BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

December 2000

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Editorial

Apart from containing information about CIM and its activities, the Bulletin will continue to publish material of general interest to mathematicians. While diversifying the types of articles included, we will try to keep the high quality of previous issues. We are grateful to Professor F. J. Craveiro de Carvalho for passing on to us several contributions he obtained for the Interview, Great Moments and Gallery sections. They will appear in this and the following issues.

The Bulletin will expand to accomodate, in each issue, a mathematical or historical article of interest to the mathematical community, as well as a generalist section on 'What's new in Mathematics', adapted from the AMS section with the same title and reprinted here with the kind permission of the AMS.

We hope the Bulletin may be a contribution to the main purpose of CIM – to act as a forum for the Portuguese mathematical research community.

Contents

Coming Events

Thematic Term on Semigroups, Algorithms, Automata and Languages

Organizers

Gracinda M. S. Gomes (University of Lisbon, Portugal), Jean-Eric Pin (University of Paris VII, France) and Pedro V. Silva (University of Porto, Portugal).

Date: May - July, 2001.

The Term is designed to make Coimbra the gathering point of researchers in the subjects of semigroup theory and automata theory during the months of May, June and July 2001. Besides providing a basepoint for the development of joint research projects, the Term includes multiple activities such as specialized schools and workshops on relevant specific subjects. Postgraduate students will be particularly welcome.

Each school consists of several 5 hour courses held by prominent researchers. The workshops include 50 minute invited lectures and a limited number of 20 minute talks on the specific topics of the workshop, proposed by the participants. Anyone wishing to present such a communication is invited to submit a 1 to 2 page long abstract before February 28 to the e-mail address term2001@cii.fc.ul.pt.

The programme of events is the following:

2-11 May: School on Algorithmic Aspects of the Theory of Semigroups and its Applications

invited lecturers: J. Almeida (Porto), C. Choffrut (Paris VII), J. Fountain (York), S. Margolis (Bar-Ilan), L. Ribes (Carleton), M. Sapir (Vanderbilt), M. Volkov (Ekaterinburg), T. Wilke (Kiel).

4-8 June: School on Automata and Languages

invited lecturers: M. Branco (Lisbon), V. Bruyère (Mons), O. Carton (Marne-la-Vallée), A. Restivo (Palermo).

11-13 June: Workshop on Model Theory, Profinite Topology and Semigroups

invited lecturers: J. Almeida (Porto), T. Coulbois (Paris VII), H. Straubing (Boston College), P. Trotter (Tasmania), P. Weil (Bordeaux).

2-6 July: School on Semigroups and Applications

invited lecturers: K. Auinger (Vienna), M. Lawson (Bangor), W. D. Munn (Glasgow), A. Pereira do Lago (São Paulo).

9-11 July: Workshop on Presentations and Geometry

invited lecturers: R. Gilman (Stevens Inst. of Tech.), D. McAlister (DeKalb), J. Meakin (Lincoln), S. Pride (Glasgow), N. Ruskuc (St. Andrews), B. Steinberg (Porto).

The venue for all events is the Observatório da Universidade de Coimbra, in the peaceful setting of Mount Santa Clara.

registration fees

may school

Before February 28: Euro 100

After February 28: Euro 120

june events

Before February 28: Euro 100

After February 28: Euro 120

july events

Before February 28: Euro 100

After February 28: Euro 120

full term

Before February 28: Euro 200

After February 28: Euro 240

In return for their fees, the participants are entitled to receive school/workshop documentation and to participate freely in the social activities, including the corresponding Term dinners, to be held on May 10, June 12 and July 10.

Accompanying persons wishing to join the social programme will pay 75% of the normal fee. Early payments can be made by international cheque addressed to "Centro Internacional de Matemática" (CIM). The cheques should be sent to:

Patrícia Paraíba,
C.A.U.L., Av. Prof. Gama Pinto 2,
1649-003 Lisboa, Portugal.

Detailed information on accommodation and travel arrangements will be supplied in the second announcement, to be issued soon.

A limited number of scholarships will provide funding for those participants in need of financial support, particularly postgraduate students. Anyone wishing to apply for support is invited to do so before December 31, mentioning the amount of funding required and justifying such a request.

In order to receive further information on the Thematic Term, the potential participant should fill out the attached preregistration form and return it by e-mail to the address term2001@cii.fc.ul.pt. The information contained in the form will be assumed to be provisional.

For more information on these events and registration forms, please visit the site

http://alf1.cii.fc.ul.pt/term2001/

The Thematic Term has the support of Fundação Calouste Gulbenkian, Fundação para a Ciência e Tecnologia, Centro de Matemática da Universidade do Porto and Centro de Álgebra da Universidade de Lisboa.

Summer School
Analytical and Numerical Methods in Non-Newtonian Fluid Mechanics

Organizers

Estelita Vaz (University of Minho, Portugal), J. Maia (University of Minho, Portugal) and K. Walters (University of Wales Aberystwyth, United Kingdom).

Date: 25-29 June, 2001.

Aims

Despite its relevance to a wide number of industries, Rheology and Non-Newtonian Fluid Mechanics are subjects that are often viewed as being of prohibitive complexity to newcomers to the field and have often not been used to the fullest possible extent. The aim of the School is, therefore, to interest young researchers into the field by helping to bridge the gap between the available theoretical tools and existing problems of a mathematical nature in industry and academia.

The School will be held in the Guimarães Campus of the University of Minho, Portugal.

Lecturers

K. Walters, University of Wales

A. R. Davies, University of Wales

M. H. Wagner, Technical University of Berlin

G. Marrucci, University of Naples

R. Keunings, Catholic University of Louvain

F. P. T. Baaijens, University of Eindhoven

Registration and fee

Registration can be performed by e-mail, fax or letter directly to the School Secretariat at:

Summer School Secretariat

Ms. Elisabete Santos

School of Sciences, University of Minho

4800-058 Guimarães, Portugal

Phone: 351 253 510 159

Fax: 351 253 510 153

e-mail: s.school@ecum.uminho.pt

The registration letter must specify the participants' name, institution, address, telephone, fax and e-mail.

The following School Fees will apply:

Before March 31, 2001: 350 Euro.

After March 31, 2001: 450 Euro.

The fee includes attendance at the lectures, the support materials (programme, book, etc.), five lunches (monday to friday), a visit to the Port Wine cellars and admission to the school dinner. The fee can be paid either by cheque payable to Summer School 2001 or by bank account transfer. The banking details are:

Bank: Caixa Geral de Depósitos

Branch: University of Minho - Azurém

NIB (account number): 0035 0130 00001457300 06

Deadlines

The absolute deadline for the receipt of registrations is April 30, 2001. The number of participants is limited to 80 and, therefore, early registration is advised.

For more information on this event, to be held in Guimarães, please visit the site

http://www.dep.uminho.pt/SummerSchool/

Advanced School on Recent Developments in Large-Scale Scientific Computing

Organizers

Filomena Dias d'Almeida (Engineering Faculty, Univ. of Porto, Portugal) and Paulo Beleza de Vasconcelos (Economics Faculty, Univ. of Porto, Portugal).

Date: 3-6 July, 2001.

Aim of the School

The aims of this advanced school are: to present the state-of-the-art methods and tools to solve large scale linear problems, namely large linear systems and large eigenvalue problems, to bring together specialist researchers on computational mathematics and to encourage the interchange of new ideas, to create a suitable environment for the participants to get acquainted and involved in today's computational mathematics research problems.

Topics

- Parallel architectures
- Performance measures, parallel programming paradigms
- Message passing paradigms
- Nonstationary iterative methods for large linear systems
- Direct methods for large sparse linear systems and preconditioners
- Large scale eigenvalue problem
- Linear algebra libraries for large scientific computations

Lecturers

Claude Brezinski, Univ. Lille, France (to be confirmed)

Jack Dongarra, Univ. of Tenesse and Oak Ridge Nat. Lab., USA

Iain Duff, CERFACS, France and RAL, UK

Joaquim Júdice, Mathematics Dep., Coimbra Univ., Portugal

Osni Marques, LBNL, USA

Francisco Moura, Informatics Dep., Minho Univ., Portugal

Orlando Oliveira, Physics Dep., Coimbra Univ., Portugal

Rui Ralha, Mathematics Dep., Minho Univ., Portugal

Program and registration fee

A detailed version of the program will be available soon.

Registration fee: Euro 100. It includes the school documentation and coffee.

Travel information and accommodation

Soon we will provide information about travel information and accommodation.

Financial support

CIM - Centro Internacional de Matemática

CMAUP - Centro de Matemática Aplicada da Universidade do Porto (applied for)

FCT - Fundação para a Ciência e Tecnologia (applied for)

FEP - Faculdade Economia do Porto

FEUP - Faculdade Engenharia da Univ. Porto

For the registration form and more information on this event, to be held in Porto, please visit the site

http://www.fep.up.pt/docentes/pjv/LSC.html

Workshop on Electronic Media in Mathematics

Organizers

F. Miguel Dionísio (IST, Technical University of Lisbon, Portugal), José Carlos Teixeira (University of Coimbra, Portugal) and Bernd Wegner (Technische Universität Berlin, Germany).

Date: 13-15 September, 2001.

