname{Im} \alpha$ and $S_i = T_i$ if $i \in \operatorname{Im} \alpha$. Define

$$D_k(T_1, \ldots, T_m) := \sum_{\alpha \in Q_{k,m}} \delta_\alpha(T_1, \ldots, T_m).$$

For instance,

$$D_2(T_1, T_2, T_3) = T_1 \otimes T_2 \otimes I_{V_3} + T_1 \otimes I_{V_2} \otimes T_3 + I_{V_1} \otimes T_2 \otimes T_3.$$

The key result that allows the above mentioned linear algebraic approach is the following theorem:

Theorem 3 *Let $A_1, \ldots, A_m$ be nonempty finite subsets of $\mathbb{F}$. Let T_i be a diagonal linear operator on V_i such that $\sigma(T_i) = A_i$, $i = 1, \ldots, m$. Then $D_k(T_1, \ldots, T_m)$ is diagonal and*

$$\sigma(D_k(T_1, \ldots, T_m)) = s_k(A_1, \ldots, A_m).$$

Using a variation of the arguments already described (for the Linear Cauchy-Davenport Theorem) we can prove:

Theorem 4 ([9]) *For p large enough we have*

$$\deg P_{D_k(T_1, \ldots, T_m)} \geq \left\lfloor \frac{\deg P_{T_1} + \cdots + \deg P_{T_m} - m}{k} \right\rfloor + 1.$$

Considering diagonal linear operators T_i in the conditions of Theorem 3, and the equality (for diagonal linear operators) between the cardinality of the spectrum and the degree of the minimal polynomial (Theorem 1), we obtain, from the previous theorem, the following result:

Theorem 5 ([9]) *Let $A_1, \ldots, A_m$ be finite nonempty subsets of $\mathbb{F}$. For p large enough we have*

$$|s_k(A_1, \ldots, A_m)| \geq \left\lfloor \frac{|A_1| + \cdots + |A_m| - m}{k} \right\rfloor + 1.$$

Bibliografia

[1] N. Bourbaki, *Elements de Mathématique*, Algèbre I, Hermann, Paris, (1970).

[2] A. Cauchy, Recherches sur les nombres, *J. École Polytech.* 9:99-116 (1813).

[3] Cristina Caldeira and J. A. Dias da Silva, The invariant polynomial degrees of the Kronecker sum of two linear operators and Additive Theory, *Linear Algebra and Appl.* 315:125-138 (2000).

[4] Cristina Caldeira and J. A. Dias da Silva, A Pollard type result for restricted sums, *Journal of Number Theory* 72:153-173 (1998).

[5] H. Davenport, On the addition of residue classes, *J. London Math. Soc.* 10:30-32 (1935).

[6] H. Davenport, A historical note, *J. London Math. Soc.* 22:100-101 (1947).

[7] J.A. Dias da Silva and Y. O. Hamidoune, A note on the minimal polynomial of the Kronecker sum of two linear operators, *Linear Algebra and its Applications* 141:283-287 (1990).

[8] J.A. Dias da Silva and Y. O. Hamidoune, Cyclic Spaces for Grassmann derivatives and Additive Theory, *Bull. London Math. Soc.* 26:140-146 (1994).

[9] J. A. Dias da Silva and H. Godinho, Generalized derivations and Additive Theory, preprint.

[10] P. Erdös and R. L. Graham, *Old and new problems and results in combinatorial Additive Theory*, L'Enseignement Mathématique, Genève, 1980.

[11] Serge Lang, *Algebra*, Addison-Wesley, New York, 1993.

[12] M. Marcus, The minimal polynomial of a commutator, *Portugaliae Math.* 25:73-76 (1964).

[13] M. Marcus and M. Shafqat Ali, On the degree of the minimal polynomial of a commutator operator, *Pacific J. Math.* 37:361-565 (1971).

[14] M. Marcus and M. Shafqat Ali, Minimal polynomials of additive commutators and Jordan products, *J. Algebra* 22:12-33 (1972).

[15] M. Marcus and M. Shafqat Ali, On the degree of the minimal polynomial of the Lyapunov Operator. *Monatshefte für Mathematik* 78:229-236 (1974).

[16] Melvyn B. Nathanson, Additive Number Theory: 1. Inverse Problems and the Geometry of Sumsets, Springer Verlag, 1996.

[17] J. M. Pollard, A generalization of a theorem of Cauchy and Davenport, *J. London Math. Soc.* 8:460-462(1974).

[18] Renato Spiegler, An application of group theory to matrices and ordinary differential equations, *Linear Algebra Appl.* 44:143-151 (1982).

[19] I. Zaballa, Controlability and Hermite indices of matrix pairs, *Int. J. Control* 68(1):61-86 (1997).

