

REPORT



Com mais de cinco séculos de história, a tipografia em Portugal nasceu através de avulsões, entre os quais, em 1496, o primeiro tratado matemático, na cidade de Leiria, sob o nome de Tipografia Matemática. Na cidade de Lisboa, há 127 anos publicamos o Almanach Perpetuum, o primeiro livro científico da imprensa portuguesa, a qual foi monumental para o império no seu dia de lançamento.

Na 1.ª metade, nomeadamente 1500 anos após a primeira livro impresso em Portugal, de primeiro volume da Tipografia Matemática, sempre diferenciadamente em 700, esta revista matemática, com 12 volumes disponíveis, passou a História da Cultura Matemática em Portugal, passando pela engenharia militar, passando por guerra de Reconquista, pela regalia dos Colégios, Académias e Universidade e ainda pela integração científica contemporânea.

A exposição está aberta de 2 de julho a 31 de outubro de 2021 no Museu Nacional da Papel em Leiria, passando ao que ainda a um sábado no Porto, em Coimbra e em Lisboa em 2022.

1. *Almanach Perpetuum*
Alonso Lainez (1496-1517)

1496 - Leiria, Museu Nacional da Papel



2. *Tratado da pratica Diuinaçõys*
Gaspard Pequeur (1497-1517)

1497 - Leiria, Museu Nacional da Papel



3. *Tratado da*
Pedro Nunes (1500)



Portuguese Mathematical Typography: 1496–1987

by José Francisco Rodrigues*

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Figure 1.—*Moinho do papel* is a modern museum on the left bank of the river Lis, in the city of Leiria, where in 1411 there was the first paper mill in Portugal.

The *Tipografia Matemática Portuguesa: 1496–1987* is a unique, rare and eloquent exhibition, which first edition took place from July 1st to October 31st 2021 in the hexacentenary *Moinho de Papel* in Leiria, a city with an ancient castle since the 12th century in the center of Portugal, residence of kings and setting of several cortes (medieval parliaments). The city gave the name to a famous pine forest (*Pinhal de Leiria*), wood supplier of the ships used in the Portuguese navigations of the 15th and 16th centuries. The exhibition was an initiative of the city of Leiria in partnership with the CIM and the Polytechnic Institute of Leiria.

The *Moinho de Papel* is an historical building on the river Lis, the first paper mill established in 1411 in Portugal, which may well have influenced the fact that Leiria was also one of the first Portuguese cities to have a typography and where 525 years ago the first scientific book, which was instrumental for navigation in the age of discoveries, was printed in the country. The second date of the title of this unprecedented exhibition corresponds to the publication, coincidentally 500 years after the first book printed in Portugal, of the first volume of *Portugaliae Mathematica* electronically composed in *TeX*. With 32 significant original works, this exhibition traverses the History of Mathematical Sciences in Portugal, through military engineering, essential in the wars of Restoration (1640–1668) after the end of the Iberian Union, through the successive reforms of Colleges, Military Academies

and Universities (1772 and 1911) and through scientific research in the 20th century.

The publication in 1496, in Leiria, of the *Almanach perpetuum*, with the astronomical tables of the Sephardi scholar Abraão Zacuto, referred to the year 1473 and translated and edited by the Portuguese José Vizinho, took place a few years after a first edition of the *Torah* of 1487, in a Hebrew typography in Faro, in the south of Portugal, and the *Tratado de Confissom* of 1489, which is the first Christian text in Portuguese language that was printed in Chaves, in the north of the country. It should be noted that the Gutenberg Bible, the first book printed in Europe, dates from 1455 and the first printing of Euclid's *Elements* in 1482 was made by the printer Erhard Ratdolt, in Venice, in a Latin edition containing the first geometric diagrams of the press.