Aims

The workshop will provide an open forum for the exchange of information and presentations on electronic media in Mathematics for mathematicians and people using mathematics in applications. Three main subject areas are to be covered:

a) Computational devices for mathematics: Mathematica, Maple and other general software packages, special packages in numerical mathematics, computational algebra, computational geometry, proof theory and their applications in mathematical research, support for teaching mathematics, support for applications of mathematics in industry.

b) Visualization and applications of CAD: visualization of geometric and physical objects, animation software, CAD-package and geometric construction.

c) Electronic information and communication: electronic publishing, preprint-servers and preprint databases, electronic document delivery, electronic access to software, literature data bases, organization of information in the web. The event will take place in Coimbra.

Workshop - From Brownian Motion to Infinite Dimensional Analysis

Organizers

A. B. Cruzeiro (University of Lisbon, Portugal) and L. Streit (University of Bielefeld, Germany).

Date: 18-22 September 2001.

Aims

The need for the development of infinite dimensional Analysis on spaces of continuous paths or of less regular objects such as distributions has become evident mainly by physical motivations (e.g. Quantum Mechanics and Quantum Field Theory). These spaces are endowed with probability measures, one of the more regular cases being the law of Brownian motion. In this case the Itô calculus provides the underlying techniques to manipulate irregular functionals of the paths and the corresponding infinite dimensional Analysis has developed intensively in the past recent decades giving rise to important results in Mathematics, but also applications outside the initial framework (e.g., Filtering and Control Theory, Financial Mathematics). More recently, special attention has been given to the geometry of (curved) spaces. The goal of the workshop is to bring together various approaches to infinite dimensional Analysis.

José Sousa Pinto

Professor José Sousa Pinto, Secretary of the CIM General Assembly since 1996, died in August 2000 after a long illness. He was a Professor at Aveiro University, and his main research interest was nonstandard analysis and generalized functions. A book by Prof. Sousa Pinto, *Métodos Infinitesimais de Análise Matemática*, was published by the Gulbenkian Foundation in 2000.

CIM Governing Bodies for 2001-2004

General Assembly

Maria Paula de Oliveira, Univ. Coimbra (Chair)
Luís Sanchez, Univ. Lisboa (Secretary)
António Caetano, Univ. Aveiro (Secretary)

Auditing Board

Estelita Vaz, Univ. Minho (Chair)
José Basto Gonçalves, Univ. Porto
Maria Paula Rocha Malonek, Univ. Aveiro

(The members of the executive board were listed in the Bulletin no. 7.)

CIM Program for 2001-2004

The new CIM executive board, chaired by Prof. Luís Trabucho de Campos (Universidade de Lisboa, Portugal), took office in July 2000. The board intends to continue CIM's activity of promoting and sponsoring several kinds of international meetings on mathematics and its applications. A central position in CIM's scientific program is occupied by the *Thematic Terms*. The subjects for the next four years are:

- Mathematics and Computation (2001)
- Mathematics and Biology (2002)
- Mathematics and Engineering (2003)
- Mathematics and the Environment (2004)

CIM Scientific Committee for 2001-2004

At the CIM General Assembly held in Coimbra on 25 November 2000, the CIM Scientific Committee for 2001-2004 was approved. Its members are:

- Jorge Almeida (Universidade do Porto, Portugal)
- Luís Trabucho de Campos (Universidade de Lisboa, Portugal), *ex officio*
- Raul Cordovil (Universidade Técnica de Lisboa, Portugal)
- John Dennis (Rice University, USA)
- João Paulo Dias (Universidade de Lisboa, Portugal)
- Ivette Gomes (Universidade de Lisboa, Portugal)
- Joaquim João Júdice (Universidade de Coimbra, Portugal)
- João Martins (Universidade Técnica de Lisboa, Portugal)
- Jacob Palis (IMPA, Rio de Janeiro, Brasil)
- Luís Moniz Pereira (Universidade Nova de Lisboa, Portugal)
- Eduardo Marques de Sá (Universidade de Coimbra, Portugal)
- Luís Sanchez (Universidade de Lisboa, Portugal)
- José Perdigão Dias da Silva (Universidade de Lisboa, Portugal)
- Hugo Beirão da Veiga (Dipartimento di Matematica Applicata "Ulisse Dini", Pisa, Italy)
- Bernd Wegner (Technische Universität Berlin, Germany)
- Efim Zelmanov (Yale University, USA)
- Enrique Zuazua (Universidad Complutense, Madrid, Spain)

CIM Associates

The current CIM Associate institutions are:

- Sociedade Portuguesa de Matemática
- Universidade de Coimbra
- Universidade do Porto
- Faculdade de Ciências da Universidade de Lisboa
- Universidade do Minho
- Universidade Nova de Lisboa
- Universidade de Aveiro
- Universidade dos Açores
- Universidade da Beira Interior
- Universidade de Évora
- Universidade de Trás-os-Montes e Alto Douro
- Instituto Superior de Línguas e Administração
- Cooperativa de Ensino Universidade Lusíada
- Universidade da Madeira
- Universidade do Algarve
- Centro de Matemática Aplicada do IST
- Centro de Investigação em Matemática e Aplicações da Universidade de Évora
- Centro de Álgebra da Universidade de Lisboa
- Centro de Matemática da Universidade de Coimbra
- Universidade de Macau
- Centro de Matemática da Universidade do Porto
- Centro de Estruturas Lineares e Combinatórias
- Instituto Superior de Economia e Gestão

CIM Publications

In 2000 the following CIM publications were issued:

- A. Sequeira (ed.), *International Summer School on Computational Fluid Dynamics*
- L. Trabucho and J. F. Queiró (ed.), *O ensino da Matemática em Portugal e assuntos relacionados*
- M. Field, *Complex Dynamics in Symmetric Systems*
- M. Golubitsky and I. Stewart, *The Symmetry Perspective: From Equilibria to Chaos in Phase Space and Physical Space*

CIM Events 2000

The CIM events in 2000 took place as planned. The Thematic Term, organized by J. A. Basto Gonçalves and I. Labouriau (Porto), was dedicated to Dynamics, Bifurcation and Biology. Its events (see the complete list, with the respective organizers, in the Bulletin no. 8) were of high level and had a great success.

Information about some of the events not included in the Thematic Term is given here, in complement to the announcements published in previous issues of the Bulletin.

Second debate on mathematical research in Portugal

This debate was held in Coimbra on 1-2 April. The Minister for Science and Technology spoke at the opening session. During the two days of debate there were panels on *Graduate studies in Portugal* (with a speech by the Secretary of State for the Universities), *Portugal's scientific system* (with a speech by the President of the Science and Technology Foundation), *The 1999 research assessment*, *Research units and university departments*, *Graduate studies in Mathematics*, *Internationalization of Portuguese mathematicians*, *Mathematics as seen by other sciences*, *Popularization of Mathematics*. CIM will publish the debate proceedings. The debate was organized by J. Soares (Minho), L. N. Vicente (Coimbra) and R. Santos (Algarve).

Mathematical aspects of evolving interfaces

This CIM/CIME Summer School, organized by P. Colli (Pavia, Italy) and J. F. Rodrigues (Lisboa, Portugal), took place in Funchal (Madeira) on 3-7 July and consisted of the following five courses:

- *A survey on the optimal transport problem*,
  by L. Ambrosio (Scuola Normale Superiore, Pisa, Italy);
- *Numerical approximation of mean curvature flow of graphs*,
  by G. Dziuk (Univ. Freiburg, Germany);
- *Dynamics of patterns and interfaces in reaction-difusion systems from chemical and biological viewpoints*,
  by M. Mimura (Univ. Hiroshima, Japan);
- *Evolution free boundary problems for parabolic equations and Navier Stokes equations*,
  by V. A. Solonnikov (Steklov Math. Inst., St. Petesburg, Russia);
- *Variational and dynamic problems for the Ginzburg-Landau functional*, by H. M. Soner (Univ. Princeton, USA).

Workshop on Partially Known Matrices and Operators

This workshop, organized by F. Conceição Silva (Lisboa), A. Leal Duarte (Coimbra), I. Cabral (U. N. Lisboa) and Susana Furtado (Porto), took place in Coimbra on 16-18 September. The invited speakers were:

- M. Fiedler (Prague, Czech Republic),
- C. Johnson (William & Mary, USA),
- P. Freitas (Tech. Univ. Lisboa, Portugal),
- J.-M. Gracia (Vitoria, Spain),
- W. Xiaochang (Texas Tech. Univ., USA),
- W. Barrett (Brigham Young Univ., USA),
- Ion Zaballa (Bilbao, Spain).

CIM Cooperation Activities

CIM has a Committee to deal with cooperation activities in Mathematics with Portuguese-speaking countries. This Committee has been chaired by Prof. J. Sampaio Martins (Coimbra).

In the year 2000, professors António Ornelas (Évora), J. C. David Vieira (Aveiro), Joaquim Madeira (Coimbra) and J. C. Tiago de Oliveira (Évora) taught courses at the *Instituto Superior de Educação* of Cabo Verde, in the city of Praia. These missions received support from Instituto de Cooperação de Portugal and Instituto Superior de Educação of Cabo Verde.

The new chairman of the Cooperation Committee for 2001 is Prof. J. C. David Vieira (Aveiro).

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include: office space, computing facilities, and some secretarial support; access to the library of the Department of Mathematics of the University of Coimbra (30 minutes away by bus); lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form

http://www.cim.pt/cim.www/cim_app/application.htm

also acessible from the CIM web page (see below).

