António Monteiro and his Modernist Essay

by **José Francisco Rodrigues***

The 1939 "ENSAIO sobre os FUNDAMENTOS da ANÁLISE GERAL" (Essay on the foundations of General Analysis), by the Portuguese mathematician António Aniceto Monteiro (1907–1980), in spite of being awarded an important prize by the Lisbon Academy of Sciences, has been unknown and ignored until its recent rediscover and its facsimile publication [AM1939]. The ENSAIO is a 130 pages typed monograph that introduces mathematical modernism and prepares a turning point in the mathematical activities in Portugal, preceding the creation of the *Centro de Estudos Matemáticos de Lisboa* in 1940, the first Portuguese research centre, affiliated with the *Faculdade de Ciências* of the Lisbon University and independently supported by the *Instituto para a Alta Cultura*, the incipient national science foundation at the time [Ro].

António Monteiro, Modernist and Mathematician

On the occasion of the centenary of his birth, the Portuguese Mathematical Society (SPM) published in 2007 a remarkable photobiography [AM_Fb2007] and a special issue of its Bulletin [AM_B2007] with the proceedings of an International Colloquium at the University of Lisbon. In the presentation of his *Works* [AM_O2008],

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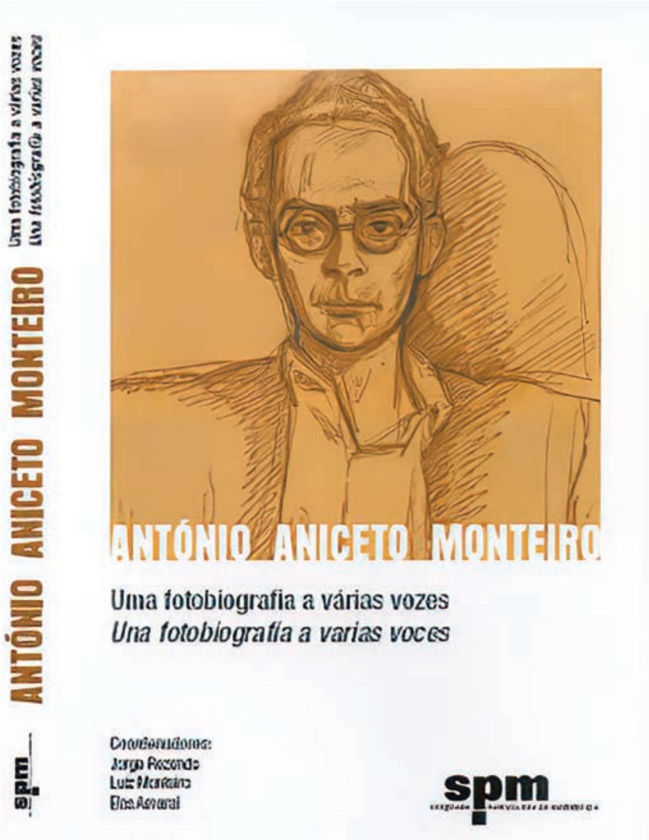


Figure 1. Cover of the photobiography [AM_Fb2007]

Figure 2. Cover of [AM_Bc2007]

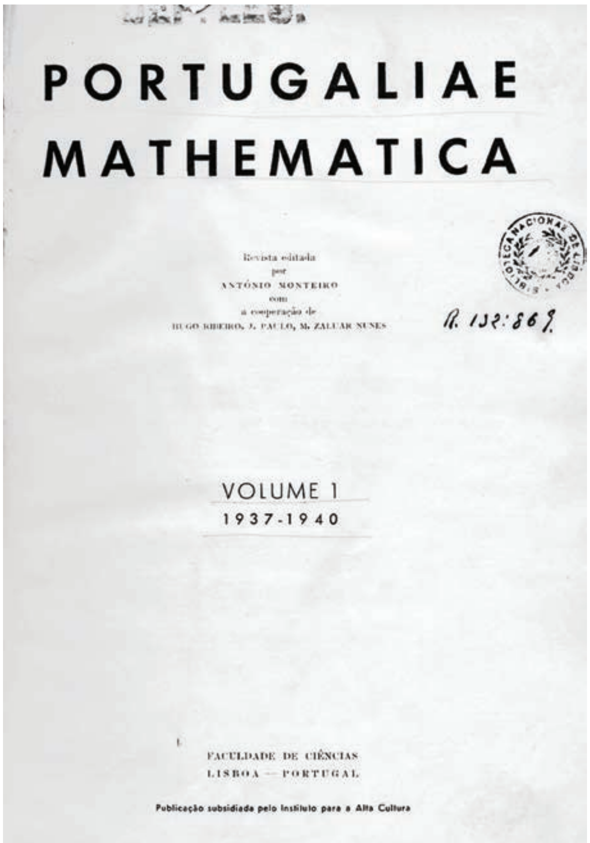
consisting of eight volumes of about 2800 pages, which does not include his 1939 Essay, Jean-Pierre Kahane, from the Paris Academy of Sciences, wrote

> The works of António A. Monteiro belong to the world history of mathematics. They cover a large variety of topics from classical analysis to topology and from advanced algebra to logic in its more modern chapters. Some of them come from courses and synthetic presentations, but the majority of them are research papers. They are presented in different styles, occasionally handwritten, and also in different languages. Despite their intrinsic value, these works are a testimony of an age and of an exceptional life. They were written, in four different countries: France, Portugal, Brazil and Argentina. Monteiro was the founder of mathematical journals and various mathematical institutions, first in Portugal, then in Latin America. He had to emigrate from Portugal because of Salazar's regime and was also affected by the military dictatorship in Argentina. His life testifies the link between the struggle for science and the struggle for freedom.

