Mathematics and Music
25 years after the Diderot Mathematical Forum

by **José Francisco Rodrigues***

At the beginning of the third millennium of our era, Leibniz's famous expression, "Musica est exercitium arithmeticæ occultum nescientis se numerare animi" (Music is a hidden arithmetical exercise of a mind unconscious that it is counting) can be taken in a broad sense in a contemporary conception of art and science. In the European universities, since the middle ages, Music was part of the studies and Portugal was not an exception. The *Studium Generale* of Lisbon, instituted by King Dinis in 1290, marking the foundation of the Portuguese University, included the medieval version of the Liberal Arts of the Trivium (grammar, rhetoric and dialectic) and the Quadrivium (arithmetic, geometry, astronomy and music).

Nowadays the classical *Mathematical Physics* area does not appear explicitly in the *Mathematics Subject Classification*, but, of course, it currently includes a wide variety of areas and sub-areas of mathematics that are contained in that classification. In contrast, the somehow older subject *Mathematics and Music* has integrated that classification, under the reference 00A65, only since 2010. On the other hand, the *Journal of Mathematics and Music* has been publishing articles on computational and mathematical approaches to music composition, analysis and theory since 2007.

The European Mathematical Society (EMS) has promoted the *Fourth Diderot Mathematical Forum* simultaneously in Lisbon, Paris and Vienna, on 3–4 December

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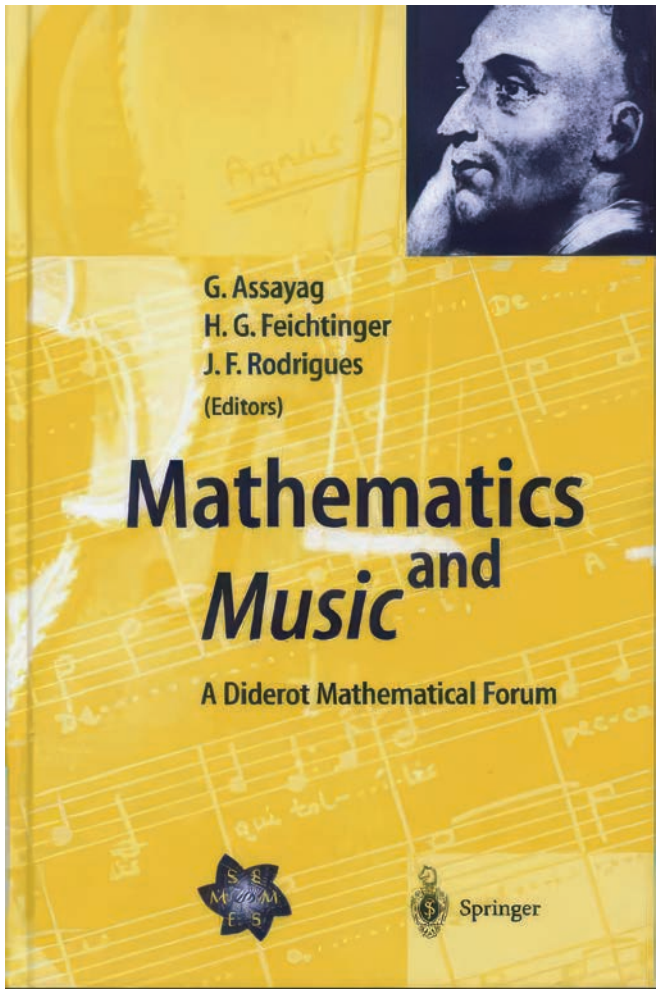
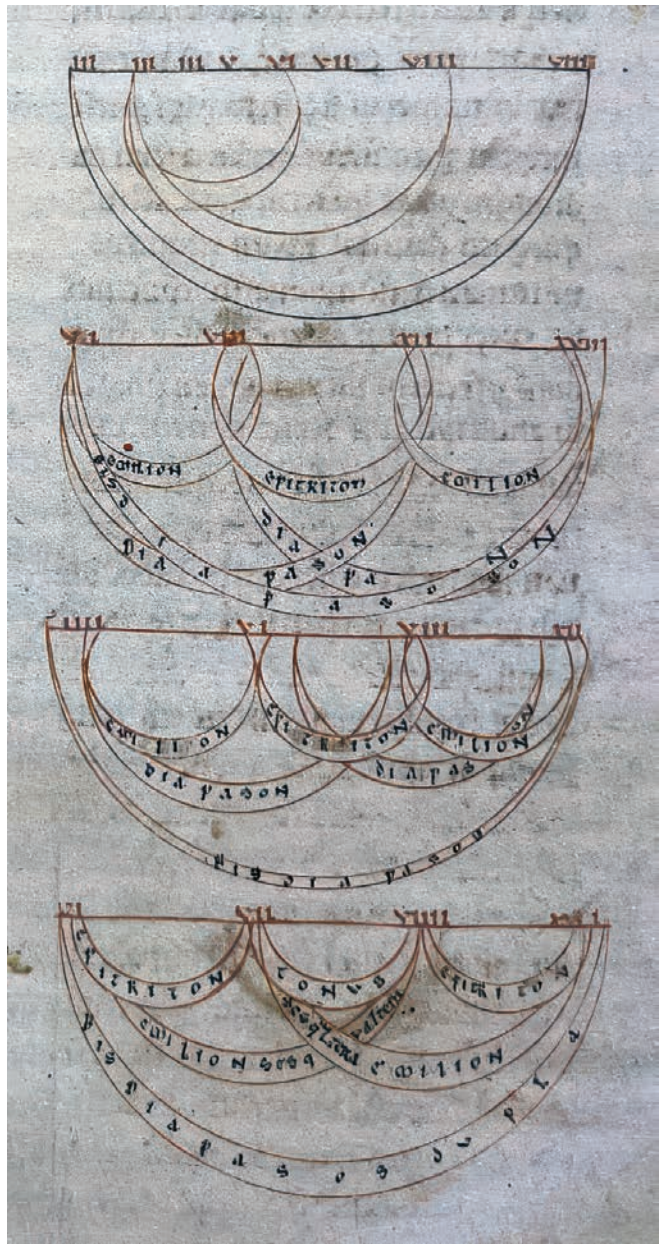