CIM on the WEB

Complete information about CIM and its activities can be found at the site

http://www.cim.pt

This is mirrored at

http://at.yorku.ca/cim.www/

Feature Article

Notes on the life and work of Álvaro Tomás

by Henrique Leitão
C. F. M. C., Universidade de Lisboa

Alvarus Thomas (fl. 1509), a Portuguese master at the University of Paris at the beginning of the sixteenth century, is still a poorly known historical figure. In his Liber de triplici motu *he presented a comprehensive and sophisticated analysis of the theory of proportions and of the science of motion of his time, in the characteristic form of the Calculatory tradition. Besides some interesting criticism of contemporary physical theories, this work is also relevant from the point of view of mathematics since Thomas achieves some surprising results in the study of infinite series.*
In this paper I summarize the present knowledge on Alvarus Thomas. I collect all biographical information currently available and present very briefly the contents of the Liber de triplici motu *adding some observations on its historical context and influence.*

Introduction

In his well-known work, *Los Matematicos Españoles del siglo XVI*, J. Rey Pastor devotes a brief chapter to the Portuguese Alvarus Thomas (or Álvaro Tomás[1]). After describing in a few pages the main mathematical contributions of the Portuguese scholar, Rey Pastor closes the chapter with the following paragraph:

> Sirvan estas notas sobre el *Liber de triplici motu* de incentivo para que alguien lo analice, tan minuciosamente como merece, y para que alguna corporación ibérica emprenda su traducción. De los eruditos portugueses esperamos que indaguen en sus archivos datos bastantes para trazar la biografia de este sutil ingenio, digno precursor de Pedro Núñez[2].

These words were written in 1926 but unfortunately such a programme of study on this "digno precursor de Pedro Nunes" remains to be accomplished.

Rey Pastor was not the first scholar to mention the work of Alvarus Thomas. In 1913, the great historian of science Pierre Duhem, in his *Études sur Léonard de Vinci* presented the first modern analysis of Thomas' work[3]. Duhem noted Thomas' erudition and brilliance, and commented that

> Les problèmes que ces maîtres et régents s'acharnent à résoudre, dont ils entrevoient parfois la solution, en dépit de leurs connaissances rudimentaires en Mathématiques, ce sont les deux grands problèmes de l'intégration des fonctions et de la sommation des séries. Et l'on se demande alors quels résultats ces hommes n'eussent point obtenus, quelle promotion ils n'eussent point imprimée aux Mathématiques s'il leur eût été donné de lire Archimède[4].

Following these pioneer studies, other historians of science, while investigating the contributions of sixteenth century authors to the development of physics and mathematics, pointed to the important role played by Alvarus

[1]For consistency with the rest of the text I will use the 'English' version of the name. The only exception is the title of this paper.

[2]J. Rey Pastor, *Los Matematicos Españoles del Siglo XVI* (Toledo: Biblioteca Scientia, 1926), p. 89.

[3]Alvarus Thomas was not, in an absolute sense, unknown to historians. His name is included in important bibliographic repertoires since the eighteenth century, such as the one by Barbosa Machado, *Bibliotheca Lusitana* (Lisboa, 1741), Vol. I, pp. 114-115, and appears in well-known nineteenth century studies: Martín Fernandez Navarrete, *Disertacion sobre la Historia de la Nautica* (Madrid: Imprenta de la Viuda de Calero, 1846), p. 126.

[4]Pierre Duhem, *Études sur Leonard de Vinci*, 3 Vols. (Paris: Hermann et Fils, 1906–1913), Vol. III, p. 543. [Information on A. Thomas in pp. 532–543].

Thomas. Of special interest is the work by the eminent historian of mathematics Heinrich Wieleitner, in which Thomas' techniques for the summing of infinite series are analysed[5]. In more recent years, scholars such as Marshal Clagett, William Wallace, Edward Grant and Edith Sylla devoted some attention to the work of Alvarus Thomas. Their studies on the significance of Thomas' contributions will be used throughout this paper. At this point it is sufficient to quote the evaluation made by William Wallace:

> At Paris [...] there can be little doubt that Thomaz was the calculator par excellence at the beginning of the sixteenth century, and the principal stimulus for the revival of interest there in the Mertonian approach to mathematical physics.[6]

Strangely, the impact of these authoritative voices in the Portuguese community has been practically nil. If we except the three brief pages that Gomes Teixeira devoted to Thomas in the *História das Matemáticas em Portugal*[7], and a short, but correct, notice in the most important Portuguese encyclopedia[8], there are no more detailed or reliable references to Thomas' scientific work, much less a careful analysis of his book. Garção Stockler, Rodolfo Guimarães and Pedro José da Cunha in their classic studies[9] do not mention Alvarus Thomas, and later historians of Portuguese science follow essentially the same pattern[10]. The most apt characterization of this state of affairs was perhaps Joaquim de Carvalho's assertion that Thomas was "[...] uma das figuras mais lamentavelmente esquecidas da nossa história científica."[11] Further evidence of the oblivion into which Alvarus Thomas has fallen in his own country is that references to his biography or work often contain innacuracies: caveat lector.

The objective of this paper is to provide a very brief introduction to the life and work of Alvarus Thomas. I do not claim to present here any new findings related to this Portuguese scholar, nor a detailed explanation of his work. However, the neglect into which this Portuguese master has fallen among his countrymen and the importance of his work justify that even such a modest project should be undertaken[12].

Biography

Biographical information on Alvarus Thomas is extremely scarce. As Rey Pastor remarked in the quotation reproduced above, this is an aspect were more work is required.

The facts of Thomas' life that can be ascertained on a documentary basis are very few and cover a time span of only about ten years[13]. The first piece of evidence we have is his book—as far as is known, his only work—published in Paris in 1509 (or 1510[14]). The complete title is: *Liber de triplici motu proportionibus annexis magistri Alvari Thomae Ulixbonensis philosophicas Suiseth calculationes ex parte declarans*, a translation of which runs as follows: "Book on the Three [kinds of] Movement, with Ratios Added, by Master Alvarus Thomas of Lisbon, Explaining in Part Swineshead's Philosophical [i.e. Physical] Calculations".

The "explicit" of the *Liber de triplici motu* states that it was "compositus per Magistrum Alvarum Thomam ulixbonensem. Regentem Parrhisibus in Collegio Cocquereti." Another document confirms that in 1513 Thomas was still teaching Arts, i.e. Natural Philosophy, in the same college.

Thus, Alvarus Thomas was born in Lisbon and acted as Master of Arts and "regens" in the *Collège de Cocqueret*

[5] H. Wieleitner, "Zur Geschichte der unendlichen Reihe im Christlichen Mittelalter", *Bibliotheca Mathematica*, Dritte Folge, Vol. 14, (1914), 150–168.

[6] W. Wallace, "Thomaz, Alvaro", in: C. C. Gillispie (Ed.), *Dictionary of Scientific Biography* (New York: Charles Scribner's Sons, 1970-1980), Vol. 13, p. 350.

[7] F. Gomes Teixeira, *História das Matemáticas em Portugal* (Lisboa: Academia das Ciências de Lisboa, 1934), pp. 95–97.

[8] F. Gama Caeiro, "Tomás, Álvaro", in: *Enciclopédia Luso-Brasileira de Cultura* (Lisboa: Editorial Verbo, 1963–1985), Vol. 17, p. 1643.

[9] Francisco de Borja Garção Stockler, *Ensaio Historico sobre a Origem e Progresso das Mathematicas em Portugal* (Paris: Officina de P. N. Rougeron, 1819); Rodolfo Guimarães, *Les Mathématiques en Portugal* (Coimbra: Imprensa da Universidade, 1900); Pedro José da Cunha, *Bosquejo Histórico das Matemáticas em Portugal* (Lisboa: Imprensa Nacional de Lisboa, 1929).

[10] To give but two examples, only the briefest of mentions is found in Joaquim Barradas de Carvalho, *Portugal e as Origens do Pensamento Moderno* (Lisboa: Livros Horizonte, 1981), p. 69; and in Luís de Albuquerque, "Matemática e Matemáticos em Portugal", in: Joel Serrão (Ed.), *Dicionário de História de Portugal* (Lisboa: Iniciativas Editoriais, 1965), Vol. IV, p. 222.

[11] Joaquim de Carvalho, "Influência dos Descobrimentos e da Colonização na Morfologia da Ciência Portuguesa do Séc. XVI", in: *Obra Completa* (Lisboa: Fundação Calouste Gulbenkian, 1982), Vol. III, p. 360. [Originally in: *Congresso do Mundo Português*, Vol. V (Lisboa, 1940)].

[12] My essential purpose is to introduce this author to a Portuguese audience. I will rely mostly on materials already published. A more extended and detailed study will be presented elsewhere.

[13] For biographical information on Thomas and indications of the original documents, see: Luís de Matos, *Les Portugais à l'Université de Paris entre 1500 et 1550* (Coimbra: Universidade de Coimbra, 1950), R. G. Villoslada, *La Universidad de Paris durante los estudios de Francisco de Vitoria, O.P. 1507–1522* (Roma: Gregorian University Press, 1938), and Luís Ribeiro Soares, *Pedro Margalho* (Lisboa: Imprensa Nacional-Casa da Moeda, 2000).

[14] Owing to a technical detail related to the academic calendar in Paris, there is some uncertainty on the precise date. Following most authors, I will use 1509. See L. R. Soares, *Op. cit.*, p. 225.

in Paris from, at least, 1510 to 1513. This *Collège* had been established in 1439, and although it never rose to the distinction of the *Collège de Montaigu* or the *Collège de Saint-Barbe*, it had among its teachers and students some leading intellectuals of the time[15].