Great Moments in XXth Century Mathematics

In this issue we present the answers of two researchers, E. C. Zeeman and Thomas J. Laffey, to the question "If you had to mention one or two great moments in XXth century mathematics which one(s) would you pick?".

Christopher Zeeman

Of course one is tempted to mention famous theorems that have earned Field's Medals, but I would like to draw attention to a moment in the early 1960's that witnessed the rebirth of the whole subject of geometric topology.

Topology grew out of analysis at the beginning of the century, became geometric in the 1910's - 1920's, then turned algebraic in the 1930's - 1940's. By the 1950's algebraic topology reigned supreme while geometric topology was all but dead. But then in the 1960's four results ushered in its rebirth, and gave rise to a great resurgence of geometric topology, as well as spinning off new subjects like differential topology, dynamical systems and chaos. The four results were:

Smale's proof of the Poincaré Conjecture in dimensions greater than or equal to 5 (bypassing the lack of proof in dimensions 3 and 4).

Mazur's (and Morton Brown's) proof of the Schoenflies Conjecture in higher dimensions (under the hypothesis of local flatness, thereby bypassing the psychological obstruction of the Alexander Horned Sphere).

Milnor's proof of the different differential structures on the 7-sphere (thus lauching differential topology).

My own unknotting of spheres in 5 dimensions (leading to the piecewiselinear unknotting of n-spheres in (n+k) - space, for all $k \geq 3$*).*

Thomas J. Laffey

I have chosen two results in Algebra which have had a profound effect.

I was still an undergraduate when my teacher, Professor Seán Tobin, announced to the class one day that a paper had just appeared by Feit and Thompson proving that all finite groups of odd order are solvable. I knew that Burnside had conjectured this but only later came to appreciate the amount of innovative techniques and arguments that had to be created to prove it—Richard Brauer in his talk in 1970 on the occasion of the award of a Fields Medal to John Thompson said in relation to the Burnside conjecture that before the Feit-Thompson paper: "Nobody did anything about it, simply because nobody had any idea how to get started". The paper contains the initial version of many of the tools used in the classification of the finite simple groups as well as techniques used in studying p-groups, p-nilpotence of finite groups etc. The fact that a paper of such extreme length and complexity successfully resolved the conjecture also encouraged mathematicians to attack other outstanding conjectures.

The development of (particularly non-commutative) ring theory and the theory of algebras has been a highlight of twentieth century Mathematics. Jacobson's Density Theorem revolutionised this area. The resulting emphasis on matrix-type rings led to the theory of PI*-algebras, central identities etc. and had a major impact on representation theory and functional analysis.*

After completing a doctorate in Sussex University under the supervision of Walter Ledermann, Thomas Laffey joined the Mathematics Department of University College Dublin in 1968 and has remained there since. He served two terms as head of department (1986-90 and 1996-99).

His principal research interests are in algebra, particularly in finite group theory and algebraic linear algebra.

He was the founding editor of the Newsletter (now Bulletin) of the Irish Mathematical Society and is currently one of the two editors of the Mathematical Proceedings of the Royal Irish Academy and a member of the editorial board of three other journals.

What's New in Mathematics

What color is my hat?

This is the crux of "the hat problem," as presented to readers of the Science section of the New York Times on April 10, 2001. The article, dispatched from Berkeley by Sara Robinson, describes the puzzle as follows:

"Three players enter a room and a red or blue hat is placed on each person's head. The color of each hat is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each person can see the other players' hats but not his own. No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats, the players must simultaneously guess the color of their own hats or pass. The group shares a hypothetical 3 million prize if at least one player guesses correctly and no players guess incorrectly. The same game can be played with any number of players. The general problem is to find a strategy for the group that maximizes its chances of winning the prize."

There is a strategy, and the surprising thing is how well it works. In fact, when the number n of players is 1, 3, 7, 15, etc. (one less than a power of 2) the strategy promises a win $n/(n+1)$ of the time. That strategy is given in terms of Hamming codes, a special class of error-correcting binary codes that sit at the intersection of electrical engineering and abstract algebra. Robinson interviewed Elwyn Berlekamp, the Berkeley math professor who worked out this strategy. She quotes him giving the following life lessons to be deduced from the problem: "The first is that it's O.K. to be wrong as long as you contrive not to be wrong alone. The other, more important lesson is a need for teamwork that goes against the grain of most mathematicians. If the evidence suggests someone on your team knows more than you, you should keep your mouth shut. Most of us assume that each player's strategy is oriented toward him getting it right, and it's not. It's the whole team."

Solitons in matter.