Monteiro was born in Moçâmedes (Angola) in 1907, the son of a Portuguese colonial army officer, he came to Lisbon, already orphan, to attend the Military College in 1917 before his graduation in Mathematics in 1930, at the *Faculdade de Ciências*, and his departure to Paris with a fellowship, where he followed courses at the *Faculté des Sciences* and seminars at the *Institut Henri Poincaré*. In 1936, he presented his thesis, *Sur l'additivité des noyaux de Fredholm*, at the University of Paris under Maurice Fréchet (1878–1973) [AM_Fb2007] and [AM_O2008]. During his stay in Paris, Monteiro assumes also his mission of studying "the organization of a Centre for Mathematical Studies which would have, among others, the objective of achieving the complete resurgence of Portuguese mathematical traditions", and, in his correspondence, he even refers to the acquisition of books for the *Instituto de Matemática* [AM_Bc2007].

After his return to Lisbon, he refused to sign a compulsory political statement in order to be integrated at the University — Monteiro would have said: "I do not accept limitations on my intelligence" — and he was thus unable to pursue in Portugal the career as a mathematician he developed in his exile in 1945 in Brazil and in Argentina from 1950 until his jubilation and removal, also for political reasons, from the *Universidad Nacional del Sur* in 1975, in Bahia Blanca, where he had been Professor Emeritus since 1972 and where he died in 1980 [Re].

Between 1937 and 1943, Monteiro's scientific and academic activity in Lisbon was carried out as a precarious inventor of scientific libraries in Portugal. In spite of financial difficulties, he was a major participant of the brief decade of the Portuguese Mathematical Movement (1936–1946), which began with the activities of


PORTUGALIAE
MATHEMATICA

Revista editada
por
ANTÓNIO MONTEIRO
com
a cooperação de
HUGO RIBEIRO, J. PAULO, M. ZALUAR NUNES

VOLUME 1
1937-1940

FACULDADE DE CIÊNCIAS
LISBOA — PORTUGAL

Publicação subsidiada pelo Instituto para a Alta Cultura

Figure 3. Frontispiece of vol. 1 of Portugaliae Mathematica.

the *Núcleo of Mathematics, Physics and Chemistry* at the end of 1936, with the founding, by Monteiro, of the journal *Portugaliae Mathematica* in 1937 (Fig.3), with the beginning of the *Seminário de Análise Geral* in 1939, the creation of the *Centro de Estudos de Matemática de Lisboa* (CEML), under his scientific direction, the *Gazeta de Matemática* and the Portuguese Mathematical Society, all in 1940.

His remarkable qualities as researcher and professor were first developed in Lisbon with the orientation of young mathematicians in the first three years of the activity of the CEML, which were marked by the influence of the modernist ideas of the 1939's "Ensaio" and culminating in the visit of Maurice Fréchet in early 1942 (Fig.4), and the successive departure of the young grantees abroad, including the first two Portuguese disciples of Monteiro, Hugo Ribeiro for the ETH in Zurich and José Sebastião e Silva for the University of Rome [AM_B2007].

He continued his activities at the *Centro de Estudos de Matemática do Porto*, created in 1942 and affiliated with the *Faculdade de Ciências* of the University of Porto, where he had a third disciple, Alfredo Pereira Gomes who also began his academic career in France and Brazil, and where he was supported by the *Junta de Investigação Matemática* (JIM). This remarkable association was created in 1943 and was sponsored by private funds during some years. JIM aimed to bring together almost all the (few) Portuguese researchers in the country, had as its primary objective "to promote the development of mathematical research" and played a very important role in funding scientific publications, particularly, the journal *Portugaliae Mathematica*, after the government stopped the initial financial support. This support to Portugaliae Mathematica was accomplished in connection with the CEML, and was limited to the first three volumes.

Figure 4. M. Fréchet, P. J. da Cunha and A. Monteiro at Faculdade de Ciências de Lisboa in early 1942 ([AM_Fb2007]).

Between 1945 and 1948, Monteiro was professor of *Análise Superior* at the University of Brazil, now the *Universidade Federal de Rio de Janeiro*, where he had a strong influence on young mathematicians. These include Maurício Peixoto, who was his co-author, Maria Laura Lopes, whose thesis of 1949 solved a question raised by Monteiro, and also Leopoldo Nachbin, who succeeded him in directing the series of monographs *Notas de Matemática* and was the author of its No. 4, the influential lectures notes on *Espaços Vetoriais Topológicos* (1948). This series, which Monteiro had founded in 1948, was published in Rio de Janeiro until 1972 and included in its No. 5 the text on *Rings of Continuous Functions* by Marshal H. Stone. It was continued with volume 48 by North-Holland and reached in 2008, already with Elsevier, the number 208 of that well known collection of *Mathematics Studies* [Ro].