The "regens" was generally a student of one of the higher Faculties (Theology, Law or Medicine) who payed for his studies by teaching Arts in a college. Indeed, it is known that Thomas enrolled at the Faculty of Medicine in 1513 and it is very likely that he studied there while teaching Arts at *Cocqueret*. He completed his *licentia* examinations in Medicine two years later and obtained his degree of doctor in 1518. In that same year he was appointed professor at the Faculty of Medicine. After 1521 his signature no longer appears in the University archives. What happened afterwards is not known.

Front page of the *Liber de triplici motu*

The fact that he was studying Medicine in 1513 and that he received a doctorate in 1518 makes us suppose that he was not yet a middle aged man at that time. On the other hand, the sound command of an impressive range of sources that he shows in his book and his teaching position in 1510 at *Cocqueret* are hard to imagine (but not altogether impossible) in a man in his early twenties. Comparing with the academic career of other Portuguese scholars in Paris at the same period, it is plausible to suppose that Thomas had arrived in Paris by 1500, as a young man of around 16-18 years of age, and that he wrote the *Liber de triplici motu* a few years after having finished his studies in Arts and before embarking on the study of Medicine. This would mean that he would have been born in Lisbon around 1480-85.

The presence of Alvarus Thomas in Paris is quite natural. After a period of lesser prominence during the fifteenth century, by the turn of the century the University of Paris had recovered the glory of past ages. It had established itself as the most reputable university in Europe, attracting students from everywhere except Italy where the local Universities disputed this leading position. It is known that Portuguese students had been sent to the University of Paris since as early as 1192. In the period 1500-1550 around 300 Portuguese attended the University of Paris[16].

In the first decades of the sixteenth century a remarkable group of Portuguese students was at Paris. Besides humanists, philosophers, and theologians, among the contemporaries of Thomas one finds men who will greatly contribute to the scientific history of Portugal. Such is the case of Pedro Margalho (1471?–1556), Francisco de Melo (1490–1536) and João Ribeiro, for example. Also outstanding was the group of Spaniards. Among others, Gaspar Lax (1487–1560), Pedro Ciruelo (1470–1554), Juan Martínez Silíceo (1486–1557), Juan de Celaya (1490–1558), were contemporaries of Thomas in Paris[17]. These Iberians would play an important role in the History of Science, which prompted a modern historian to comment that

> Among the many foreigners at Paris at the turn of the sixteenth century, no group is more interesting than that of the Spaniards and the Portuguese[18].

The history of the intellectual relations between these men is, to a great extent, still to be made. The study of their influence in the Iberian Peninsula is also a desideratum. To a greater or lesser extent these men seem to have been influenced by the Scottish nominalist John Major (1467–1550) who was, at the beginning of the sixteenth century, the leading intellectual figure in Paris. Major pontificated in what was perhaps the most important of the colleges of the University of Paris at the time, the *Collège de Montaigu*, but his pupils would eventually occupy chairs in all other colleges, thus extending his influence to the whole of the University of Paris. There is no evidence of Thomas being directly associated with Major or of having been his direct disciple, but no doubt he benefited from the intellectual environment around the Scottish master.

Even surrounded by men of great intellectual prestige, Thomas seems to have been a leading figure. One of his contemporaries considered him to be superior to Pierre d'Ailly[19] and modern historians confirm Thomas' intellectual position among his peers[20].

[15]See: André Tuilier, *Histoire de l'Université de Paris et de la Sorbonne* (Paris: G.-V. Labat, 1994).

[16]See: L. de Matos, *Op. cit.*

[17]Hubert Élie, "Quelques maîtres de l'Université de Paris vers l'an 1500", *Archives d'histoire doctrinale et littéraire du moyen âge*, **18** (1950–51) 193–243.

[18]Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison: University of Wisconsin Press, 1959), p. 655.

[19]Such is the opinion of Gregoire Bruneau in his letter to Thomas printed at the end of the *Liber de triplici motu*.

[20]Villoslada, after extolling the famous Juan de Celaya, mentions Thomas saying that "El maestro lusitano era, por su ecletismo, su erudición y dialética invencible, gemelo de Celaya e superior a él como matemático". Villoslada, *Op. cit.*, p. 190.

The Calculatory Tradition

To understand the scientific contribution of Alvarus Thomas, a brief excursion into the achievements of medieval to late fifteenth century mechanical science is necessary. A complete description of these ideas is, of course, well beyond the scope of this paper. The interested reader is therefore directed to the relevant bibliography on this subject[21].

By mid fourteenth century the study of motion—a central and always problematic question in the *corpus* of aristotelian physics—was radically changed due to the contributions of a group of men at Merton College, in Oxford. In a period of about twenty years, the successive appearance of a number of texts on proportions and ratios, motion, and logical rules applied to physical questions, heralded a new approach to problems of natural philosophy. Of these, the most important were: Thomas Bradwardine, *De proportionibus velocitatum* (1328), William Heytesbury, *Regulae solvendi sophismata* (1335), John Dumbleton, *Summa logicae et philosophiae naturalis* (1349), Richard Swineshead, *Liber calculationum* (ca. 1350). Instead of pursuing an analysis of motion in the traditional categories of act and potency, these men adopted a formal and highly speculative analytical approach which considered motion essentially as a ratio. Their analysis of motion included detailed discussions on the possible types of motions (*uniformiter, uniformiter difformis, difformiter difformis*, etc.), the description of each of these different types of motion, and an inspection of the origin of each motion. These studies were abstract, without reference to any natural event or artifact, and made extensive use of logical techniques originally developed in other intellectual pursuits such as the study of language.

The original context of these discussions was the much debated question of the "intensio and remissio formarum", which is, basically, the question of how qualities varied in intensity. To the Oxford "Calculators" —such was the designation by which they came to be known, and Swineshead "the Calculator"—variations of velocity, that is, local motion, were treated as variations in the intensity of a quality, in the same way as color changes its hue or a body becomes warmer. But the problems they addressed had a much broader context than merely the question of understanding local motion (*motus localis*); calculatory techniques were also used in medicine and theology, for example. From the perspective of the history of mechanics, the contributions of the Merton school have been summarized thus by one of the most competent historians of medieval science:

> From the discussions of these four men at Merton emerged some very important contributions to the growth of mechanics: (1) A clear-cut distinction between *dynamics* and *kinematics*, expressed as a distinction between the *causes* of movement and the spatial-temporal *effects* of movement. (2) A new approach to speed or velocity, where the idea of an instantaneous velocity came under consideration, perhaps for the first time, and with a more precise idea of 'functionality'. (3) The definition of a uniformly accelerated movement as one in which equal increments of velocity are acquired in equal periods of time. (4) The statement and proof of the fundamental kinematic theorem [...][22]

These are no small intellectual accomplishments. Although to a modern reader the texts that these men produced are certainly prolix, confused and difficult to follow—a critique that some contemporaries also made— underneath this complexity lies an exceptional ability to seize upon and extract the mathematical features of the problem of motion. What is perhaps their greatest feat—sometimes considered the most outstanding medieval contribution to physics—was the statement and demonstration of the so-called "Mean Speed Theorem" for uniformly accelerated motion. In modern terms, this theorem asserts that a body in uniformly varied motion during a certain interval of time will traverse the same distance as a body with a uniform velocity equal to the instantaneous velocity at the middle instant, in the same interval of time. The power of this theorem lies in equating, for the purpose of calculating the distance traversed, an accelerated motion with a uniform motion. This theorem was proved by means of many different geometrical and numerical arguments and became the cornerstone of the studies of motion by the *Calculatores*.

The Calculatory tradition evolved significantly when it arrived on the Continent. In Paris, the ideas and techniques of the Merton approach were incorporated into the more realistic framework which had been worked out by fourteenth century thinkers such as Jean Buridan and Nicole Oresme. A salient feature of the Parisian achievements was the introduction of the notion of *impetus* in the analysis of motion.

[21] Good introductory works to this subject are: Edward Grant, *Physical Science in the Middle Ages* (Cambridge: Cambridge University Press, 1977); David C. Lindberg (Ed.), *Science in the Middle Ages* (Chicago & London: The University of Chicago Press, 1978); A. P. Juschkewitsch, *Geschichte der Mathematik im Mittelalter* (Leipzig: Teubner, 1964). For more detailed studies, see the work of Marshal Clagett cited before and also the following: Curtis Wilson, *William Heytesbury: Medieval Logic and the Rise of Mathematical Physics* (Madison: University of Wisconsin Press, 1956); Edith D. Sylla, *Oxford Calculators and the Mathematics of Motion, 1320–1350*, Harvard Dissertations in the History of Science (Garland, 1991); H. Lamar Crosby, Jr., *Thomas of Bradwardine. His Tractatus de proportionibus. Its significance for the Development of Mathematical Physics* (Madison: University of Wisconsin Press, 1955).

[22] M. Clagett, *Op. cit.*, p. 205.

The Contribution of Alvarus Thomas

The *Liber de triplici motu* is a sophisticated and technically complex piece in the *corpus* of the Calculatory tradition. The three types of motion mentioned in the title are local motion, augmentation and alteration. The book opens with a detailed discussion on the theory of proportions, in which the author presents systematically some of the most important results. The second part of the book is a discussion about motion. In this second part the author addresses the questions of "De motu locali quoad causam", "De motu locali quoad effectum", "De motu augmentationis", "De motu alterationis"[23].