Book engraving from *Etymologiae*, by Isidore of Seville, belonging to the *Manuscrito de Santa Cruz* (12.° 17, fl.31 at *Biblioteca Pública Municipal do Porto*) with divisions of the monochord, in the medieval style of *De Institutione Musica* by Boethius (c. 475–524).

1999, which consisting of several conferences in parallel and, as a main feature of that Forum, a joint teleconference among the three cities, which took place under the theme "The relations between Mathematics and Music are natural or cultural relations?" The series of those *EMS Forums* took the name of the French philosopher Denis Diderot, who was the editor of the *Encyclopédie*, that he co-founded with the mathematician Jean D'Alembert (who also wrote on music), where he wrote "C'est par les nombres et non par les sens qu'il faut estimer la sublimité de la musique. Etudiez le monocorde." (It's by numbers and not by the senses that one should evaluate the sublimity of music. Study the monochord.), in the very classical Pythagorean tradition.

The contributions presented at that 1999 pioneering conference on *Relationships between Mathematics and Music* were published in 2002 in the influential book [MM] covered three complementary directions: "Historical Aspects", the topic addressed in Lisbon at the *Fundação Calouste Gulbenkian*; the "Mathematical Logic and Music Logic in the 20th century" at the IRCAM-*Centre Georges Pompidou*, in Paris, and the "Mathematical and computational methods in Music", at the *University of Vienna*.