Appointed Professor at the *Universidad Nacional de Cuyo*, San Juan, Argentina, in 1950, Monteiro founded the *Revista Matemática Cuyana* with M. Cotlar and E. Zarantonello, in 1955. Although he was invited to the University of Buenos Aires, Monteiro, with some of his new Argentine disciples, moved in 1957 to the recently created *Universidad Nacional del Sur*, in Bahía Blanca, where he founded the Mathematical Institute, the Mathematical Library, new series of monographs and developed research on Algebraic Logic and Latices. In 1974 he was appointed an honorary member of *Unión Matemática Argentina*, due to his remarkable intellectual influence. According to the Argentine mathematician Eduardo Ortiz, Monteiro belongs "to an old tradition of Argentinian progressive and independent thought to which the country owes some of its most valuable achievements" [AM_PM1980].

Although he spent a sabbatical year in Europe, during 1969–1970, visiting several universities in France, Romania, Belgium, Italy and England, he did not return to Lisbon until March 1977, with a scholarship of the National Institute of Scientific Research. There he resumed his research for about two years at the Centre for Mathematics and Fundamental Applications (CMAF), the direct successor of the CEML he had directed thirty seven years ago. During this period, he supervised his fourth Portuguese disciple, M. Isabel Loureiro, and wrote the extensive work *Sur les Algèbres de Heyting Symétriques*, which, in 1979, was awarded the Gulbenkian Prize for Science and Technology and was published in *Portugaliae Mathematica* [AM_PM1980], in a volume posthumously dedicated to him. In a letter dated June 5, 1978 sent to Alfredo Pereira Gomes, his former disciple from Porto, then professor at the University of Lisbon, despite his state of health, Monteiro wrote "I am really satisfied with the results of my scientific activity in Portugal. This is mainly due to the Centro de Matemática (CMAF), which provided me with free time to study" [AM_PM1980].

On his return to Bahía Blanca, where he had residence with his family, Monteiro died on 29 October 1980 in the country of his second exile. In a letter to his Argentine friend he wrote: "That's life dear Ortiz. One uses and spends oneself on tasks that cannot be finished: and yet one begins with enthusiasm and dedication, because hopes and certainties are never lost. Sadnesses of Bahia Blanca! on the margins of the Napostá; between winds and storms in which the earth drowns us, I see Lisbon distant - memories of my childhood!" [AM_B2007].

The forerunner 1939's "Ensaio sobre os fundamentos da análise geral"

In the Foreword of his Ensaio Monteiro wrote:

> The General Analysis was founded at the beginning of this century by Maurice Fréchet, with the aim of generalising the differential and integral calculus for those functions where the independent variable — and possibly the function itself — are elements of any nature (. . .) having as goal the study of the correspondences between variables of any nature.

In fact, Fréchet was a pioneer in proposing, in his 1906 thesis under the guidance of Jacques Hadamard (1865–1963), an abstract approach to mathematical analysis based on general structures: *class* (L), spaces with convergence; *class* (E), spaces with *écart*, i.e., with distance, renamed metric spaces by F. Hausdorff, in 1914, as a subclass of topological spaces, and *class* (V), a generalization of spaces (E) provided with neighborhoods (*voisinages*). These notions were evolving and Fréchet set out his results in the influential book *Les Espaces Abstraits* [F], which strongly marked Monteiro's early mathematical activity and, in particular, his essay. Monteiro shared the view of Norbert Wiener (1894–1964), who visited Fréchet in Strasburg in 1920 and later wrote about him: "more than anyone else who had seen what was implied in the new mathematics of curves rather than points (…) One of the specific things which attracted me in Fréchet was that the spirit of his work [spirit of abstract formalism]".

The 1939 essay clearly reflects the "spirit of abstract formalism", which Monteiro absorbed from Fréchet and had anticipated Bourbaki in France, but unfortunately has remained unpublished and lost in the archives of the Lisbon Academy of Sciences until recently. It consists of four chapters: Abstract Set Theory (13 p.); Abstract Algebra (52 p.); Abstract Topology (26 p.) and Abstract Analysis or General Analysis (38 p.) and corresponds to

Figure 5. Announcements of the course and the seminar by A. Monteiro, already at the CEML, attached to the Faculdade de Ciências of Lisbon [AM_Fb2007].