The influence of Swineshead's *Liber Calculationum* is clear, but Thomas' exposition is more systematic and better organized. The first impression any reader has is the extent of Thomas' knowledge. His sources for mathematics range from the older Nicomachus or Boethius to the very recent edition of Euclid by Bartholomeus Zambertus (Venice, 1505). He is at ease with the Englishmen Swineshead, Bradwardine and Heytesbury, but also with Parisians such as Oresme, and with the Italians (Paul of Venice, James of Forli, etc.). The Portuguese master is in the exceptional position of knowing both the formal techniques of the Merton approach, the conceptual tradition of the Parisian school, and the Italian contributions[24].

But the contribution of Alvarus Thomas would not be correctly described by mentioning simply his role as the catalyst of the Merton tradition in Paris. From the perspective of his mathematical accomplishments the *Liber de triplici motu* contains remarkable results. Since it is impossible to even review its contents, I will simply comment on some aspects that relate to summing infinite series.

Thomas follows strategies typical of the Calculatory tradition, and by ingenious and complex use of the Mean Speed Theorem he manages to establish some suprising results. The approach used by Thomas can be better understood by using present day terminology. The reader can imagine that one is considering a modern graph with velocity represented in the vertical axis and time in the abscissas[25]. In such a depiction, a motion at constant velocity is represented by a horizontal line, and a uniformly accelerated motion by a straight line with some finite slope. A generic motion will be represented by some curve. In all cases, the total space traversed by the mobile is given by the area below the curve. Alvarus Thomas considers different types of motion. The question he tries to answer is inspired by the Mean Speed Theorem, but now for these much more complicated motions: Given a certain complex motion, what should the uniform velocity be such that a body moving with this constant velocity traverses, in the same time, the same distance as the body following the more complex motion?

Thomas cannot address the problem in the most general terms, but he considers complex motions that correspond to a division of the time axis in a geometrical progression. In each interval the velocity is assumed to be constant or uniformly accelerated. By this judicious construction and using the Mean Speed Theorem, Thomas is then able to calculate the total space traversed by the mobile and the corresponding uniform velocity which would make it traverse the same distance in the same time. It is not difficult to realize that, from a mathematical point of view, Alvarus Thomas is calculating the sum of an infinite series.

One of the motions considered by the Portuguese master corresponds to the series

$$1 + 2x + 3x^2 + 4x^3 + \cdots$$

Thomas is able to show that the sum of this series is equal to the square of the sum of the series:

$$1 + x + x^2 + x^3 + \cdots$$

In a typical Calculatory spirit, Thomas will stretch his techniques to the limit, considering motions progressively more complex. With this he is able to obtain remarkable mathematical results. For example, he shows that the series

$$1 + \frac{2}{1}x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + \cdots$$

is bounded above by

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

and bounded below by

$$1 + 2x + 3x^2 + 4x^3 + \cdots = \frac{1}{(1-x)^2}.$$

[23] This is just a rough indication of the parts of the book. The *Liber de triplici motu* is divided into various parts, and each of these in chapters. It has 141 fls., printed in two columns in a small gothic font. I have used the copy at the Biblioteca Nacional, Lisbon.

[24] M. Clagett comments that "Thomas not only used the English works, but he was conversant with the Italian commentaries and paraphrases. He, then, has at his command the whole medieval mechanical tradition", *Op. cit*, p. 657.

[25] Although such a graphical representation can be traced back to Oresme, there are no such figures in the *Liber de triplici motu*, and even numbers appear only very rarely. The reader must bear in mind that the original context for which the book was written seems to have been the verbal disputations in *Sophismata*. In a fundamental study, Edith Sylla remarked that "it is the disputative context that apparently motivates Alvarus to many of his mathematical results". These are important questions which unfortunately cannot be pursued here. In any case, Sylla concludes that this disputative context had a positive impact on Alvarus' mathematical physics. See: Edith D. Sylla, "Alvarus Thomas and the role of Logic and Calculations in sixteenth century Natural Philosophy", in: S. Caroti (Ed.), *Studies in Medieval Natural Philosophy* (Florence: Olschki, 1989), 257-298.

Rey Pastor observed that several of the series analysed by Thomas would cause many difficulties even to present-day students. He is able to sum series such as,

$$1 + x + ax^2 + bx^3 + a^2x^4 + b^2x^5 + \cdots$$

or even,

$$1 + \frac{3}{2}\frac{1}{2} + \frac{5}{4}\frac{1}{2^2} + \frac{9}{8}\frac{1}{2^3} + \cdots$$

Naturally, there is no detailed inspection of the criteria of convergence of the series studied, nor an attempt at rigorous definitions. Nevertheless, Thomas is aware that while some of the series he proposed can be summed, others cannot, either because it is technically very difficult (or impossible) or because the partial sums of terms increase very rapidly.

Conclusion

At this point one would like to assess the influence of Thomas' book. Such an evaluation cannot, in general, be made since the impact and fate of Calculatory techniques in sixteenth century Europe is a question which has been investigated only recently. But I believe that enough evidence has already been collected to substantiate the claim that Thomas' book was well known and influential. Several of his contemporaries quote the book. Pedro Margalho, Pedro de Espinosa and Diego de Astudillo all include praise to Alvarus Thomas in their works. Juan de Celaya, who was a fellow teacher of Thomas at *Cocqueret* does not cite the Portuguese by name, but his *Expositio (...) in octo libros phisicorum Aristotelis*[26] was certainly inspired by the *Liber de Triplici Motu*. One other indication of a wide dissemination of the *Liber de triplici motu* is the number of extant copies. According to Wieleitner it is a "liber rarissimus", an opinion that several other historians have echoed[27]. But a cursory search trough the catalogues of major libraries around the world revealed to me more than 20 copies still exist today. For a work written in 1509 it is a significant number which hints at wide dissemination.

In what concerns the study of infinite series, Alvarus Thomas is the high point of an intellectual tradition which had reached its limits. The discursive approach to these mathematical problems would soon be abandoned, and forgotten, with the development of the much more powerful algebraic approaches of the seventeenth century. Jakob Bernoulli's *Tractatus de seriebus inifinitis* (1689) ushers in a new world in the study of infinite series. But in the study of local motion, the book of Alvarus Thomas was perhaps of much more relevance. It has been plausibly argued that the *Liber de Triplici Motu* may have been influential in the scientific formation of Domingo de Soto (1495–1560), either directly or via Soto's teacher in Paris, Juan de Celaya. A possible intellectual connection between Thomas and Domingo de Soto is of the utmost historical significance since it is known today that Soto was the first author to have argued that the free fall of bodies is a motion *uniformiter difformis* with respect to time. That is, in modern terms, that in free fall the body traverses spaces in direct proportion to the squares of the times of fall[28].

After the investigations of William Wallace, it is today agreed that Galileo was well acquainted with the results of the tradition of the *Calculatores* after Domingo de Soto. It is very likely that it was from this knowledge that Galileo first noticed the correct law for the free fall of bodies, which he presented at the beginning of the seventeenth century[29]. In this sense, the role of Alvarus Thomas, as a leading figure in the Calculatory tradidion that ultimately led to Galileo's outstanding contributions, needs to be noted. But there is more to interest Portuguese readers in this fascinating story. Galileo's knowledge of the ideas and techniques of the *Calculatores* was drawn from his study of the lecture notes of the Jesuit professors at the Roman College. In the efficient Jesuit network of colleges the Calculatory tradition underwent a diffusion originating in the Iberian Peninsula—a direct consequence of the return to the Peninsula of former students at Paris, and in particular, of the teaching of Domingo de Soto. In fact, a substantial number of manuscripts from Jesuit colleges confirms that in the last decades of the sixteenth century the terminology and notions of the *Calculatores* were being used in the analysis of the nature of motion in Portugal[30].

[26] [Juan de Celaya], *Expositio magistri ioannis de Celaya Valentini in octo libros physicorum Aristotelis, cum questionibus eiusdem secundum triplicem viam beati Thomae, realium, et nominalium* (Paris, 1517).

[27] Wieleitner gives a detailed bibliographical description of the Munich copy and mentions that it displays, written by a later hand, the inscription *Liber rarissimus*; Rey Pastor, certainly drawing from the German scholar, calls it "un libro rarissimo" and likewise Gomes Teixeira says that "o livro de Álvaro Tomaz é muito raro".

[28] Domingo de Soto, *Super octo libros physicorum Aristotelis questiones* (Salamanca, 1545). For biographical information on Soto, see: V. Beltrán de Heredia, *Domingo de Soto: Estudo Biográfico Documentado* (Salamanca: Biblioteca de Teólogos Españoles, 1960); For the physical questions see the many papers by William A. Wallace. One of his later works contains references to prior writings: William A. Wallace, "Domingo de Soto's "laws" of motion: Text and context", in: Edith Sylla and Michael McVaugh (Eds.), *Texts and Contexts in Ancient and Medieval Science* (Leiden: Brill, 1997) 271–303.

[29] Galileo Galilei, *Galileo's early notebooks: the physical questions*, a translation from the Latin with historical and paleographical commentary [edited by] William A. Wallace (Notre Dame: University of Notre Dame Press, 1977); William A. Wallace, *Galileo and His Sources: The Heritage of the Collegio Romano in Galileo's Science* (Princeton: Princeton University Press, 1984).