That *Almanach* is a landmark of the beginning of the culture of Mathematical Sciences in Portugal through the influence and use of the art and knowledge of navigation, namely in the first ocean voyages of Vasco da Gama to India and Pedro Álvares Cabral to Brazil. It was used in the following century in the preparation of *Reportórios dos Tempos*, the popular time calendars and almanacs also used in astrology, one of which had the collaboration of a certain Gaspar Nicolas. This Portuguese mathematician published in 1519, in Lisbon, a *Tratado da pratica Darismetyca*, which is a book of a technical and utilitarian nature about the rules of arithmetic, also “for overseas



Figure 2.— A glimpse of the nine showcases and the posters containing the 32 books and the eight biographies of Portuguese mathematicians of the Leiria exhibition.



Figure 3.—*Almanach Perpetuum*, by Abraham Zacut (1452-1515), printed in 1496 in Leiria by Abraham d’Ortas.

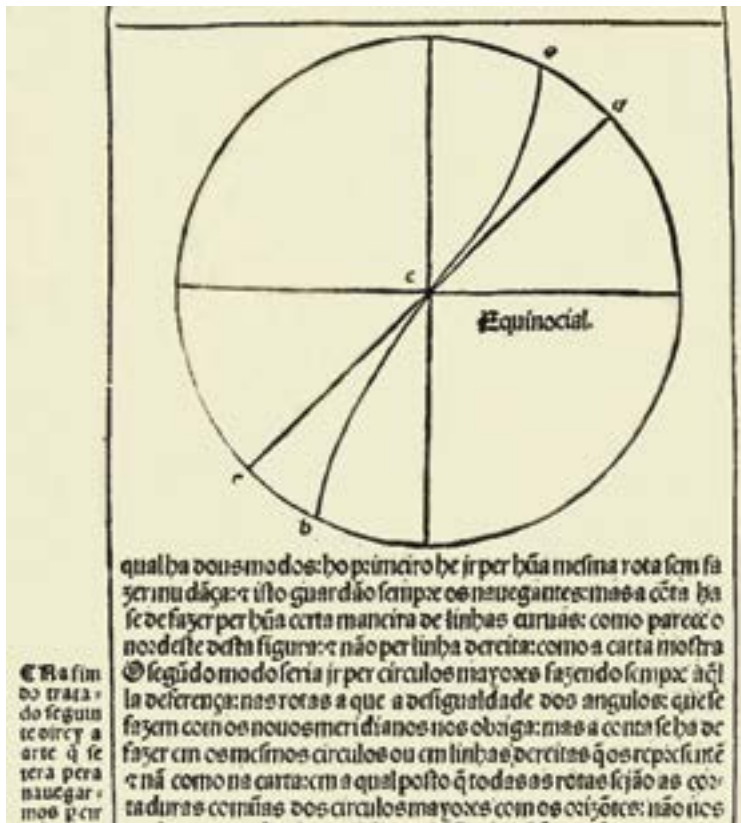


Figure 4.—The Pedro Nunes' representation of the rhumb line *acb* and of the great circle *dce* was published in 1537 in the *Tratado da Sphera*, printed in Lisbon by Germão Galhardo.