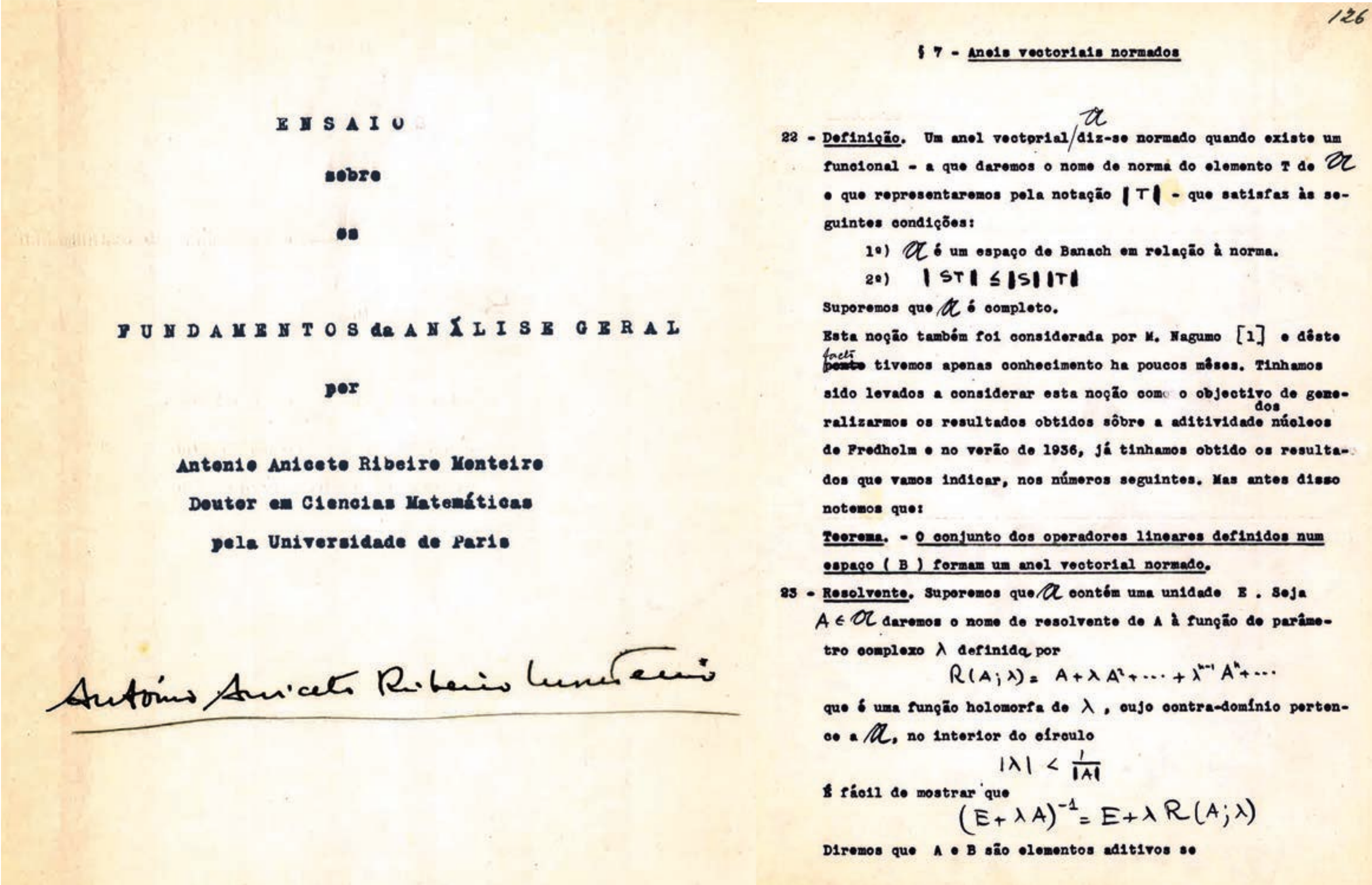
a programmatic plan, which was put into practice immediately by himself, with a Course in1939 and in the Seminar of General Analysis in 1940 (Fig. 5), already within the scope of the recently created *Centro de Estudos Matemáticos de Lisboa* [Ro].

In the first chapter, considering the Set Theory as a chapter of General Analysis, Monteiro characterized it as the theory that "occupies itself with the properties of the sets of points that remain invariant in relation to the group of biunivocal transformations", such as Abstract Algebra, "one of the most recent chapters of modern mathematics", which deals with the properties "which remain invariant in relation to the group of isomorphisms (biunivocal correspondences which respect the operation considered)" and, in the third chapter, Topology, also a chapter of the General Analysis, as the theory "which studies the properties of the sets of points which remain invariant in relation to the group of bicontinuous transformations or homeomorphisms." Besides the influence of the French school, Monteiro, who had followed the Julia's seminars in Paris since 1933 until 1936 (on Groups and Algebras, Hilbert Spaces and Topology), was also well aware of the contemporary mathematical developments of the Polish and the Russian schools in Set Theory and Topology and of the German school in Algebra.

Finally in the fourth and main chapter, Monteiro introduced "the notion of algebraic-topological space — which we can define as a space where there is simultaneously an algebra and a topology", with the aim of dealing with the "study of invariant properties for a topo-isomorphism, that is, by a biunivocal correspondence that is simultaneously a homomorphism and an isomorphism", especially in what he called "analytical spaces", that is, those for which the algebraic operation is continuous. Among these, he introduced the perfectly decomposable abelian topological groups, for which he proved "a theorem of structure", which, being "analogous to Banach's and Cantor-Bernstein's theorems" (about the equivalence of two sets with the same power), establishes, in particular, that if two of those groups "have the same algebraic dimension they are topo-isomorphic". The new notion of algebraic dimension is a generalization of the "linear dimension of a (vector) space of type (F) recently introduced by Banach" in his classic 1932 book on *Théorie des opérations linéaires*. Culminating a modern synthesis of some algebraic-topological structures, including topological groups, normed rings and Banach spaces, Monteiro generalized the results of his thesis on the additivity of Fredholm kernels, obtaining necessary and sufficient conditions for the additivity of the resolvents within the rings of linear operators in Banach spaces (Fig. 6).