Solitons, or solitary waves, were first discovered as surface waves in canals. They manifest solutions of the nonlinear wave equation which have the remarkable property of maintaining their form unchanged as they propagate. Eran Sharon, Gil Cohen and Jay Fineberg, three members of the Racah Institute of Physics in Jerusalem, have a "Letter to Nature" in the March 1, 2001 issue where they show how perturbations to a crack front in a brittle material result in long-lived and highly localized waves ('front waves') with many of the properties of solitons. They conclude that (presumably novel) nonlinear focussing processes, "perhaps analogous to processes that occur in classical soliton formation, are at play."

God, Stephen Wolfram, etc.

What has Stephen Wolfram, alumnus of Eton and Oxford, veteran of Argonne, CalTech, and the Institute for Advanced Study, MacArthur Fellow at age 21, been doing since his release of Mathematica ("the most popular scientific software ever made") in 1988? He has been planning the complete mathematization of science, and the overhaul of mathematics itself, through his work on cellular automata. This from a long essay by Michael S. Malone, in the online Forbes ASAP for November 27, 2000, entitled "God, Stephen Wolfram, and Everything Else." Cellular automata go back to Von Neumann, but gained wide fame through John Conway's game "Life". How will Wolfram bring about his revolution? To a mathematician the article does not offer any useful clues. The one specific example given, the pattern of markings on a Textile Cone Shell, fits into perfectly conventional science, but it is not clear whether this example is to be taken literally or not, i.e. whether this remark is relevant. A piece appearing in Forbes, and containing statements like "Everything from cars to cartoons, from farms to pharmaceuticals, may reflect the richness of the natural world as seen through Wolfram's cellular automata" and "Within 50 years, more pieces of technology will be created on the basis of my science than on the basis of traditional science," inevitably sounds more like the publicity for an IPO than the presentation of news about current scientific research. The beautiful and moving initial image (the 2-billion-tile rose generated from black and white squares laid according to "half a dozen ... arbitrary rules") typifies the essay. We do not know if the rose is fact or metaphor. We have no way of judging if the tremendous technical developments hinted at are fact or science fiction. "A New Kind of Science," Wolfram's magnum opus on the topic, is promised for sometime this year.

Unknotting the unknot.

The February 9, 2001 issue of Science has a nice piece by Charles Seife entitled "Loopy Solution Brings Infinite Relief." The subject is the recent discovery, by Jeff Lagarias (AT&T) and Joel Haas (U.C. Davis), of an upper bound on the number of Reidemeister moves required to remove all the crossings from the projection of a topologically unknotted curve. The three Reidemeister moves are elementary local changes in the projection; they correspond to moves you might actually make trying to unsnarl a tangle. So there is finally an upper bound on just how long it might take to do it. Seife: "Finite numbers, however, can still be ridiculously large. All Lagarias and Hass guarantee is that if a knot crosses itself n times, you can untangle it in no more than $2^{100,000,000,000\ n}$ Reidemeister moves. In other words, if every atom in the universe were performing a googol googol googol Reidemeister moves a second from the beginning of the universe to the end of the universe, that wouldn't even approach the number you need to guarantee unknotting a single twist in a rubber band. ... Still, [Lagarias] says, just showing that a limit exists may inspire future researchers to whittle it down to a reasonable size. (Macedonian swordsmen need not apply.)"

How do fish swim?

"The dynamics of swimming fish and flapping flags involves a complicated interaction of their deformable shapes with the surrounding fluid flow."This is the beginning of a "letter to Nature"(14 December 2000) from a Courant Institute/Rockefeller University team headed by Jun Zhang. Their research used flexible filaments in a flowing soap film 3-4 microns thick. In particular they report that beyond a certain critical length the system becomes bi-stable, with both a "stretched-straight state"and a "stable flapping state"possible. The stable flapping state has an especially simple mathematical form: "Unlike a simple pendulum, the undulation is well fitted by a travelling harmonic wave with a spatially varying envelope: $y(x,t) = f(x)\ \sin(2\pi\nu t+2\pi x/\lambda)$. Here, $y(x,t)$ is the horizontal displacement of the filament from the centre-line, measured at a vertical distance x from the fixed point for time t. $f(x)$ is a spatial envelope function (increasing monotonically from the fixed point), ν is the flapping frequency and λ the wavelength."An interesting final point: "Swimming offers alternatives comparable to the bistability of our filament. The stretched-straight state is the analogue of a glide, whereas the flapping state is analogous to swimming. Efficient propulsion uses the natural oscillations of the swimmer, which in the filament is a property mediated by stiffness."A web presentation of this research is available.

Jock Math.