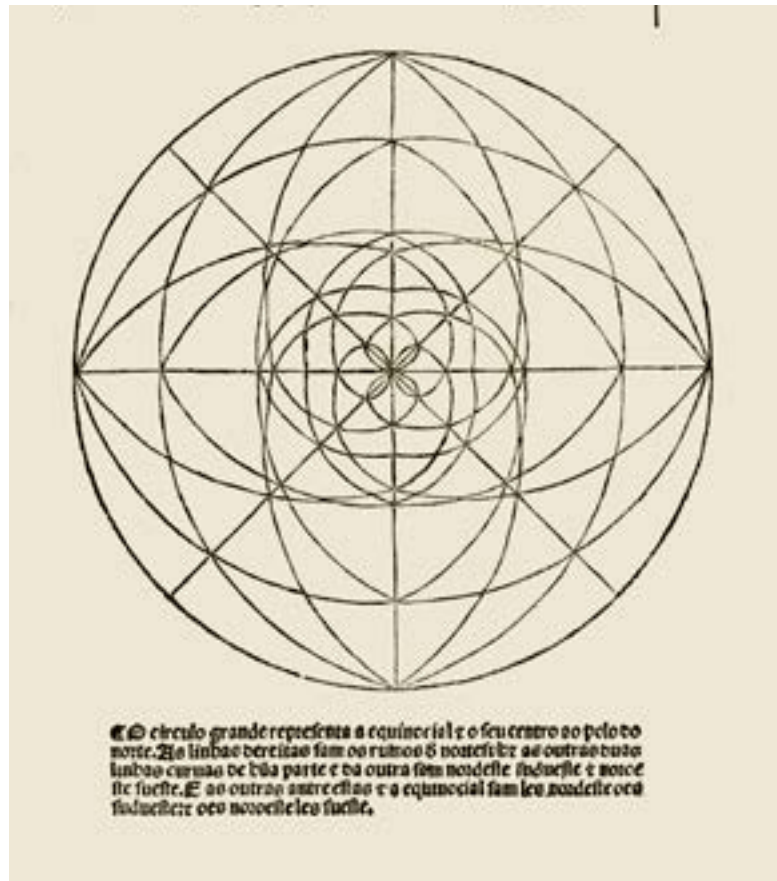


Figure 5.—The first polar projection of the rhumb line in the Pedro Nunes' *Tratado da Sphera* is the beautiful rosette composed of projections of loxodromes with azimuths with angles of 45° and 67.5° .

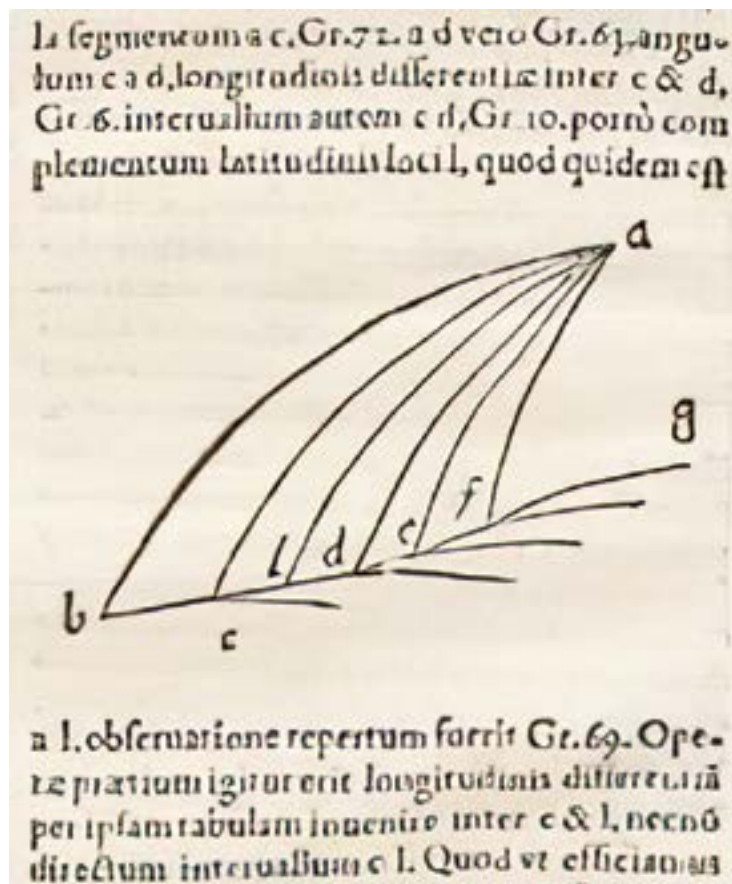


Figure 6.—The broken line *bcdef* (noniodrome) is the 1566's approximation of the loxodrome proposed by Pedro Nunes for the construction of nautical tables used by Mercator in his *Mappa-mundi* of 1569 and Edward Wright in 1596, which is in page 28 of the second edition of *De arte atque ratione navigandi*, printed in Coimbra, by António Mariz.

trade", and which had ten reeditions in the following two centuries.