The first of five texts on the historical topic of [MM], Proportion in Ancient and Medieval Music, by M. P. Ferreira, deals with the Pythagorean theory, from the Greek heritage and the Latin world to the late-medieval France and the decline of proportional thinking. The chapter by E. Knobloch on *The Sounding Algebra: Relations Between Combinatorics and Music from Mersenne to Euler* highlights the role of the Mersenne's *Harmonia Universalis* (1636) for the baroque music, when "to compose" was equivalent with "to combine", up to the Euler's contributions, with a reference to the Mozart's *Musicaliaches Würfelspiel* (Musical game of dice). The third article, by B. Scimemi, explains *The Use of Mechanical Devices and Numerical Algorithms in the 18th Century for the Equal Temperament of*

the Musical Scale, from Zarlino and Tartini up to Strähle and Schröter. In the next article, J. Dhombres recalls Lagrange, *"Working Mathematician", on Music Considered as a Source for Science*, and the last one in this first part of that book presents some *Musical Patterns*, by W. Hodges and R. J. Wilson, with illustrations of mathematical devices, like canon, expansion, retrograde motion and inversion, used in music writing by composers.

The seven contributions in [MM] from the Paris meeting, going in general into an anti-Pythagorean direction, started with *Questions of Logic: Writing, Dialectics and Musical Strategies*, by F. Nicolas, followed with *The Formalization of Logic and the Issue of Meaning*, by M.-J. Durand-Richard, with *Musical Analysis Using Mathematical Proceedings in the XXth* Century, by L. Fichet, with *Universal Prediction Applied to Stylistic Music Generation*, by S. Dubnov and G. Assayag, evolving into *Ethnomusicology, Ethnomathematics, The Logic Underlying Orally Transmitted Artistic Practices*, by the ethnomusicologist M. Chemillier, or into cognitive musicology with *Expressing Coherence of Musical Perception in Formal Logic*, by M. Leman. The last chapter on *The Topos Geometry of Musical Logic*, by G. Mazzola, searches for connections between the logic of musical composition and analysis with abstract algebraic geometry and logic structures.

In the last part of [MM], J.-C. Risset in *Computing Musical Sound* shows how mathematics is the pervasive tool of the computational craft of musical sound up to real-time musical performance, while E. Neuwirth gives an overview on The *Mathematics of Tuning Musical Instruments — a Simple Toolkit for Experiments*, the computer musicologist X. Serra describes *The Musical Communication Chain and its Modeling*, using contributions from music, electrical engineering, psychology and physics, and, completing the book, G. De Poli and D. Rocchesso review some of the most important *Computational Models for Musical Sound Sources* based on physical models and mathematical descriptions of sound sources, which are natural extensions of the classical cooperation and interaction between science and music.

In the comprehensive and positive review of the book of that fourth Diderot Forum, S. Perrine in [P] acknowledges "a new alliance between music and mathematics" and states his conviction "that other Mathematics and Music initiatives need to be taken, and that there is no lack of topics to be covered".

In that year of 1999 that preceded the World Mathematical Year, WMY2000, and in fact to announce it, the Portuguese magazine of scientific culture COLÓQUIO/CIÊNCIAS published a series of articles dedicated to the interactions of Mathematics and Music. A first one [R] was a brief general introduction to the theme and to a series of seven conferences held in Lisbon, one per month, from January until July of 1998, at the *Fundação Calouste Gulbenkian* in Lisbon, organised with the collaboration of the *Centro de Matemática e Aplicações Fundamentais* of the University of Lisbon, which gave origin to a full