You may wonder, halfway through the season, if your team has a mathematical chance of winning the league. If your sport is soccer ("football"), then well may you wonder. It turns out that this is an NP-complete problem, equivalent to the notorious traveling salesman problem, and therefore computationally as hard as a problem can get. This information comes from a piece by Justin Mullins in the January 27, 2001 New Scientist, entitled "Impossible Goal" and explaining this recent discovery, due to Walter Kern and Daniël Paulusma of the University of Twente, and also, independently, to Thorsten Bernholt, Alexander Gülich, Thomas Hofmeister and Niels Schmitt of the Dortmund University Computer Science department. "Fans had a much easier time in the days when teams got 2 points for a win and 1 for a draw. Kern and Paulusma have shown that this is mathematically simpler than a travelling salesman problem, and the time to solve it increases more slowly as it gets bigger. The switch a few years ago to 3 points for a win turned it into an NP-hard problem. "

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An Interview with Gareth A. Jones

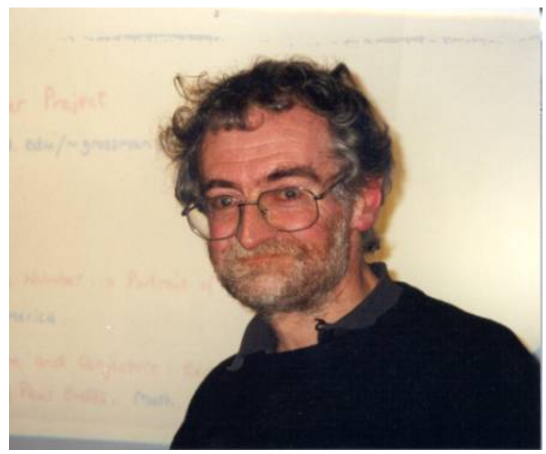
I acquired this piece of information from Nigel Hitchin back in 1986. Let me see if I get it right. You were colleagues in Oxford and during your first year you were taught by Michael Atiyah. What course was it and how was he as a teacher for students just starting university?

Nigel Hitchin and I were students together at Jesus College, Oxford, from 1965 to 1971. Oxford, like Cambridge, consists of several dozen colleges, each containing a few hundred students. Weekly individual teaching is provided by college tutors, but lecture courses and examinations are organized by the University departments. In my time, the mathematics tutors at Jesus College, Edward Thompson for Pure Mathematics and Christopher Bradley for Applied Mathematics, had excellent reputations as teachers, so the standard of mathematics in the college was very high. There were eight mathematics students there in my year: among them, Nigel Hitchin went on to do great work in geometry with Atiyah and Donaldson, while Lyn Thomas, after getting a doctorate in quantum theory, became one of the leading figures in the Operations Research community.

The first-year algebra lectures were given by Michael Atiyah: he was clear, precise, and very fast! His course started at a rather elementary level, with sets, functions, equivalence relations, and so on, but he very soon accelerated, covering as much material on groups, rings, fields and vector spaces as most university courses do in two years. He lectured with such energy and enthusiasm that it was impossible not to be inspired by him: I already loved algebra, and this course confirmed my view of the subject. As students, I don't think we initially realised how great a mathematician he was: I remember asking Edward Thompson for help with one of Atiyah's exercises, and Thompson puffing on his pipe for a while and then saying "You know, Michael is generally reckoned to be rather clever". A few months later (in 1966), Atiyah was awarded a Fields Medal, and then we knew! We had some other excellent lecturers too: Charles Coulson for applied mathematics, and later on, Ian Macdonald for algebraic geometry, though not all of the lecturing was uniformly good.

Before your Oxford days how was your life? I think you come from Wales. Did you realize at an early age that Mathematics was the science you wanted to devote your life to?

Before Oxford, I lived in Cardiff, the capital of Wales. My father was a railway traffic controller, and my mother had been a librarian. They both regretted that they never had the opportunity to go to university, and were very proud when my sister and I did.



Gareth A. Jones

Although I considered other subjects like architecture and physics, I really knew from about 15 years old that I wanted to be a mathematician. As is so often the case, it was good teachers who influenced me, especially one called Howard Williams, who spent many hours giving me individual help. What attracted me to the subject was its elegance, consistency and objectivity. I can still remember the feeling of excitement on finding a really neat solution to a problem: for instance, evaluating

$$I = \int_0^{\pi/2} \sin\theta/(\sin\theta + \cos\theta)\, d\theta$$

by using symmetry to see that

$$I = \int_0^{\pi/2} \cos\theta/(\cos\theta + \sin\theta)\, d\theta$$

and then adding. (It increased the pleasure to discover later the famous story in which the young Gauss used a similar idea.)

You stayed on in Oxford to work for a PhD. Your research was in pure Group Theory and Peter M. Neumann was your supervisor. It may be a romantic view but I *always think of you as a sort of grand-son of the famous triad HNN* [1]*...*

In my final year as an undergraduate, I decided to specialise in commutative algebra, topology and group theory. The group theory lectures were given by Graham Higman, with Peter Neumann running the problem classes. Higman's lectures contained a mixture of classical material and his own recent research, which was very wide-ranging. The course was very interesting but also hard work: after each lecture, several of us would spend a whole afternoon together, going through our notes and trying to understand them. This was excellent training for research, and after a few months I knew that I wanted to do a doctorate in group theory. At the time I (unjustifiably) found Graham Higman rather daunting, but Peter Neumann, who was much younger and had a great sense of fun, seemed more accessible, so I was delighted when he agreed to supervise me.