Printing also played an important role in the development of the mathematical theory of navigation through the works of Pedro Nunes, with the edition in Portuguese of his *Tratado da Sphera*, in 1537, which presented the first mathematical conceptualisation of the rhumb line, later called loxodrome. With the enlarged reedition of Nunes' fundamental work *De arte atque ratione navigandi*, where the Portuguese mathematician developed original methods, in particular for the approximation of the rhumb line, which laid the foundations for the elaboration of nautical tables and the cartographic projection made in 1569 by Mercator. That broken line invented by

PRINCIPIOS MATHEMATICOS
 PARA INSTRUCCAO
 DOS ALUMNOS DO COLLEGIO
 DE
 SAO LUCAS,
 DA REAL CASA PIA DO CASIELLO
 DE
 SAO JORGE:
 OFFERECIDOS
 AO SERENISSIMO SENHOR
D. JOAO,
 PRINCIPE DO BRAZIL:
 COMPOSTOS PELO DOUTOR
JOSE ANASTACIO DA CUNHA,
 DE ORDEM DO DESEMBARGADOR DO PAÇO,
DIOGO IGNACIO DE PINA MANIQUE,
*Intendente Geral da Policia da Corte,
 e Reis. &c., &c., &c.*


 LISBOA
 NA OFFIC. DE ANTONIO RODRIGUES GALHARDO,
 Impressor do Emocissimo Senhor Cardinal Patriarca,
 ANNO M.DCC.LXX.
*Com licença do Real Mesa da Consielha Geral
 para o Ensino, e Confirmação da Livros.*

PRINCIPIOS MATHEMATICOS.

LIVRO VIII. Definição I.

SERIE convergente chamam es Mathematicos aquella, cujos termos são sucessivamente determinados, cada um pelo numero dos termos precedente, de sorte que sempre a serie se possa continuar, e finalmente venha a ser indifferente o continua-la ou não, por se poder desprezar sem erro notavel a somma de quantos termos se quizee ajuntar aos já escritos ou indicados: e estes ultimos indicam-se escrevendo &c. depois dos primeiros dois, ou tres, os quantos se quizar: he porém necessario que os termos escritos mostrem como se poderia continuar a serie, ou que isso se faisa por outra via.

Proposição I.

Serie de proporçoes continuas he convergente, se o primeiro he maior que o segundo. Sejam A e B duas quaisquer grandezas homogeneas, mas $A > B$; e $A + B + B \frac{B}{A} + B \frac{B}{A} \times \frac{B}{A} + B \frac{B}{A} \times \frac{B}{A} \times \frac{B}{A} + \&c.$ a serie. Escreva-se e em lugar de $\frac{B}{A}$, a serie será $A (1 + e + e^2 + e^3 + e^4 + \&c.)$. Seja O huna grandez $< A$, e tal que se possa desprezar sem erro notavel. Na serie $1 +$

PRINCIPIOS MATHEMATICOS.

LIVRO XV. Definições.

I. **S**E huna expressão algebrica de huna valor, quando outra expressão admittir huns só, chamam-se esta constante, e aquella, variavel.
II. A variavel que podê sempre admittir valor maior que qualquer grandez que se proponha chamam-se huna infinita; e a variavel que podê sempre admittir valor menor que qualquer grandez que se proponha, chamam-se huna infinitesima.
III. Se o valor de huna expressão A depender de outra expressão B , chamam-se A função de B , e B sua de A .

Advertencias.

I. As principaes letras do alfabeto denotam ordinariamente constantes e as ultimas x, y, z variaveis, variavel e funçoes.
II. A funçao denota-se escrevendo immediatamente antes da sua alguma letra grega maiuscula, que não seja occureta no alfabeto latino; e huna mesma letra B representa huna mesma funçao a por exemplo, se x denotar x^3 , $e \Delta x$, $(x + \Delta x)^2$, x denotará x^3 , $e \Delta x$, $(x + \Delta x)^2$.

N

III.