[30] See the works by W. Wallace: "Late sixteenth-century Portuguese manuscripts relating to Galileo's early notebooks", *Revista Portuguesa de Filosofia*, 51 (1995) 677–698; "Domingo de Soto and the Iberian roots of Galileo's science", in: Kevin White (Ed.) *Hispanic Philosophy in the Age of Discovery* (Washington DC: Catholic University of America Press, 1997), 113–129.

Great Moments in XXth Century Mathematics

In this issue we present the answers of two researchers, Gudlaugur Thorbergsson and Mark Pollicott, to the question "If you had to mention one or two great moments in XXth century mathematics which one(s) would you pick?".

Gudlaugur Thorbergsson

The most dramatic mathematical discoveries of the twentieth century might be the Gödel incompleteness theorem and the proof of Fermat's last theorem.

Instead of elaborating on these outstanding contributions, which are both far from the area that I have specialized in, I would prefer to reflect on what has impressed me from a more personal point of view, restricting myself to my own area of research.

We have witnessed enormous growth in Differential Geometry in the last quarter of a century. Gromov and Yau are clearly in the center of this development. Yau's solution of the Calabi conjecture is maybe the single most important result with important consequences in other areas like Algebraic Geometry. Gromov's most influential contribution might be his paper on pseudo-holomorphic curves which started a line of research in Symplectic Topology that recently culminated with a solution of the Arnold conjecture. One should also mention his highly original theory of hyperbolic groups and his numerous contributions to Riemannian Geometry.

If I look at the whole century and not only the few years that I have been able to witness personally, the theory of semi-simple Lie algebras and Lie groups and their representations comes to my mind. This is a theory that started in the last decades of the nineteenth century with the work of Killing and Cartan and took its final shape in the twentieth century with contributions of Cartan and Weyl and many others. First of all the beauty and intricate structure of this theory is fascinating. Then it is also central in so many areas of Mathematics that it certainly deserves to be considered as one of the the truly important contributions to Mathematics in the twentieth century.

Gudlaugur Thorbergsson was born in Melgraseyri, Iceland, having graduated from the University of Iceland. His postgraduate studies were done at the University of Bonn and he held positions at Bonn, IMPA - R. J. and University of Notre Dame. He is currently at the University of Köln. An important part of Professor Thorbergsson's research is in submanifold geometry. Recently he contributed a survey on isoparametric hypersurfaces and their generalizations to the *Handbook of differential geometry*, Vol. I, published by North-Holland.

Mark Pollicott

It is interesting that the XXth century began with a number of problems and conjectures which have helped to shape mathematical research and for 100 years (for example, the Hilbert problems and the Poincaré Conjecture) and concluded with the surprising solution of a far older conjecture (Wiles' proof of the Fermat Conjecture).

However, if I was asked to suggest some development of lasting importance and greatest impact then I might propose the work of Alan Turing on Mathematical Logic in the 1930s.

His conceptual development of what is now popularly known as a universal Turing machine anticipated the modern computer. His description of an abstract machine which can be made to carry out complicated tasks *through a combination of simple instructions inserted on a tape is easily recognised now as a description of a programmable computer (with the tape as the programme).*

Turing's motivation came from one of the most abstract of ideas in mathematics: The problem of "decidability" (Hilbert's second problem) which relates to whether, for a given well formulated mathematical problem, a solution necessarily exists or not. In the context of the Turing machine, given a finite set of instructions, it may be impossible to decide whether the machine would continue forever, or stop in some finite time.

The first practical application of Turing's ideas was during the second world war. Turing worked for the British Government Code and Cypher School on decoding military transmissions encoded by the german "Enigma" machines, developing practical decoding machines based on his original abstract ideas. The second important application of his ideas was the construction of the first programmable computer in Manchester under the aegis of Max Newman, in the late 1940s.

Mark Pollicott has held positions at the universities of Edinburgh and Warwick, as well as visiting positions at IHES, MSRI and IAS (Princeton). He was an Investigador Auxiliar of INIC from 1988-92, whereafter he took up a Royal Society University Fellowship at Warwick. He presently holds the Fielden Professorship in Pure Mathematics at Manchester University, England.

What's New in Mathematics

Race to settle Catalan conjecture: it's people vs. computers

Ivars Peterson reports in the December 2, 2000 Science on recent progress towards the resolution of this 150-year-old conjecture. Catalan noted that $8 = 2^3$ and $9 = 3^2$ are consecutive integers and conjectured that they were the only set of consecutive whole powers. This translates to the Fermat-like statement that the equation $xp - yq = 1$ has no whole-number solutions other than $3^2 - 2^3 = 1$. Recently Maurice Mignotte (Strasbourg) had given upper bounds on possible values of p and q; now Preda Mihailescu (ETH, Zürich) has shown that p and q must be a pair of "double Weiferich primes." Only six pairs are known, and, as Peterson reports, "a major collaborative computational effort has been mounted" to find more. You can help: volunteer at Ensor Computing's Catalan Conjecture page. Or you can join mathematicians who "are betting that a theoretical approach will beat out the computers."

Incompressible is incomprehensible

Why are some things so hard to understand? Jacob Feldman of the Rutgers Psychology Department has an answer, reported in the October 5, 2000 Nature. He found in a large set of experiments that for human learners, "the subjective difficulty of a concept is directly proportional to its Boolean complexity (the length of the shortest logically equivalent propositional formula)-that is, to its logical incompressibility." For example a concept which encodes as (A and B) or (A and not B) is equivalent to A and (B or not B), i.e. to A and so can be compressed to Boolean complexity 1. Whereas (A and B) or (not A and not B) cannot be compressed and has complexity 4. Subjects were asked to extract the concepts from sets of examples and non-examples. Main conclusion: "For each concept, learning is successful to the degree that the concept can be faithfully compressed." Feldman reflects on his result: "In a sense, this final conclusion may seem negative: human conceptual difficulty reflects intrinsic mathematical complexity after all, rather than some idiosyncratic and uniquely human bias. The positive corollary though is certainly more fundamental: subjective conceptual complexity can be numerically predicted and perhaps explained."

New encription algorithm

A new Federal encryption algorithm was reported in the October 20, 2000 Chronicle of Higher Education. The article, by Florence Olsen, relates how the Commerce Department, after a 4-year search, has declared the new federal standard for protecting sensitive information to be Rijndael, an algorithm named after its inventors Vincent Rijmen and Joan Daemen. The two Belgians beat out 20 other entries, including teams from IBM and RSA. The new encryption algorithm, of which no mathematical details were given, can be made stronger as more powerful computer processors are developed. This was an entry requirement for the competition. According to Raymond G. Kammer of NIST, which managed the selection process, it should be good for about 30 years, "that is, if quantum computing doesn't manifest itself in five or six years."

Updating Ramanujan

The June 17, 2000 issue of Science News has a very complete and satisfying piece by Ivars Peterson about the recent discovery of new Ramanujan-type partition congruences. The n-th partition number p(n) is the number of different ways of expressing n as a sum of positive integers less than or equal to n. So $p(5) = 7$, as is easy to check, but these numbers grow very rapidly with n. Ramanujan discovered for example, that $p(5n+4)$ is always a multiple of 5. (Thus $p(4) = 5$, $p(9) = 30$, $p(14) = 135$, $p(19) = 490, \ldots$). He also discovered similar congruences involving the primes 7 and 11. No one knew if those were all the possible partition congruences and if so, what was so special about 5, 7 and 11. Peterson recounts how Ken Ono, a number theorist at Penn State and Wisconsin-Madison, became interested in the problem and how he ended up proving that in fact there exist infinitely many partition congruences, work reported in the January 2000 Annals of Mathematics. Ono only gave one example: $p(an+b)$ is always a multiple of 13, where $a = 594 \times 13$ and $b = 111247$. (This gives an idea of why such congruences had not been found before!) His work was complemented in a remarkable way by Rhiannon L. Weaver, an undergraduate at Penn State, who developed an algorithm and used it to generate over 70000 new examples. Peterson quotes Ono: "It is now apparent that Ramanujan-type congruences are plentiful. However, it is typical that such congruences are monstrous."

Double Bubbles

In the March 17, 2000 Science is a piece by Barry Cipra: "Why Double Bubbles Form the Way They Do," and reporting on the recent solution of the Double Bubble Conjecture. The problem was to give a mathematical proof that the most economical way to enclose two contiguous given volumes is by a combination of three spherical surfaces, just as shown in John Sullivan's pictures. The solution, by Michael Hutchings of Stanford University, Frank Morgan of Williams College and Manuel Ritoré and Antonio Ros at the University of Granada, proceeds by showing that "any other, supposedly area-minimizing shape can be ever so slightly twisted into a shape with even less area."

Squeeze in a few more?