The clarity and novelty with which the new abstract ideas are described and put into practice by Monteiro in his *Ensaio* is remarkable and it represents a significant progress, certainly independent and unknown to the collective of mathematicians who, under the name of N. Bourbaki, were creating the *Éléments de Mathématique* which would only start publishing a year later in Paris. In his autobiography, André Weyl (1906–1998), one of the founders and most influential mathematicians of this collective, recorded the spirit of the time by writing [W, p.114]:

> In establishing the tasks to be undertaken by Bourbaki, significant progress was made with the adoption of the notion of structure, and of the related notion of isomorphism. Retrospectively these two concepts seem ordinary and rather short on mathematical content, unless the notions of morphism and category are added. At the time of our early work these notions cast new light upon subjects which were still shrouded in confusion: even the meaning of the term "isomorphism" varied from one theory to another. That there were simple structures of group, of topological space, etc., and then also more complex structures, from rings to fields, had not to my knowledge



ENSAIO

sobre

os

FUNDAMENTOS da ANÁLISE GERAL

por

Antonio Aniceto Ribeiro Monteiro

Doutor em Ciencias Matemáticas

pela Universidade de Paris

Antonio Aniceto Ribeiro Monteiro

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§ 7 - Aneis vectoriais normados

22 - Definição. Um anel vectorial $\mathfrak{A}$ diz-se normado quando existe um funcional - a que daremos o nome de norma do elemento T de $\mathfrak{A}$ e que representaremos pela notação $|T|$ - que satisfaz às seguintes condições:

1º) $\mathfrak{A}$ é um espaço de Banach em relação à norma.

2º) $|ST| \leq |S|\,|T|$

Suporemos que $\mathfrak{A}$ é completo.

Esta noção também foi considerada por M. Nagumo [1] e dêste facto tivemos apenas conhecimento ha poucos mêses. Tinhamos sido levados a considerar esta noção como o objectivo de generalizarmos os resultados obtidos sôbre a aditividade dos núcleos de Fredholm e no verão de 1936, já tinhamos obtido os resultados que vamos indicar, nos números seguintes. Mas antes disso notemos que:

Teorema. - O conjunto dos operadores lineares definidos num espaço (B) formam um anel vectorial normado.

23 - Resolvente. Suporemos que $\mathfrak{A}$ contém uma unidade E . Seja $A \in \mathfrak{A}$ daremos o nome de resolvente de A à função do parâmetro complexo λ definida por

$$R(A;\lambda) = A + \lambda A^2 + \cdots + \lambda^{k-1} A^k + \cdots$$

que é uma função holomorfa de λ , cujo contra-domínio pertence a $\mathfrak{A}$, no interior do círculo

$$|\lambda| < \frac{1}{|A|}$$

É fácil de mostrar que

$$(E + \lambda A)^{-1} = E + \lambda R(A;\lambda)$$

Diremos que A e B são elementos aditivos se

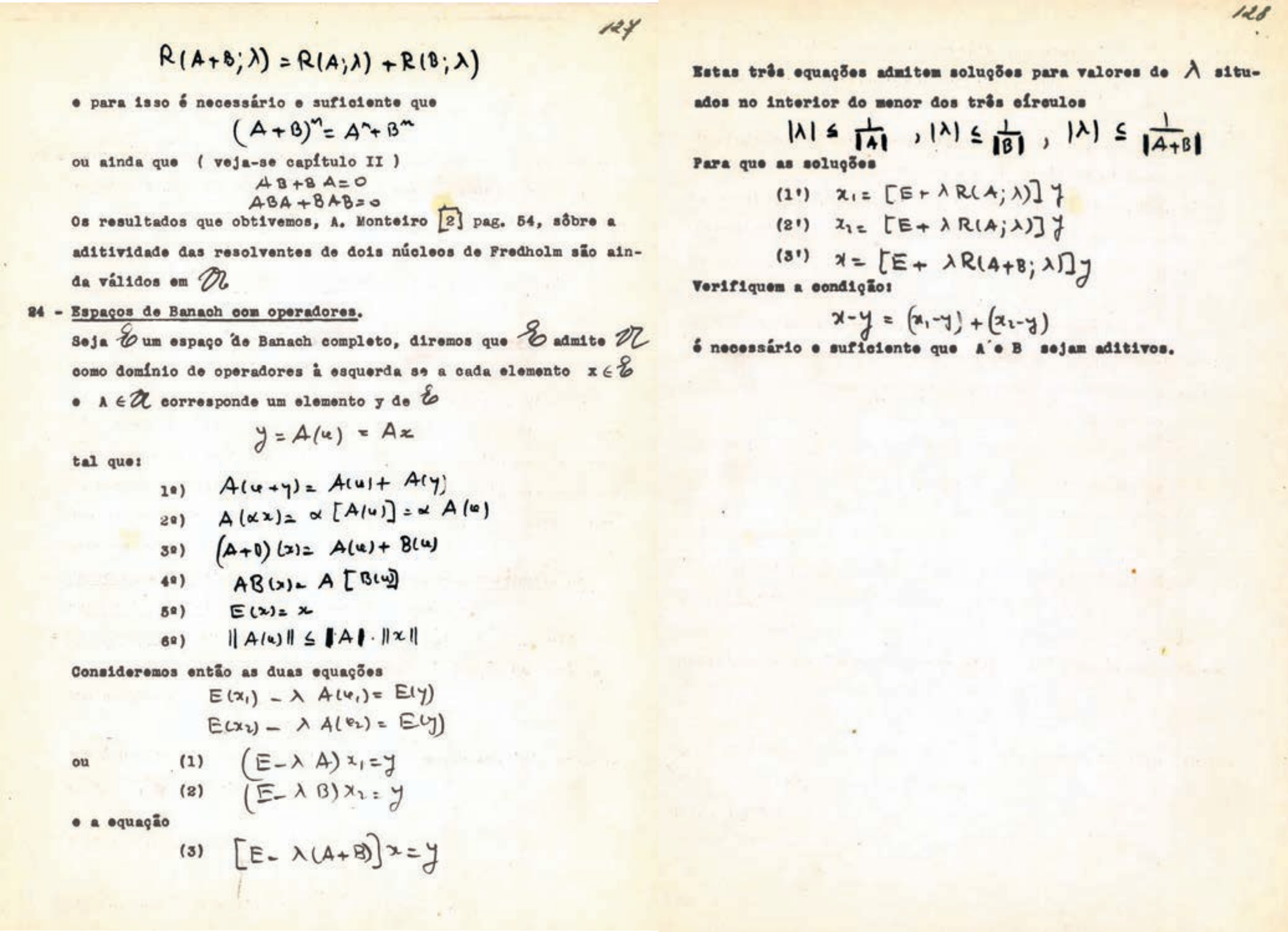
Figure 6. Pages of the Essay with Monteiro's original result [AM1939].