Figure 7.— The *Principios Mathematicos*, by José Anastácio da Cunha, printed in 1790, in Lisbon by António Rodrigues Galhardo, contains the first modern definitions of convergence of a series, in Chapter 8, and of the differential, in Chapter 15, respectively.

Pedro Nunes in 1566, called the *noniodrome* and chosen as the logo of the exhibition, is the natural way to approximate the loxodrome that corresponds to the method of Euler to integrate differential equations and was used numerically by Edward Wright in the secants' method to construct nautical tables in 1596.

Printing was instrumental also in the teaching and practical use of mathematics for military architecture, navigation and artillery in Europe and, in particular in Portugal with the *Methodo Lusitanico* (1680) and *O Engenheiro Portuguez* (1728), after the Restoration of Portuguese independence. Mathematics book printing continued in Portugal for the navy and the army schools, throughout the 18th and 19th centuries, among others, with the translations of the notable books by Lagrange (1798) and Lacroix (1812), the latter at the *Impressão Régia* (Royal Printing House) in Rio de Janeiro, which was the capital of the kingdom from 1808 until 1821, as well as with other original texts, such as the interesting *Carta Físico-Mathematica sobre a theórica da pólvora em geral e o comprimento das peças em particular* (1769), by José Anastácio da Cunha.

After the reforms of Marquis of Pombal, head of the government in the age of the Portuguese Enlightenment, the printing press would serve teaching with the pub-

lication of higher mathematics textbooks, initially with translations of foreign authors, such as Euclid's *Elementos* (1768), for the *Colégio dos Nobres* in Lisbon, several Bézout's textbooks for the first Faculty of Mathematics of the University of Coimbra, reformed in 1772, and, for the following century, the *Curso Completo de Mathematicas Puras* (1838 and 1839) by Francoeur. The first exception is the original and remarkable *Principios Mathematicos* (1790), by José Anastácio da Cunha, the military mathematician and "lente penitenciado", penitentiated professor at the University of Coimbra, whose innovations place him among the eminent predecessors of the 19th century reform of the Infinitesimal Calculus, namely with the modern criteria of convergence of series and the rigorous definitions of infinitesimal and differential, more than three decades before Cauchy.

The publication of university original textbooks by Portuguese authors would only continue a century later in Porto, with the *Curso de Analyse Infinitesimal* (1887), by F. Gomes Teixeira, which, reedited and enlarged, became the reference Portuguese treatise at the beginning of the 20th century. Only from the middle of that century did this university practice resume, illustrated by the classical *Curso de Álgebra Superior* by J. Vicente Gonçalves (Coimbra, 1933) and the modern *Lições de Análise Infini-*



Figure 8.—The *Memórias* of the Academy of Sciences of Lisbon, started their publication in 1797, by its own typography, with an interesting applied mathematics article on the Kepler’s problem on the volume of barrels.

tesimal by F.R. Dias Agudo (Lisbon, 1973), among others.

The Lisbon Academy of Sciences, created in late 1779 and endowed with a printing press, began publishing its *Memórias* in 1797 with an article on applied mathematics, worthy of its motto “*Nisi utile est quod facimus stulta est gloria*” (If what we do is not useful, glory is in vain), continuing in a new series after the period of *Regeneração*, in the 1850s, attempted to develop the country economically and modernise it, and creating the first Portuguese scientific journal, the *Jornal de Sciencias Mathematicas Physicas e Naturaes* (1867), where few articles of mathematics appeared, including original articles by Daniel da Silva.