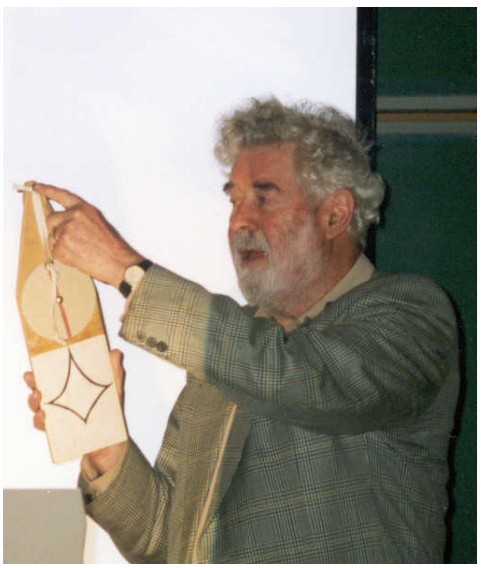
Kepler conjectured in 1611 that the most efficient way to pack equal-sized spheres (for example, identical oranges) in a box was to use the face-centered cubic configuration. It took a long time to settle this question to everyone's satisfaction. This finally happened two years ago, when Thomas Hales showed that the density of the face-centered cubic arrangement (approximately 74%) could not be improved upon. Then the question was considered, suppose the spheres are packed at random, like balls being poured into a container. Was there a maximum density for a random packing? Different experiments led to different estimates of this number, leaving a confusing situation. Charles Seife reports in the March 17, 2000 Science on the solution to this problem. There is no such number, and looking for it "makes no more sense than searching for the tallest short guy in the world." Random packings achieved with gentler and gentler pressure on the spheres can get arbitrarily close to Kepler's limit (and as they do so, they become more and more ordered). Seife is reporting on results recently published by S. Torquato, T. M. Truskett and P. G. Debenedetti, of the Complex Materials Theory Group at Princeton University, in the Physical Review Letters.

How to win $1,000,000 - the hard way

An Associated Press story, picked up by the March 26, 2000 Seattle Times, reports that Faber & Faber and Bloomsbury Publishing are offering a million bucks to whoever can prove that every even number is the sum of two primes. Simple? $2 = 1+1$, $4 = 3+1$, $6 = 3+3$, $8 = 3+5$, ...$98 = 79+19$, $100 = 97+3, \ldots$ but the problem has been open since its proposal in 1742. The stunt is in connection with the release of "Uncle Petros and Goldbach's Conjecture," by Apostolos Doxiadis. The million dollar assertion is in fact Goldbach's Conjecture. Good luck.

An Interview with E. C. Zeeman

As part of my homework for this interview I read a portuguese translation of the interview you gave to Lewis Wolpert for BBC, Radio 3[31] *Some of the questions I am going to formulate are based on that interview and I want to express my debt to him.*



E. C. Zeeman

Professor Zeeman, at 7 you were fascinated when your mother showed you how to solve a problem using the unknown x. I'm sure that during your mathematical career some of the results you proved must have given you a similar feeling.

Which were the peaks of your research?

When my mother showed me at the age of 7 how to use x for an unknown it was a revelation to me. However, I think the feeling of revelation that you get when someone reveals something to you is different from the feeling of exhilaration that you get when you discover something for yourself. Revelation can be wonderful, but exhilaration can be even better!

I can distinctly remember a few revelations such as understanding limits rigorously for the first time (and hence calculus), or understanding the complex numbers as the algebraic closure of the reals, or using groups and fields to show the insolubility of the quintic, or proving the knottedness of knots, or understanding Newton's proof of elliptic orbits, and much later realising that Newton's equations are contained in the symplectic structure of a cotangent bundle, or understanding Mather's proof of Thom's theorem on elementary catastrophes.

When I began proving my own theorems each one seemed the best at the time, but in retrospect I suppose I am particularly fond of having unknotted spheres in 5-dimensions, of spinning lovely examples of knots in 4-dimensions, of proving Poincaré's Conjecture in 5-dimensions, of showing that special relativity can be based solely on the notion of causality, and of classifying dynamical systems by using the Focke-Plank equation. And amongst my applications of catastrophe theory I particularly liked buckling, capsizing, embryology, evolution, psychology, anorexia, animal behaviour, ideologies, committee behaviour, economics and drama.

When he introduced you as the 1992/93 Johann Bernoulli Lecturer, Floris Takens mentioned that you served as a flying officer in R. A. F. during World War II. I presume it must have been between high school and university.

What are your recollections of that experience and how did it affect your mathematical path?

I served in the Royal Air Force during the war from 1943 to 1947 (between the ages 18 - 22). I was a navigator on bombers, trained for the Japanese theatre, but that was cancelled because they dropped the atomic bomb a week before we were due to fly out. Since the death rate was 60% in that theatre it probably saved my life, but at the time I was disappointed not to see action, although relieved not to have to bomb Japan, the land of my birth.

The air force was a rewarding experience, a breath of

[31] E. C. Zeeman's interview was part of a set of interviews which were later published in a volume. Such a volume was translated into portuguese and published with the title *"Uma Paixão pela Ciência"* by *Edições Salamandra*, with Lewis Wolpert and Alison Richards as authors.

Another interview with E. C. Zeeman, conducted by Steen Markvorsen, was published in the EMS Newsletter in December 1998.

freedom that allowed my self-esteem to recover from the prison of boarding school. It enabled me to realise that I loved mathematics, and wanted to do that more than any other career. I was unashamedly happier my first day back as a student than my last day as an officer in the air force. Of course by then I had forgotten all my mathematics, and so it set me back 5 years in my mathematical career, but then who cares now that I am 75 and still at it. I am grateful to the air force for providing an opportunity for personal development, and for enabling me to laugh at myself slightly as an academic ever since.

And yet "They (the problems one sets about solving) are rarely solved", I quote from the interview mentioned at the beginning.

Were there problems of which the solution eluded you? Do you still think about them from time to time?

Of course the solutions to many problems have eluded me, and I still think about them from time to time. A good mathematician probably has 25 failures to each success. The important thing is that new ideas keep coming.

One of my favourite failures is the 3-dimensional Poincaré Conjecture, which I spent the first year of my research thinking about, and which is still unsolved today. Another little hobby is to try and rediscover Fermat's own original proof of his last theorem, at least for $n = 3$, without using complex numbers (which he is unlikely to have used). I have done half of it.

At the moment I am busy trying to unfold some difference equations in higher dimensions using alien techniques from dynamical systems, algebraic geometry and number theory. Last month I managed to prove a theorem that I conjectured 25 years ago about Eudoxus' theory of proportion. I suspect that Eudoxus was able to take ratios of ratios, which Euclid was not able to do in Book 5 (nor in Book 6, in spite of Definition 5, which is a later blemish added by other writers) because he had fouled up Eudoxus' beautiful abstract approach by, ironically, introducing the Euclidean algorithm too soon.

"Among students the good ones are automatically good and it is not possible to improve the bad ones' performance". You are talking about Maths students. I agree with you and it brings to my mind the following question.

What do you think of Mathematical Education as a scientific discipline?

There are two different meanings to the word "discipline". The first meaning is my definition of an academic discipline as a corpus of works of genius that a student can study without the interference of the lecturer. In this sense mathematics is a discipline, as are also physics, chemistry, biology, literature, etc. But mathematical education is not.

This became sharply clear to me once at Warwick. Each year the Mathematics Institute there runs a year-long symposium, with some 80 long-term visitors, in topics like topology, groups, dynamical systems, algebraic geometry, etc. One year we debated whether to run a symposium on mathematical education, and tried out a pilot week to examine the potential, but it transpired that there was not enough material: it was not an academic discipline.

On the other hand vocational apprenticeship to the profession of mathematical teaching needs discipline if the student is to master the necessary techniques. And such discipline needs to be taught, needs specialists to teach it, and needs to be supported by research on curriculum reform and the analysis of learning techniques.

"Deep down I am a geometer and geometry is very clear. ...Proofs are rigorous and very satisfying from the aesthetic viewpoint". This is something you said. On the other hand René Thom is well known for statements such as "I do not think that a mathematician's vocation is to prove theorems" or "One can always find imbeciles to prove theorems". Such different approaches to Mathematics and however parts of your works overlap very significantly.

How did that come about? It would be unthinkable to interview you and not bring up Catastrophe Theory...

Thom is quite witty, and he occasionally talks rubbish because he loves being provocative. At the same time he is the greatest genius I have had the privilege to know well. He is the fountainhead of many wonderful ideas. Sometimes he does not bother to be rigorous, nor to get down to the nitty-gritty of proofs, whereas I do. I like to rework and repolish a proof until it is in its simplest rigorous form.

Thom occupies a position halfway between mathematics and philosophy. He was reluctant to get his hands dirty predicting experiments, lest the potential failure of those predictions detracted from the purity of his theory. He quoted the unfortunate example of D'Arcy Thompson who got all his theoretical ideas right but all his experimental predictions wrong, and said he did not want to be caught the same way.

I, on the other hand, occupied a position halfway between mathematics and science. I wanted to get my hands dirty, and make predictions, and get the experimentalists to test them, because I knew that the scientific community would never take a theory seriously unless it was capable of being tested experimentally. And I was gratified that several of my predictions were confirmed. Some were refuted, and others remain to be tested.

Since we occupied different positions Thom and I complemented each other. We met over the mathematics and the theory in between, and our collaboration turned out to be very fruitful.

In the 60's when you were in your early forties you founded the Mathematics Institute and Research Centre at Warwick. It must have meant a lot of paper and administrative work and surely it must have affected your mathematical output.

Do you have any regrets?

It is true that the founding of the Mathematics Institute and Research Centre at Warwick was a big administrative load, that prevented me from doing much research in topology during the first 5 years 1964 - 69 (while I was 39 - 44). But I certainly had no regrets, because founding Warwick was one of my best and most rewarding achievements. And it made me into a much broader mathematician. During 1968/9 I learnt all about dynamical systems by running a symposium on it for a year, with many of the world leaders including Smale and Thom coming for long periods. Then in 1969/70 I had the good fortune to spend a sabbatical year with the latter at the IHES in Paris, where I learnt all about catastrophe theory. So I was very fortunate to get in on the ground floor of such beautiful new subjects.