Group theory was very active in Oxford in the late 1960s, under Higman's leadership: counting research students, research fellows, visitors and permanent staff, there must have been at least twenty at a time working on it. The big challenge in finite group theory then was the classification of finite simple groups, and people like John Conway, Don Higman and Charles Sims would come to Oxford and discuss the sporadic simple groups they were constructing. Graham Higman was also interested in infinite groups, especially combinatorial group theory, embedding theorems and decision problems, so we got a very wide education. Atiyah was increasingly influential in Oxford, and I also used to go to his seminars, and those of his visitors, like George Mackey, though few of the other group-theorists did this.

Peter Neumann is the son of the group-theorists Bernhard and Hanna Neumann (who, together with Graham Higman, introduced HNN extensions), and for my diploma dissertation he suggested one of the open problems in Hanna's book on varieties of groups, about identical relations in finite simple groups; this was a good problem to start my research on, because it forced me to spend my first year learning about varieties and finite simple groups, two very different topics. For my D.Phil. (Oxford's version of a Ph.D.), I worked for the next two years on finite permutation groups, applying techniques of Burnside, Schur and Wielandt to groups of prime-power degree.

The algebra research students, mostly supervised by Graham Higman and Peter Neumann, formed a very lively and sociable crowd; they included Peter Cameron, who is now a leading figure in permutation groups and combinatorics, and my future wife Mary Tyrer, working in combinatorial group theory. We held a weekly Junior Algebra Seminar, in which we would take it in turns to give seminars on our particular interests; this was excellent training for a lecturing career, as we could learn from our mistakes without too much embarrassment. We also had plenty of less formal activities, such as punting on the River Cherwell, or squeezing into someone's car and driving out into the beautiful countryside around Oxford, with the latest Beatles record blasting out of the radio.

Over the years your work has spread from Group Theory to several other areas which interfere with it. An important part of it is in the theory of Dessins d'enfants*. The theory was initiated by Grothendieck but I have some idea that you started working on it independently. Was that not so?*

When I finished my thesis, in 1971, I got a lectureship at Southampton. Before then, it had been relatively easy to get academic positions in the UK, but suddenly the expansion of higher education stopped, and it became almost impossible; I think my year were among the last who were reasonably successful in doing this. Mary got a fellowship at New Hall, Cambridge, and I spent most of the 1970s commuting between there and Southampton.

The Mathematics Department at Southampton was totally unlike what I had been used to at Oxford. There were about 18 pure mathematicians, mostly working in differential geometry or topology. There was no real group theory, though a number of pure and applied mathematicians needed to use the subject, and it was made clear to me when I was appointed that I was expected to collaborate with them, rather than concentrate on pure group theory. The need to learn about my new colleagues' specialities, together with the strain of commuting, slowed down my research, but I gradually absorbed a great deal of useful mathematics. Having been trained as a group-theorist put me in a strong position to do this: in almost every case, symmetry played a fundamental role, so that some form of group theory could be applied to the problems.

Two of these collaborations proved particularly rewarding. Keith Lloyd and I have applied techniques from graph theory and permutation groups to problems in mathematical chemistry, and gradually others, such as Mikhail Klin in Beer-Sheva and Reinhard Pöschel in Dresden, have also been involved. Equally fruitful has been my collaboration with David Singerman on maps on surfaces, now more fashionably called *dessins d'enfants*. Around 1970, Norman Biggs, who was briefly at Southampton, wrote a few papers showing how

[1] HNN stands for Higman-Neumann-Neumann, that is, Graham Higman, Bernhard Neumann and his wife Hanna Neumann, famous for their work in Group Theory

maps on surfaces could be described by permutations (an idea originating with Hamilton). He then abandoned the subject, and became a leading authority on algebraic graph theory, but David and I were convinced that there was a significant theory waiting to be discovered. We published a number of papers, and supervised research students in this area, but nobody in the UK took much notice: the subject was too inter-disciplinary for those times, involving a mixture of combinatorics, permutation groups, Fuchsian groups, and Riemann surfaces. In the mid-1980s we were excited to discover that Grothendieck, with some of his colleagues at Montpellier, had also been working on these ideas, and had found some surprising links with Galois groups and Teichmüller spaces. He wrote out a sketch of his ideas, but then withdrew from active mathematics, leaving others to work out the details. There was a very important conference at Luminy in 1993, with people like Fried, Ihara, Itzykson and Serre involved, and now it's quite a thriving subject.