The first Portuguese periodical exclusively dedicated to mathematics, the *Jornal de Sciencias Mathematicas e*

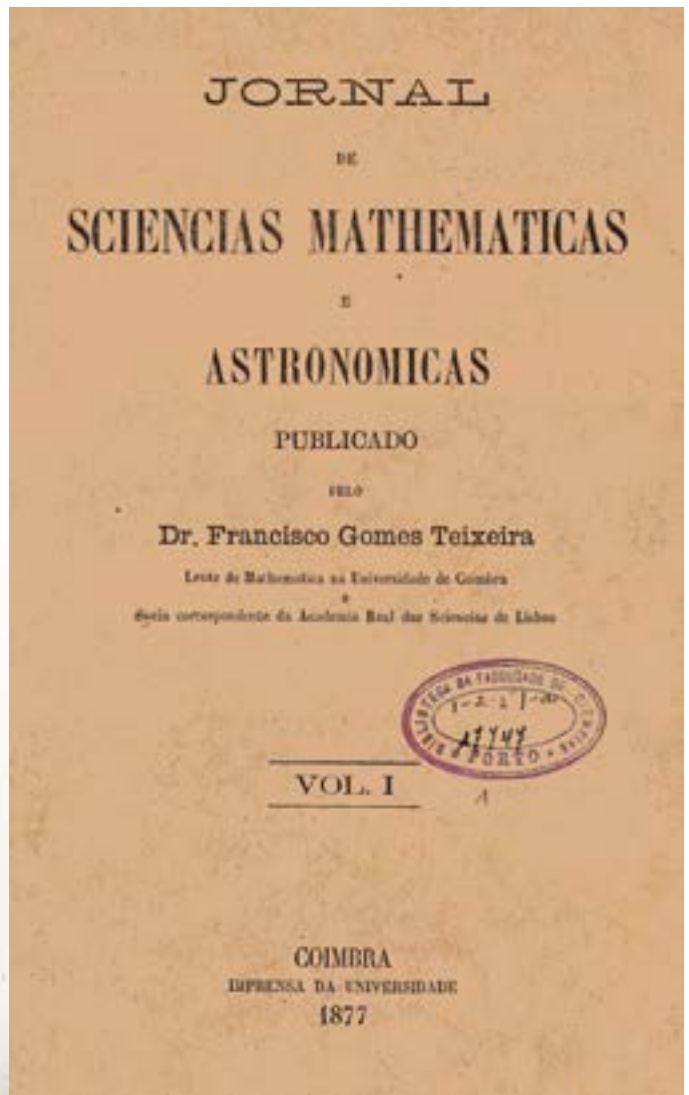


Figure 9.—The first periodic Portuguese research journal, *Jornal de Sciencias Mathematicas e Astronomicas*, was published by Francisco Gomes Teixeira and was printed in 1877 by the press of the University of Coimbra.

Astronomicas by F. Gomes Teixeira, started publication in 1877 in Coimbra, at the University Press. Later its publication was transferred to Porto and was integrated in 1905 in the *Annaes Scientificos da Academia Polytechnica do Porto*, and only more than thirty years later Portugal had a new mathematical journal. Francisco Gomes Teixeira was the most active and fruitful Iberian mathematician of the 19th century, who corresponded with numerous European mathematicians of his time. He was also the author of the remarkable and unsurpassable *Traité des Courbes Spéciales Remarquables Planes et Gauches*, in three of the seven volumes of its *Obras* with over 1300 pages, and he was the first rector of the University of Porto, between 1911 and 1918.

The foundation of the scientific journal *Portugaliae*



Figure 10.—The first original book on the popularisation of mathematics, *Conceitos Fundamentais da Matemática*, had several edition after its first publication in Lisbon in 1941, integrated in the relabel collection of scientific culture Biblioteca Cosmos, funded by Bento de Jesus Caraça.



Figure 11.—The first volume of *Portugaliae Mathematica*, published by António Monteiro, started to be printed in Porto in 1937 and it was completed in 1940 in Lisbon.

Mathematica in 1937 in Lisbon, by António Aniceto Monteiro, who returned the year before from Paris where he had completed his doctoral degree under Maurice Fréchet, started a modernist movement also in Science with a certain “mathematical effervescence”, which lasted a short decade in Portugal. It was followed by the *Gazeta da Matemática*, in 1940, which was later printed by the *Tipografia Matemática*, and by the creation of the first research mathematical center in the country, the *Centro de Estudos Matemáticos de Lisboa*, also in that year. That unique typography was established in 1945 and had a remarkable, although limited, activity in the Portuguese mathematical press for more than three decades.