been said by anyone before Bourbaki, and it was something that needed to be said.

That was relevant to be said, and Monteiro also knew it and wrote it very clearly not only in the Preface of his *Ensaio*, that he delivered the 4th February 1939 at the Academy of Sciences of Lisbon, but also throughout its four chapters, which substantial contents coincide in great portions with those of the first four issues of Bourbaki's treaty, published in Paris in 1940 and 1942.

In fact, if the initial objective of those young mathematicians from the *École Normale Supérieur* of Paris, who founded the Bourbaki group in 1935, was to write a new course on Differential and Integral Calculus in the form of a modern treatise on Mathematical Analysis to replace the classical *Cours d'analyse* of the old French school, they evolved into an axiomatic and abstract presentation of "*les structures fondamentales de l'analyse*". The first four fascicules begin the Éléments de Mathématique: Livre I — Théorie des Ensembles (Fascicule de résultats), 1939; Livre II — Algèbre (Structures algèbriques), 1942; Livre III — Topologie Génerale (Chap.I, Structures topologiques; Chap.II, Structures uniformes), 1940; Livre III — Topologie Génerale (Chap.III, Groups topologiques; Chap.IV, Nombres réels), 1942.

The 45 pages booklet on Set Theory, although dated 1939, has the printing date of February 1940, and begins by explaining the "*mode d'emploi de ce traité*", which "takes the mathematics at the beginning, gives complete demonstrations and, in principle, does not suppose any particular mathematical knowledge, but only a certain habit of mathematical reasoning and a certain power of abstraction". In the English translation of the 1970 profound and enlarged revision of the Set Theory fascicule, one can read: "the axiomatic method allows us, when we are concerned with complex mathematical objects, to separate their properties and regroup them around a small number of concepts: that is to say, using a word which will receive a precise definition later, to classify them according to the *structures* to which they belong." The second book, on Algebra, published in 1942, has about 160 pages and contains the first chapter of algebraic structures. It is a synthesis of modern algebra which is considered as a result "above all of the work of the modern German school" and recognizes the 1930 book by van der Waerden, also used by Monteiro in his 1939 *Ensaio*, as a source of inspiration.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$$R(A+B;\lambda) = R(A;\lambda) + R(B;\lambda)$$

e para isso é necessário e suficiente que

$$(A+B)^n = A^n + B^n$$

ou ainda que (veja-se capítulo II)

$$AB + BA = 0$$
$$ABA + BAB = 0$$

Os resultados que obtivemos, A. Monteiro [2] pag. 54, sôbre a aditividade das resolventes de dois núcleos de Fredholm são ainda válidos em $\mathfrak{M}$

24 - Espaços de Banach com operadores.

Seja $\mathcal{E}$ um espaço de Banach completo, diremos que $\mathcal{E}$ admite $\mathfrak{M}$ como domínio de operadores à esquerda se a cada elemento $x \in \mathcal{E}$ e $A \in \mathfrak{M}$ corresponde um elemento y de $\mathcal{E}$

$$y = A(x) = Ax$$

tal que:

1º) $A(x+y) = A(x) + A(y)$

2º) $A(\alpha x) = \alpha [A(x)] = \alpha A(x)$

3º) $(A+B)(x) = A(x) + B(x)$

4º) $AB(x) = A[B(x)]$

5º) $E(x) = x$

6º) $\|A(x)\| \leq |A| \cdot \|x\|$

Consideremos então as duas equações

$$E(x_1) - \lambda A(x_1) = E(y)$$
$$E(x_2) - \lambda A(x_2) = E(y)$$

ou (1) $(E - \lambda A)x_1 = y$

(2) $(E - \lambda B)x_2 = y$

e a equação

(3) $[E - \lambda(A+B)]x = y$

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Estas três equações admitem soluções para valores de λ situados no interior do menor dos três círculos

$$|\lambda| \leq \frac{1}{|A|}, \quad |\lambda| \leq \frac{1}{|B|}, \quad |\lambda| \leq \frac{1}{|A+B|}$$

Para que as soluções

(1') $x_1 = [E + \lambda R(A;\lambda)]y$

(2') $x_2 = [E + \lambda R(A;\lambda)]y$

(3') $x = [E + \lambda R(A+B;\lambda)]y$

Verifiquem a condição:

$$x - y = (x_1 - y) + (x_2 - y)$$

é necessário e suficiente que A e B sejam aditivos.