These days with research assessment exercises all over the place some people tend to think that only research work is important for Mathematics and its development. You have written a couple of excellent textbooks and have, for instance, produced a translation of Jean-Pierre Serre's " Complex semisimple Lie algebras" [2]. *It seems you do not share that opinion ...*

Research assessment exercises now play a major role in British academic life: each department undergoes a rigorous examination every few years, with major financial rewards and penalties for success and failure. There is no doubt that some mechanism has to be used to direct limited resources towards those most capable of using them effectively, but the current system is producing serious distortions and injustices, and there is a general consensus that something simpler and fairer must be introduced. One effect has been to place too great an emphasis on research papers and grants, and to devalue more scholarly activities such as editing journals, organizing conferences, writing text-books, etc. Fortunately, research output is mainly judged by its quality, rather than its quantity, and it is enough to publish an average of one good paper each year, preferably in a prestigious journal. My view is that one shouldn't allow one's career to be too strongly influenced by these forces, and that one should concentrate on what one does best.

In my case, I have always enjoyed expository writing, including survey articles, text-books and encyclopedia contributions. I've written a couple of undergraduate text-books with David Singerman and with my wife Mary, based on the lecture-notes for courses we have taught at Southampton, and I hope to publish one or two more in the next few years. I also translated Serre's book on complex semisimple Lie algebras, partly to learn the subject properly, since the classification of finite simple groups, around 1980, meant that one couldn't really understand the groups without the Lie algebras; another reason was to study Serre's style, which I've always admired for its simplicity and directness.

This time you are in Portugal to give a talk in connection with the exhibition of a video on Paul Erdös. Paul Erdös is sometimes referred to as a "problem solver" and his work does not appear to command the same respect and admiration as, say, Milnor's, Grothendieck's or Atiyah's. That is perhaps unfair. What do you think?

Paul Erdös was loved and respected throughout the mathematical world. He lived a nomadic life, with no permanent position or home, travelling between conferences and visits to research colleagues, many of whom were glad to tolerate his rather demanding nature (for a few days, at least) in order to achieve the honour of a joint paper with him. He published over 1500 papers, many of them deeply influential, with nearly 500 collaborators, whereas most mathematicians would be proud to publish 100 in their lifetime.

Erdös won several major prizes, such as the Cole and Wolf Prizes (characteristically giving away most of the money for charitable causes), but nevertheless many feel that his achievements were insufficiently recognised at the highest levels. The classic instance of this is the fact that when he and Selberg found an "elementary" proof of the Prime Number Theorem, it was the latter who got a Fields Medal and a position at the Princeton Institute for Advanced Studies, not Erdös. Perhaps his idiosyncratic approach to mathematics, preferring to tackle problems rather than build theories, was out of step with the prevailing view of the subject. His legendary ability to enter new areas, such as dimension theory, and solve difficult problems without absorbing masses of theory, cannot have endeared him to specialists in those areas. One of his main fields of activity was combinatorics, and even today, despite its rich structure and wide applicability, this subject is often looked down upon as lacking in depth (*"Graph theory is the slums of topology"*, in a famous phrase); perhaps the problems are too easily stated for the guardians of jargon, though the solutions (in Ramsey Theory, for instance) are often notoriously difficult to obtain.

Mathematics is fertile enough to allow many different talents to flourish, ranging from "Bourbakiste" system-builders to "Hungarian" problem-solvers. Although Paul Erdös began his mathematical career nearly 70 years ago, it is still rather early to judge his influence; however, I predict that some of his results and techniques (such as

[2]Springer Verlag, 1987

the probabilistic method) will in future be regarded as among the greatest achievements of 20th-century mathematics.

Outside Mathematics what are your interests? I know you are a keen jogger. Did you not take part in the London Marathon several times?

Outside Mathematics, my main interests are now my family (my son Peter is studying History at Oxford, my daughter Elizabeth hopes to study Electronic Engineering, and Mary is still active in Mathematics), and also running. I try to run about 10km each day, preferably at lunch-time, as a break from the morning's work. I compete regularly, in road and track races, from 800m upwards, and I've represented Wales several times. I've run seven marathons, my best time being 2:26 in London, 1990, but now lack of time for training forces me to concentrate on shorter distances. I used to play a lot of chess, and as a student I came 2nd in the British Junior and Welsh Senior Championships; however, taking chess seriously is too much like doing research in mathematics, and far too time-consuming, so I only play casually now.

(Questions and picture by F. J. Craveiro de Carvalho)

Gareth A. Jones was born in Cardiff, Wales, where he lived until the age of 19 when he won a scholarship to study Mathematics at Jesus College in Oxford. After six years in Oxford he obtained a DPhil for work on finite permutation groups. He was supervised by Peter M. Neumann and also benefited from Graham Higman's strong research leadership in Group Theory.

After Oxford he moved to Southampton where he has been ever since and where he is currently Professor of Pure Mathematics.