During that short decade some mathematical activity flourished in Portugal. For example, the remarkable little

book *Conceitos Fundamentais da Matemática*, by B. J. Caraça, had a first edition in 1941 and was the first Portuguese book aimed at the popularization of mathematics in a perspective of the “integral culture of the individual”. More advanced publications, like two doctoral theses printed at the *Tipografia Matemática*, are illustrated in the exhibition with the *Publicação #18* of the *Centro de Estudos Matemáticos do Porto*, by A. Pereira Gomes, which was the first modern PhD thesis in a Portuguese university, or *As Funções Analíticas e a Análise Funcional*, by José Sebastião e Silva, both published in *Portugaliae Mathematica*, respectively in 1946 and in 1950. In this work, J. Sebastião e Silva starts deep contributions to Functional Analysis, which will lead him to introduce in 1955 an important class of the locally convex spaces as

GAZETA DE MATEMÁTICA

PUBLICADA POR

A. MONTEIRO, B. CARAÇA, H. RIBEIRO, J. PAULO, M. ZALUAR

1.º ANO - N.º 1 - PREÇO DESTA NÚMERO: 3000 em JANEIRO 1940
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Figure 12.—The first issue of *Gazeta de Matemática*, published in Lisbon in 1940, was mainly dedicated to the first years university students in science, engineering and economics and became later the Mathematics Magazine of the Portuguese Mathematical Society

Figure 13.—The volume 44 of *Portugaliae Mathematica*, was published in its fiftieth anniversary already by the Portuguese Mathematical Society in 1987 in Lisbon, and it was already composed in *TeX*.

inductive limits of an increasing sequence of normed spaces with compact inclusions, later called the Silva LN^* -spaces.

After five centuries of existence, the mathematical typography of movable type, with its specific and distinctive aspects, such as tables, figures, diagrams and mathematical formulae, gave way to electronic publishing driven by the *TeX* program, created by Donald Knuth in 1978. This electronic composition system was adopted by the American Mathematical Society five years later and was used in 1987 by the *Sociedade Portuguesa de Matemática* (SPM) in the publication of the fiftieth anniversary volume 44 of *Portugaliae Mathematica*, the last book of this exhibition.

This first edition of this exhibition, with these 32 significant works of Portuguese mathematical typography, many of them being rare books and containing unknown or still poorly known very interesting pages, grouped in nine thematic showcases, allowed for an illustrated visit to the main milestones of five centuries of the History of Mathematics in Portugal. It is expected that more editions of the *Tipografia Matemática Portuguesa: 1496-1987* will be organised in the next years by the Universities of Porto, Coimbra and Lisbon in collaboration with the CIM and the SPM.