Bourbaki's third book, dedicated to General Topology, is in fact the second to be published in 1940 and consists of two chapters dealing with structures of another kind which "give a mathematical sense to the intuitive notions of limit, continuity and neighbourhood". The Chapter I, on topological structures, begins with open sets, to define topological space, and bases the notion of convergence on the concept of filter, obtaining the complete equivalence between neighborhood, open set and the topology of convergence. The Chapter II deals with uniform structures, which makes it possible to extend the structure of the metric spaces introduced by Fréchet in 1906, and to generalise to the uniform spaces important results, in particular of compactness and completeness. Chapters III and IV of General Topology were published in 1942. Chapter III, starting with the definition of a topological group, develops the theory based on filters and their convergences and on the properties of uniform structures, and concludes with some topics on topological rings and fields. Chapter IV introduces the group of the real numbers, proves the usual topological properties and basic results on series and on numerical functions, ending with an extensive and fairly complete twelve-page historical note. Those three chapters are more innovative and have a broader scope then the corresponding two last chapters od Monteiro's *Ensaio*.

However, comparing the structure of the four chapters and the respective sections of António Monteiro's essay, delivered on February 4, 1939 at the Lisbon Academy of Sciences, with the contents of these first four fascicules by Bourbaki, which are a work of another dimension and with another ambition, we are surprised by the coincidence of their sequencing and even by the overlapping of many of their contents. Naturally Monteiro absorbed in Paris, during his stay between 1931 and 1936, the new ideas and the most recent results of modern mathematics. Monteiro´s objectives had, in a completely different scale and context, some parallelism with the ambitious programme of the Bourbaki collective, but he could not know either the plans or the contents of the *Éléments de Mathématiques*. However, although Monteiro had never been a "bourbakist" or revealed sympathies for the work of Bourbaki's disciples, we dare to consider that his remarkable and forgotten *Ensaio* is, in fact, a forerunner of the great project of that collective author, which is also characterised by a remarkable modernism and structuralism [Ro].

The influence of the contents of the *Ensaio* and its

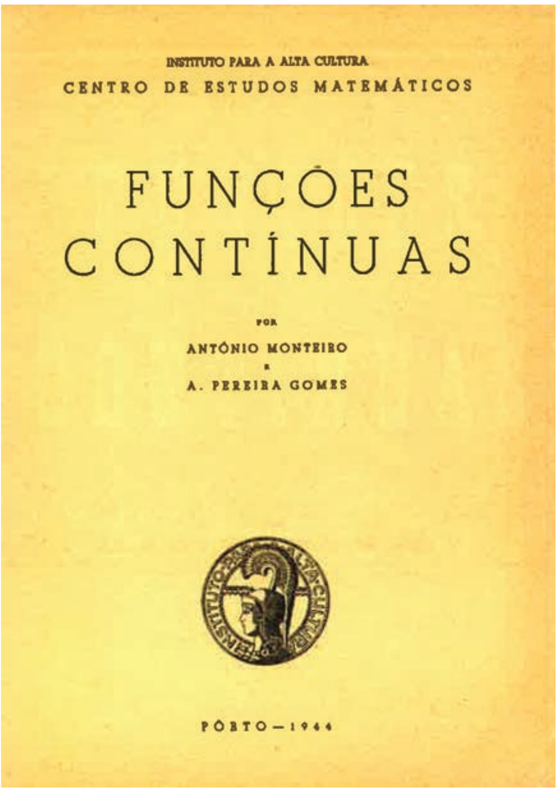
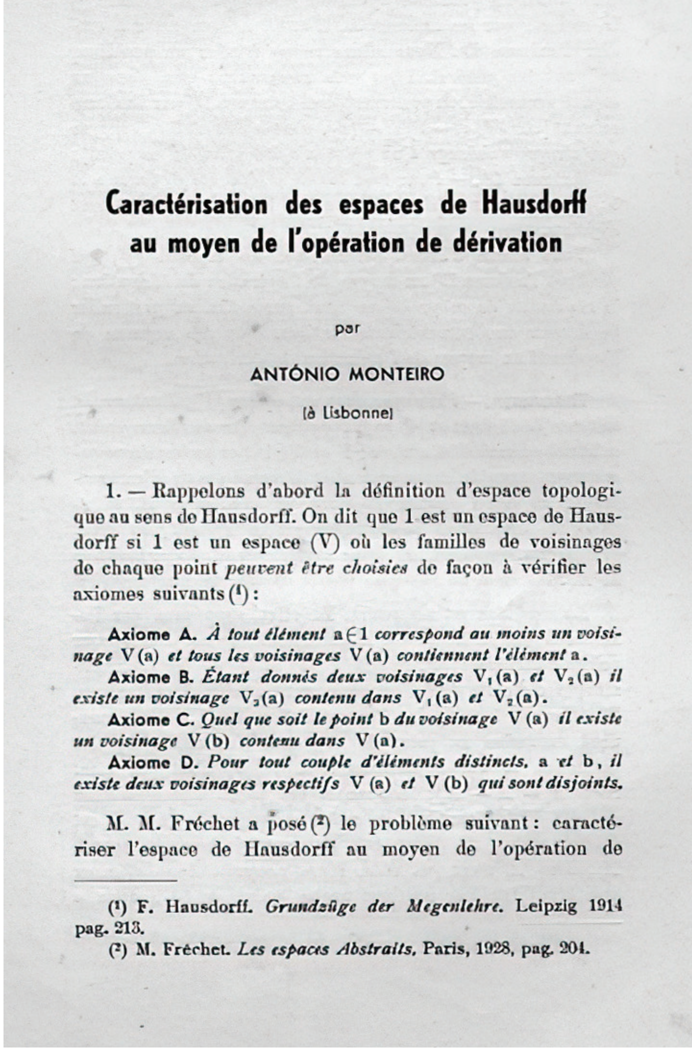