Professor Jones has written three textbooks, one in collaboration with David Singerman and two with his wife, the group-theorist Mary Jones. He also contributed a long article on *Symmetry* to Walter Ledermann's *Handbook of Applicable Mathematics*.

Gallery

João Farinha

Prof. João Pereira Dias, summarizing the beginning of João Farinha's academic life, wrote: "...in 1934 he graduated in Mathematics in Coimbra with distinction". After mentioning his "ceaseless teaching work", he added: "Recruited as an Assistant in 1950, the School of Sciences showed its trust the very same year by giving him full charge of several courses; and four years later his position at the School was definitively established with the *Very Good* mention given to his brilliant doctoral examination".

Of those 16 years of "ceaseless work", I followed closely the last six, probably the most important: I met João Farinha in August 1944.

Having finished high school, I was going to stand for the university admission examination. Aware of my mathematical deficiencies, I went to look for the most reputed teacher of mathematics in Coimbra, who then lived in a strange *República*: its name was "Lactarium Paradoxorum", possibly because most of its members had already graduated, or were old enough for it.

A 12-year friendship began that day. I recall the warning he gave me and a cousin of mine: "I can teach you, but I can't promise to be very assiduous because I'm about to be married". The frequency of classes indeed suffered from this. My cousin, who was better prepared, passed the examination; I failed.

In October I began my mathematics studies with João Farinha, with three 2-hour classes every week. These tutorials quickly transcended mathematics, entering into history, philosophy, sports, literature, music and politics. My mother once asked him how he managed to keep me focused, to which he answered: “It’s simple: I noticed that Luís has a 45-minute attention span. So, after that, we stop the mathematics, we talk about something else for a while, and then we resume”.

It was during those breaks that he taught me to see the beauty of mathematics, it was during those breaks that I learned who Aniceto Monteiro was, that Pierre Curie was a man with a capital M, that friendship goes beyond age and opinion. I learned to know, or know better, people like the painter Mário de Oliveira or the sculptor Aureliano Lima. I learned from him the need for rigour even in apparently minor details (my students know this!).

Once, before my Algebra examination, he gave me a test. He began by asking me to define a function of bounded variation, a concept he made me study in Vicente Gonçalves’ book. I started: “Take a function $\psi(x)$ continuous and differentiable in the interval $[a, b]$”. “Wrong”, he said coldly. “How can that be? I haven’t said anything”. “Yes, but the little you said was wrong: you said $\psi(x)$ and you wrote $\varphi(x)$”. It was a detail – but he knew that in the Algebra examinations of the time you could fail because of such a “detail”.

My sympathy for António Sérgio made him smile, and he tried to correct my sometimes vaguely romantic opinions: it was from him that I learned the deep truth of the Shakespearean aphorism, “There are more things and heaven in earth, Horatio, than are dreamt of in your philosophy”, and I came to find much of what he taught me when I read Isaiah Berlin a few years ago, for instance “total liberty for wolves is death to the lambs...”

In 1947 I went to live in Switzerland but we kept in touch. In one of my holidays I brought him a book that had made an impression on my naive marxism. I don’t recall the author’s name, but the subject was marxism and mathematics. He read a few pages and said: “It’s all very well, but this doesn’t explain Évariste Galois”, and he proceeded to tell me about Galois.

In 1949 or 1950, during the summer holidays, I went with him and his parents-in-law to Lisbon to attend the first public musical performance by his wife, in the National Conservatory. The train had a long stop at Entroncamento and the two of us went for a walk on the platform. He told me then, and I was among the first to know (the very first was surely Luís de Albuquerque), that his situation *vis-à-vis* the University was to be profoundly changed.

His trust in me shows the dimension of the man: I was a naive 23-year old, inexperienced and therefore intransigent and dogmatic. But João knew that he could and should tell me, because he knew me, because I was his disciple more than his student, and unreservedly his friend.

A few years passed, and I, already an engineer, needed to improve my knowledge of statistics. I asked for his help and for several weeks I did some serious study of statistics. One day he told me: “I was asked to give a course on statistics to engineering students: you should follow this course”. I did, and I still remember the *Sala dos Gerais* overflowing with students, and João with his way of talking, head tilted, voice low, eyes half-closed, the exposition crystal clear, all present following, as under a spell, the words that shed light on the apparent confusion of curves and formulae.

At the end of term, João asked me if I could make available the notes he had written during his private tutorials to me, since no one had taken any notes and the exams were approaching. One of the students, a long-time friend, explained to me later: “Everything was so clear that no one thought of taking notes...”

He was an exceptional teacher. Apart from the love of mathematics, I owe to him the rejection of myths and received ideas, even the most respectable, the cult of friendship, and the courage to have different ideas even if politically or culturally incorrect. At a time in which the need for individual self-assertion has become almost mandatory, it feels good to remember the extreme modesty of João Farinha, on display even in his doctoral examination.