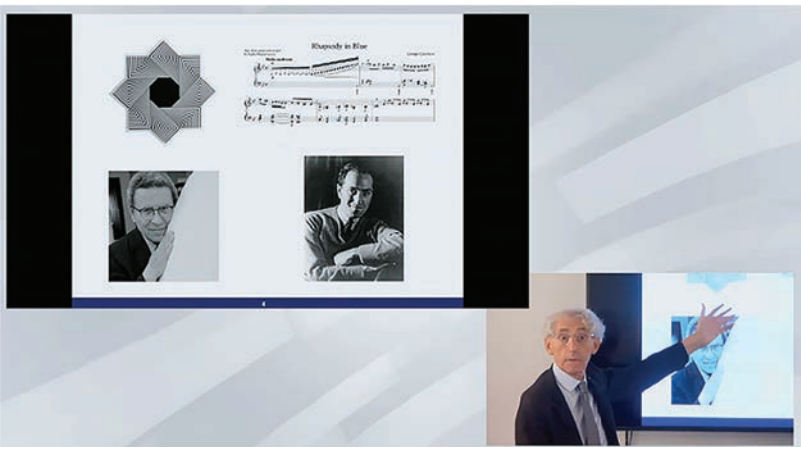
The Tonnetz explained by Andreas Matt to the author in Heidelberg the 6 July 2019.

issue of the COLÓQUIO/CIÊNCIAS magazine [CC]. Later, in 6–7 October 2006, the *Centro de Matemática da Universidade do Porto* organised a meeting on Música e Matemática, which included two concerts hold in *Casa da Música* of Porto and produced an interesting book [B] with thirteen contributions.

Actually, the article *A Matemática e a Música* [R] had the concept of an ambitious exhibition for the WMY2000 behind it, which proved impossible because it had too high a budget. The concept was based on the historical and conceptual connections and required, in addition to instruments and physical objects, several interactive exhibits to be developed in collaboration with an enthusiastic group of international experts.

Essentially the exhibition concept was composed in four movements connecting music with four mathematical areas: *Pythagorean Arithmusic*, based on the classical proportions and numbers related to harmonies (musical and celestial); *Algebra of Tones*, from the different temperaments to combinatorics and the musical symmetries; *Harmonisation of Analysis*, on the nature of the propagation of sound and the construction of instruments and *Digital Musurgia*, where, in the computer era, it is possible to produce music by calculating numbers. Indeed, if today we have mastered numerisation in the analysis and synthesis of musical sound, if we have begun to outline the mathematisation of certain musical structures and computers allow us to hear mathematical calculations and structures, i.e. paraphrasing Saccheri, we have *Pythagoras ab omni naevo vindicatus sive Conatus arithmeticus quo stabiliuntur prima ipsa universæ musicæ principia* (Pythagoras freed from all taint or the arithmetical attempt to establish the first principles of all music) and we can continue to agree with the Greek philosopher of the 4th century BCE Aristoxenus of Tarentum and accept that the justification of music lies in the pleasure of hearing it and enjoying it.

In this century, the increase of events, articles and books on mathematics and music is showing that, indeed,



The mathematician Alfio Quarteroni showing how to play Morandini's pictures at Villa Toeplitz (RISM), Varèse, in 2021.

there is no lack of interesting topics to be covered, as it was observed before in several forums. In particular, the recent exhibition *LaLaLab–The Mathematics of Music* [LLL], organised by the *Imaginary*, which has the *Mathetisches Forschunginstitut Oberwolfach* as a shareholder, was shown in Heidelberg (May 2019–December 2020) and has about two dozen of interesting interactive exhibits, free and available online. This successful exhibition was based over music theory, current research in the connection of mathematics and music and also over art and entertainment, including artworks, talks and concerts.

Besides Music, as an artistic expression, Mathematics also interacts with other arts, like Architecture, Painting or Sculpture. An interesting recent paper [GQC], by P. Gervasio, A. Quarteroni and D. Cassani, using a mathematical method based in Fourier and Wavelet transform to extract similarities between image signals and audio signals, allowed, by minimizing a certain distance, not only to associate a given painting from a specific artist with music tracks, with the possibility of choosing the "optimal" one in a certain sense, but also "to generate the new music, is the most similar one to the painting chosen, in terms of intrinsic features". Those authors have also developed an original (and free) applet, which was applied to play some artworks of the Italian artist Marcelo Morandini.