Your mathematical career has been showered with lots of awards and other forms of recognition: A knighthood, an F. R. S. fellowship, the Senior Whitehead prize, a Forder lectureship, book dedications. . . .

Was it important for you to have achieved such a recognition?

Of course I was very pleased to receive such recognitions, although I never set out to achieve them - I merely did what I liked best in teaching and research. The awards proved useful in that they enabled me to go ahead and do further things.

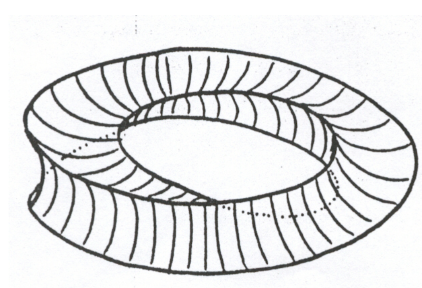
I was elected a Fellow of the Royal Society primarily for my work in geometric topology, which helped to resuscitate that subject in the 60's, and partly for my work in dynamical systems and catastrophe theory. The Whitehead Prize and Forder Lectureship were for both research and teaching. I attach great importance to teaching, and at Warwick I insisted that it should be given as much importance as research, which is one of the reasons why the Warwick Mathematics Institute remains so robust today. I was given the Royal Society Faraday Medal for my contributions to the public understanding of science, in particular for giving the Royal Institution Christmas Lectures in 1978, out of which grew the Mathematics Masterclasses for 13-year-olds (which have now been flourishing for 20 years and have spread to 50 centres around the country). My knighthood was probably for four things: my research, founding Warwick, creating masterclasses, and heading an Oxford College.

Your wife is a jeweller (I think she even coined the term "bracelet" in "bracelet umbilic"), you are now in Portugal to give a talk in connection with a video of which the title is "Geometry and Perspective"...

Are you interested in Art? Do you have a favourite painter, a favourite sculptor? I'm tempted to mention Barbara Hepworth or Henri Moore but that is a bit too obvious perhaps...

My wife Rosemary is indeed a jeweller and makes beautiful very feminine enamelled jewelry. Although she has never been a mathematician, yet she loved geometry at school, and so I try to explain geometrical things to her from time to time.

I coined the term "umbilic bracelet" when I tried to explain to her the natural stratification of the 4-dimensional space of real cubic forms in two variables. The elliptic and hyperbolic umbilics form the two open strata, and are separated by the parabolic umbilics, which form a codimension-1 stratum, which is a cone on the bracelet; the bracelet itself being a bundle over S^1 with fibre a triangular hypocycloid and group Z_3.



Yes, I am very interested in art. My favourite painters are from the Renaissance: Masaccio, Giovanni Bellini, Piero della Francesca, Botticelli, Leonardo, Filippino Lippi and Raphael; and (later) Vermeer, Ingres, Velasquez and Turner. Favourite sculptors include the Pisanos, Donatello, Michelangelo and Rodin, as well as individual pieces of sculpture like Djhutmose's unfinished quartzite head of Queen Nefertiti from Armana (the Cairo one rather than the Berlin one), the Egyptian wooden harp head from the Louvre, Myron's Diskobolos and Greek wrestlers from the 5th century BC, the Winged Victory of Samothrace, and (more modern) Boccioni's "Unique forms of continuity in space", Duchamp-Villon's "The great horse", Teddy Hutton's "Pregnant Woman", and Makonde sculptures from Tanzania and Mozambique. Modern painters I like include Rodolfo de Sanctis, Gordon Onslow-Ford, Edith Smith, Joe Brotherton, Peter Edwards and Picasso (although some of his work is junk). Your suggestions of Barbara Hepworth and Henry Moore have topological appeal but they do not make my spine tingle or move me to tears, as do the sculptures listed above.

(Questions and picture by F. J. Craveiro de Carvalho)

Sir Erik Christopher Zeeman is one of the great XXth century mathematicians. His university studies were at Christ's College, Cambridge and he also received his PhD from Cambridge.

Professor Zeeman spent most of his career in Cambridge, Warwick (where he founded the Mathematics Department and Research Centre) and Oxford.

His election to the Royal Society of London in 1975, the *Senior Whitehead Prize* in 1982, the first *Forder Lectureship* of the London Mathematical Society in 1987 and the Royal Society's *Faraday Medal* in 1988 are some of the honours he received. He was also knighted in 1991.

Professor Zeeman is the author of the video *Geometry and Perspective* based on the *Royal Institution Christmas Lectures* he gave in 1978.

Bento de Jesus Caraça – The Man and his Time

We will not be talking about Bento de Jesus Caraça *the mathematician*, who is sufficiently well-known.

We will be talking about Professor Bento Caraça *the man*, who, from his birth in 1901, fought for survival and was only saved by a miracle. A life which only lasted 47 years, but which was enough to enrich his era and to bequeath to us a cultural and ethical legacy of the highest and incomparable value.

Bento Jesus Caraça

Of the many great personalities who marked national life over the last century, Bento Caraça was particularly noted for the greatness and universality of his messages and for his courage, even his spirit of sacrifice, in defending them.

He was made to pay dearly for this defiance, respectable though it was. Bento Caraça was mercilessly persecuted by the police under the dictatorship: he was imprisoned at Aljube, he lost his professor's chair where he was a teacher like no other, and he suffered much economic hardship, whilst his health was at risk.

But the ultimate shock for the Professor was his expulsion from his university teaching post, in 1946, when he was professor at the Instituto Superior de Ciências Económicas e Financeiras, an institute for which he had so much affection and which owed so much to him.

On his own merit and as an exceptional measure, Bento Caraça was appointed 2nd Assistente of the 1st Group of Chairs of Mathematics at the ISCEF at the early age of 19, and when he was only 23 he was appointed *Professor Extraordinário*. Five years later, in 1928, he was appointed *Professor Catedrático*.

As a result of his training, he was particularly interested in economic issues and introduced methods of Econometrics in Portugal. In 1938, with his fellow professors Mira Fernandes and Beirão da Veiga, he founded the *Centro de Estudos da Matemática Aplicados à Economia*, of which he was President and immediately afterwards, with other mathematicians, he launched the *"Gazeta de Matemática"*.

Following these efforts to provide and innovate economic knowledge in Portugal, Bento Caraça, in the final period of his life, encouraged a group of young economists, all of whom were his ex-students, to launch a specialised publication, in a country in which information and knowledge were notoriously scarce and mishandled.

Hence the appearance of *"Revista de Economia"* in 1948, in which the opening article in the first issue *"Sobre o Espaço de Capitalização"* was written by Bento Caraça.

A cultured and very sensitive man, the author of a book as up-to-date and inspired as *"A Cultura Integral do Indivíduo"*, he lived the problems which affected Portuguese society as if they were his very own. The fact is that this society, which cultivated obscurantism and anti-democratic ideas which he deplored, was the same society which was at the basis of his own humble origins, as the son of poor farm workers from the Alentejo region.

A feverish worker - as if he foresaw his early demise - he faced all manner of adversities and disenchantment, without ever wavering, because reason was on his side, together with the love and satisfaction at having fulfilled his duty.

This position as citizen, master and friend, lover of Nature and all that is beautiful, combining reason and heart in an exceptional manner, was a constant in the life of the Professor.

In his modest life, rich in moral and cultural concerns, teaching and mathematical research occupied a special place. In his classes, which he gave in a unique style and which were revolutionary in educational terms, he captivated his students through his fascinating way of presenting subjects. This soon transformed Professor Caraça into a great idol, beloved not only among his students but among the whole academic community.

This general feeling can be observed, for example, in the commentaries of Professor Sebastião e Silva, another great mathematician, on his book *"Lições de Álgebra e Análise"*: "For the first time, mathematics has been presented by someone who lives the profession with the soul of an apostle and of an artist."

As a writer, communicator and polemist, he favoured biographies of great, universal names, of inspiring examples and acts, such as Romain Rolland, Rabindranath Tagore, Evariste Galois, Leonardo da Vinci, Galileo Galilei and others. He also maintained a notable polemic with António Sérgio, another great name of the 20th century, in the magazine *"Vértice"*, conducted by both with utmost elegance.

On another level of his activities, involving cultural, civic and political institutions and undertakings, Bento Caraça was unable to remain indifferent the existing socio-political situation, marked by odious dictatorship.

The overt politics of Professor Caraça in this context were, as we have seen, focused mainly on the culturalisation of the individual, on teaching and on the defence of major democratic values.

To promote this political and cultural process, there were social and artistic meetings, conferences and debates, most of which took place at the *"Voz do Operário"* and, in particular, at the *"Universidade Popular Portuguesa"*, which was a meeting place for the city "intelligentsia" at the time, and of which Bento Caraça was the President for many years.

On a similar level, another prodigious activity, due to the responsibility it demanded, was his commitment to the project *"Biblioteca Cosmos"*, undoubtedly one of the finest and most significant cultural achievements of this century and which was conceived and organised by Bento Caraça.

Over a period of less than eight years, this publisher brought out over 114 titles, of great cultural interest and unique in Portugal, agitating and mobilising the best collaborators in Portugal.

Bento de Jesus Caraça died on 18 June 1948. It was astonishing to see the crowds of people of all social classes who joined together spontaneously in the streets of Lisbon to pay their last heartfelt respects to the Master, to the citizen, to the great Friend.

Ulpiano Nascimento
Economista

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