He seldom talked about himself. He never mentioned the full marks obtained in his mathematics degree, preferring his athletical “exploits”and his adventures as a radio sports announcer in the first broadcast of a football match in the old Santa Cruz field.

Once, under direct questioning by me, he said: “I was arrested the day after my graduation. Still recovering from the late-night dinner, I was taken to the Caxias prison, where I stayed for six months without ever being interrogated. I walked in the prison yard with the others and I played a lot of football. One day I was called from the cell and told to pack my things and leave. When I asked why I had been arrested they answered: “Don’t ask questions, just leave.”When I got out I could not find a place in any public school. I was left with private tutorials”.

I can’t talk about the mathematician, and even about the professor I didn’t say much. But I can’t help remembering the days when all of João's activity were the rational mechanics examinations, and a queue formed at his door: the first time I saw this I was worried, thinking that something had happened to him, but an older student sent me on my way with the words: "*Caloiro*, go away, this is a mechanics examination".

I don't know if he might have been a great mathematician, as Vicente Gonçalves thought, but I do know he was a great professor of mathematics and a Master in the old sense of the word. And for that maybe, but above all for his human quality and his friendship for me, I miss him very much, even today when I am older than João was when we first met.

We said goodbye in front of the house of Luís de Albuquerque: João was leaving for Paris and I was going to Lisbon to restart my professional life. We never met again.

The academic career of João Farinha

After finishing high school in Castelo Branco, he enrolled in the School of Sciences of the University of Coimbra in 1927/28. He graduated in Mathematics in 1934 with distiction.

Prevented from teaching in public schools until 1949 for political reasons, he became a teacher at private schools in Coimbra, among them the S. Pedro *Colégio*, with students of all ages.

At the same time he became the most reputed of private tutors for university mathematics courses, and his rational mechanics classes were legendary. As a result of that activity, he published in 1946 a book of problems on Algebra and Analytic Geometry.

In 1949 he became 2nd Assistant at the School of Sciences of the University of Coimbra. He got his doctoral degree in 1954 with a mark of 18 out of 20 and was made 1st Assistant on May 28 of the same year.

In 1956 he became a member of the Center for Applications of Mathematics to Nuclear Energy, established and led by Prof. Manuel dos Reis. In February 1957 he received a grant from the Gulbenkian Foundation to do research at the Institut Henri Poincaré and the Collège de France. He was in Paris when he died shortly afterwards.

The variety of subjects he taught as a Professor, from Infinitesimal Calculus to Higher Analysis, Celestial Mechanics, Probability Theory and even Machine Design, is perhaps due to the first 15 years of his activity, in which, because of his communication abilities and his extended mathematical culture, he had been requested to help students and entire classes in all kinds of subjects.

João Farinha

He died in the city where one of the mathematicians he most admired, Évariste Galois, had lived. Galois' fate echoes in the words of Vicente Gonçalves, João Farinha's professor and friend, who, after mentioning the limitations of time that his teaching activities imposed on his research, wrote: "...it was difficult for him to advance. In spite of everything, he advanced. When his position was already honourable, death came and felled him."

Publications by João Farinha

O teorema dos resíduos e o cálculo da soma de uma série, Gazeta de Matemática, nos. 44-45, 1950.

Sobre um caso de convergência de fracções contínuas de elementos complexos, Gazeta de Matemática, no. 50, 1951.

Sobre dois teoremas de Pincherle, Revista da Faculdade de Ciências de Coimbra, vol. 21, 1952.

Sur les limites des zéros d'un polynôme, Revista da Faculdade de Ciências de Lisboa, 2a. série, A, vol. 3, 1953.

Fracções contínuas ascendentes periódicas, Revista da Faculdade de Ciências de Coimbra, vol. 22, 1953.

Sobre a convergência nas fracções contínuas de elementos complexos, Doctoral thesis, Coimbra, 1953.

Sur la convergence de $\Phi a_i/1$, Portugaliae Mathematica, vol. 13, 1954.

Quelques propositions concernant les zéros d'un polynôme, Revista da Faculdade de Ciências de Lisboa, 2a. série, A, vol. 4, 1954-55.

Sur la moyenne arithmétique, Revista da Faculdade de Ciências de Coimbra, vol. 23, 1954.

Une condition de convergence uniforme, Revista da Faculdade de Ciências de Coimbra, vol. 23, 1954.

Sur la probabilité maximum d'accord de deux états, Revista da Faculdade de Ciências de Coimbra, vol. 23, 1954.

Sobre o valor preferível de uma série de observações, Associação Portuguesa para o Progresso das Ciências, XIII Congresso Luso-Espanhol. Tomo III, Coimbra 1956.

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