Recently, the Institute of Mathematical sciences of the National University of Singapore organised a conference at the Yong Siew Toh Conservatory of Music, during the week 19-23 February 2024, celebrating the seminal event that took place in 1999 simultaneously in Lisbon, Paris and Vienna. The *Mathemusical Enconters in Singapore: a Diderot Legacy* consisted of about twenty two talks on different topics, such as, Mathematical and Computational approaches (Day 1), Machine Learning, Generativity, Interaction (Day 2), Computational Physiology/Medicine (Day 3), Education, Learning and Creativity (Day 4) and a Student Session during the morning of the fifth day. In addition, there were five concerts and recitals at the Conservatory Concert Hall and a Round Table, in hybrid format, on the Diderot Forum legacy and future perspec-

P. Codognet, J. F. Rodrigues, H. Chew and G. Assayag during the round table at the Mathmusical Encounters in Singapore, the 22 February 2024.

Moreno Andreatta introducing the interactive applet *Tonnetz* at the Atlantic Journey in Berlenga Island.

tives with an open online discussion with members of the Society for Mathematics and Computation in Music.

Immediately after the 9th International Congress on Mathematics and Computation in Music (MCM2024), that had taken place a the University of Coimbra, the 18–21 June 2024, the *Centro de Matemática, Aplicações Fundamentais e Investigação Operacional*, of the *Faculdade de Ciências da Universidade de Lisboa*, in collaboration with the *Centro Internacional de Matemática*, the National Agency for Scientific and Technological Culture – *Ciência Viva*, the *Academia das Ciências de Lisboa* and the *Associação Amigos da Berlenga*, organised an *Atlantic Journey on Mathematics and Music* at the Berlenga island the 22 June 2024. This small meeting was an extraordinary opportunity to get acquainted with some of the interactions of mathematics and music, not only through the six talks, covering from *Mathematics and Music in Historical Context*, by J. F. Rodrigues, *Music and Symmetries — from Bach to Jazz*, by C. Simões, *Symmetries and other mathematical beauties in music*, by E. Amiot, and *Conceptualising Tonality: Algebraic versus Statistical Approaches*, by T. Noll, and *Around theTonnetz*, by M. Andreatta, up to a talk and installation on *The MatheMusical Virtual Museum*, by G. Baroin, complemented with a Round Table on *How to make an Exhibition on Mathematics and Music?*, moderated by C. Florentino and R. Vargas, the Director of *Ciência Viva*, and the remote participation of D. Ramos, Chief Content Officer of the *Imaginary*. Possible future plans for 2026 were discussed in the Round Table.

References

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[P] Perrine, S. Mathematics and music. a Diderot Mathematical Forum. *The Mathematical Intelligencer* 27, 69–73 (2005). https://doi.org/10.1007/BF02985844

[GQC] Gervasio, P., Quarteroni, A. and Cassani, D. *Let the paintings play*, arXiv:2206.14142, 2022, arXiv:2206.14142v3

[R] Rodrigues, J. F. *A Matemática e a Música*, COLÓQUIO/CIÊNCIAS n. 23, Julho 1999, pp. 17-32.

[CC] COLÓQUIO/CIÊNCIAS n. 24, Novembro 1999 (in Portuguese): I. Grattan-Guiness, *Mozart 18, Beethoven 32: Hidden shadows of integers in classical music*; E. Behrends, *Music: Structure and Chance*; G. Assayag, *Mathematics, the number and the computer*; G. Mazzola, *Listen to Mathematics?*; C. Simões, *The order of numbers in 20th century music*; B. Scimemi, *Musical counterpoint and geometric transformations*; J.P. Bourguignon, *Is it possible to hear the shape of a drum?*

[B] Encontro Música e Matemática — Actas, CMUP – Centro de Matemática do Porto, 2006.

[LLL] https://www.imaginary.org/exhibition/la-la-lab-the-mathematics-of-music