Figure 7. The study on Continuous Functions, published by the CEM of Porto in 1944.



Caractérisation des espaces de Hausdorff au moyen de l'opération de dérivation

par

ANTÓNIO MONTEIRO

(à Lisbonne)

1. — Rappelons d'abord la définition d'espace topologique au sens de Hausdorff. On dit que 1 est un espace de Hausdorff si 1 est un espace (V) où les familles de voisinages de chaque point *peuvent être choisies* de façon à vérifier les axiomes suivants [1]:

Axiome A. *À tout élément* a ∈ 1 *correspond au moins un voisinage* V(a) *et tous les voisinages* V(a) *contiennent l'élément* a.

Axiome B. *Étant donnés deux voisinages* V_1(a) *et* V_2(a) *il existe un voisinage* V_3(a) *contenu dans* V_1(a) *et* V_2(a).

Axiome C. *Quel que soit le point* b *du voisinage* V(a) *il existe un voisinage* V(b) *contenu dans* V(a).

Axiome D. *Pour tout couple d'éléments distincts,* a *et* b, *il existe deux voisinages respectifs* V(a) *et* V(b) *qui sont disjoints.*

M. M. Fréchet a posé [2] le problème suivant: caractériser l'espace de Hausdorff au moyen de l'opération de

(1) F. Hausdorff. *Grundzüge der Megenlehre.* Leipzig 1914 pag. 213.
(2) M. Fréchet. *Les espaces Abstraits.* Paris, 1928, pag. 204.

Figure 8. A paper on general topology by A. Monteiro, published by CEM of Lisbon in *Portugaliae Mathematica*, **1** (1940), 333–339, and cited in the Kelley's book [K].

author is notorious. With the intense mathematical research activities with a small group of students during only three years at the CEML, in Lisbon, and about one year at the CEMP, in Porto, he started together an outline of an ephemeral Portuguese School of General Topology, which influence extended to Rio de Janeiro. In the classic 1955 book *General Topology* [K], the American mathematician J. L. Kelley, in his Foreword, not only thanks Hugo Ribeiro, but also cites in his Bibliography three articles by him and two by Monteiro, all published in *Portugaliae Mathematica* between 1940 and 1945, a note to the C.R. Acad. Sc. Paris by A. Pereira Gomes and the monograph in Portuguese by L. Nachbin on Topological Vector Spaces, both published in 1948 [Ro].

References

[AM_PM1980] *Portugaliae Mathematica*, vol. 39 (1980), Edition of Sociedade Portuguesa de Matemática (1985), dedicated to A. A. Monteiro under the direction of Alfredo Pereira Gomes.

[AM_Bc2007] *Boletim da Sociedade Portuguesa de Matemática*, Nº Especial, Proceedings Centenary Conference, Faculdade de Ciências da Universidade de Lisboa, June 2007, edited by Luís Saraiva.

[AM_Fb2007] *António Aniceto Monteiro, uma fotobiografia a várias vozes*, Sociedade Portuguesa de Matemática, Lisboa 2007, Editors: Jorge Rezende, Luiz Monteiro and Elza Amaral.

[AM_O2008] *The Works of António A. Monteiro, Vol.s I-VIII*, Edited by Eduardo L. Ortiz and Alfredo Pereira Gomes, Fundação Calouste Gulbenkian, Lisbon and The Humboldt Press, London 2008.

[AM1939] António Monteiro, "*Ensaio sobre os Fundamentos da Análise Geral*", Memórias Academia das Ciências de Lisboa , vol. XLVII n.2 (2020), 27–169.

[F] M. Fréchet, *Les Espaces Abstraits et leur théorie considérée comme Introduction à l'Analyse Générale*, Gauthier-Villars, Paris, 1928.

[K] J. L. Kelley, *General Topology*, Van Nostrand, New York, 1955.

[Re] J. Rezende, Blogue <*António Aniceto Monteiro*> http://antonioanicetomonteiro.blogspot.com/

[Ro] J. F. Rodrigues, *O Ensaio de 1939 de António Monteiro, O seu contexto e a sua importância, Memórias Academia das Ciências de Lisboa*, vol. XLVII n.2 (2020), 9–26.

[W] A. Weil, *The Apprenticeship of a Mathematician*, Springer, Basel, 1992.