- EXPOSITOR 1**
TABELAS MATEMÁTICAS PARA A NAVEGAÇÃO/REGRAS ARITMÉTICAS PARA O COMÉRCIO ULTRAMARINO
1 — 1496 - *Almanach Perpetuum*, Abraão Zacuto
2 — 1519 - *Tratado da pratica Darismetyca*, Gaspar Nycolas
- EXPOSITOR 2**
MATEMÁTICAS DA NAVEGAÇÃO, DA OBSERVAÇÃO DOS CÉUS E DA CARTOGRAFIA
3 — 1537 - *Tratado da Sphera*, Pedro Nunes
4 — 1542 - *De Crepusculis*, Pedro Nunes
5 — 1573 - *De arte atque ratione navigandi*, Pedro Nunes
- EXPOSITOR 3**
ENSINO E USO PRÁTICO/MILITAR DA MATEMÁTICA ANTES/DEPOIS DA RESTAURAÇÃO
6 — 1634 - *Elementos Mathematicos*, Ignacio Stafford
7 — 1680 - *Methodo Lusitanico*, Luis Serrão Pimentel
8 — 1728 - *O Engenheiro Portuguez*, Manuel de Azevedo Fortes
9 — 1838 - *Carta Físico-Mathematica de 1769*, José Anastácio da Cunha
- EXPOSITOR 4**
ENSINO NO COLÉGIO DOS NOBRES/UNIVERSIDADE/COLÉGIO DE SÃO LUCAS/ACADEMIAS MILITARES
10 — 1768 - *Elementos*, Euclides
11 — 1773 - *Elementos de Arithmetica*, É. Bézout
12 — 1790 - *Principios Mathematicos*, José Anastácio da Cunha
13 — 1798 - *Theorica das Funções Analyticas*, J. L. Lagrange
14 — 1812 - *Tratado Elementar de Calculo Differential*, S. F. Lacroix
- EXPOSITOR 5**
SOBRE A UTILIDADE/FUNDAMENTO DAS CIÊNCIAS MATEMÁTICAS PARA A "GLÓRIA NÃO SER Vã"
15 — 1797 - *Memórias da Academia Real das Sciencias de Lisboa*, Tomo 1
16 — 1851 - *História e Memórias da Academia R. das Sciencias de Lisboa*, vol.III
17 — 1867 - *Jornal de Sciencias Mathematicas Physicas e Naturaes*, Tomo I, nº 3
- EXPOSITOR 6**
PUBLICAÇÕES PERIÓDICAS COM E DE ARTIGOS DE CIÊNCIAS MATEMÁTICAS
18 — 1856 - *O Instituto*, vol. 4, *Ensaio sobre Os Principios da Mechanica*, J. Anastácio da Cunha
19 — 1877 - *Jornal de Sciencias Mathematicas e Astronomicas*, vol. 1, publicado por F. Gomes Teixeira
20 — 1955 - *Revista da Faculdade de Ciências, Universidade de Lisboa*, 2ª série, A, vol. IV
- EXPOSITOR 7**
MANUAIS DE MATEMÁTICAS SUPERIORES NA TRANSIÇÃO DO SÉC. XIX PARA O XX
21 — 1838 - *Curso Completo de Mathematicas Puras*, tomo primeiro, L.-B. Francoeur
22 — 1887 - *Curso de Analyse Infinitesimal*, Francisco Gomes Teixeira
23 — 1933 - *Curso de Álgebra Superior*, José Vicente Gonçalves
24 — 1973 - *Lições de Análise Infinitesimal, II. Cálculo Integral em \mathbb{R}^n* , Fernando R. Dias Agudo
- EXPOSITOR 8**
DA COMPILAÇÃO HISTÓRICO-EXPOSITIVA, À DIVULGAÇÃO E À INVESTIGAÇÃO MATEMÁTICA
25 — 1909 - *Traité des Courbes Spéciales Remarquables*, Tomo II, F. Gomes Teixeira
26 — 1926 - *Fundamentos de Geometria Diferencial*, Aureliano de Mira Fernandes
27 — 1941 - *Conceitos Fundamentais da Matemática*, vol. 1, Bento de Jesus Caraça
28 — 1946 - *Publicações do Centro de Estudos Matemáticos do Porto*, nº 18
- EXPOSITOR 9**
PERIÓDICOS DA SOCIEDADE PORTUGUESA DE MATEMÁTICA
29 — 1940 - *Portugaliae Mathematica*, vol. 1, Fundada por António Monteiro
30 — 1937 - *Gazeta da Matemática*, 1º ano nº 1
31 — 1950 - *As Funções Analíticas e a Análise Funcional*, José Sebastião e Silva
32 — 1987 - *Portugaliae Mathematica*, vol. 44, 1º composto em TeX



Figure 14.—The list of the 32 original books of the Leiria exhibition was organised in nine thematic topics, one per showcase: <https://www.ipleiria.pt/sdoc/tipografia-matematica-portuguesa-1496-1987/>
The online Catalogue, in Portuguese, can be read in <https://online.anyflip.com/yuqqw/btnw/mobile/index.html?1624885994427>