Portuguese texts on the calculus in the 18th century

by **João Caramalho Domingues***

Having appeared at the end of the seventeenth century, the calculus (differential and integral calculi for Leibniz, method of fluxions for Newton) was undoubtedly the most important branch of pure mathematics in the eighteenth century. This importance was recognized at the time:

"one of the most wonderful inventions in mathematics, which not only raised geometry almost to its highest peak, but also expanded the other disciplines to such an extent that one would have to write entire books if one wanted to specify the benefits of this calculus" [Zedler, 1731–1754, vol. 5, col. 190].

"Of all the discoveries that have ever been made in the sciences, there is none as important, nor as fruitful in applications, as that of infinitesimal analysis" [Bossut, 1784, lxxii].

This is also a common view among modern-day historians:

"Considered broadly, mathematical activity in the eighteenth century was characterized by a strong emphasis on analysis and mechanics. The great advances occurred in the development of calculus-related parts of mathematics and in the detailed elaboration of the program of inertial mechanics founded during the Scientific Revolution" [Fraser, 2003, 305].

"The Enlightenment in Mathematics is defined by the level achieved in the mastery of the new differential and integral calculus [...] The mark of the modernity of a work is the use made of the calculus, and undoubtedly a work whose content does not include the calculus can be said to be outdated" [Ausejo & Medrano, 2010, 26].

For all its centrality in eighteenth-century European mathematics, the adoption of the calculus in Portugal was slow [Domingues, 2021; to appear]. Before 1760, only a few isolated cases can be found of Portuguese individuals knowing about the calculus, and in each case one may wonder how profound was such knowledge. In the 1760s there were a couple of attempts at introducing the calculus into mathematical teaching, but they were not fruitful. The first successful case of the calculus being taught in Portugal occurred only as a result of the 1772 reform of the University of Coimbra, which created a Faculty of Mathematics (in section 2 we will look at the textbook used in that context).

1 Jacob de Castro Sarmento's explanation of fluxions (1737)

The very first text in Portuguese about the calculus was very short and non-technical.

It was written by Jacob de Castro Sarmento (born Henrique de Castro, 1691–1762), a "New Christian" physician who escaped to London in 1721 fleeing the

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Inquisition and there converted to Judaism [Goldish, 1997]. In 1737 he published in London, but in Portuguese, a book on the Newtonian theory of tides which also includes an eulogy of Newton and a glossary of scientific words. Sarmento's treatment of tides does not involve any calculus, but the largest entry in the glossary is precisely "fluxions" (the Newtonian equivalent of differentials) [Sarmento, 1737, 129–131]. In two and a half pages, Sarmento expounds the Newtonian point of view of a line being generated by the motion of a point, a surface being generated by the motion of line, and a solid being generated by the motion of a surface; the velocity of each of these motions is the *fluxion* of the line, surface, or solid, while the line, surface, or solid is the *fluent* of that fluxion. The Direct Method of Fluxions is used to find the fluxion of any quantity, given the fluent (which is the quantity itself); the Inverse Method of Fluxions is used to find the fluent, given the fluxion. Finally, the direct method is useful in drawing tangents, solving problems of maxima and minima, etc.; while the inverse method is useful in calculating arclengths, areas and volumes. They have also plenty of use in physics and astronomy.

In short, this is a concise explanation for a layperson, not an introduction to the subject, by any means.

2 Bézout's textbook

2.1 The first translation (1774)

It was only as a consequence of the 1772 reform of the University of Coimbra that an introductory text on the calculus was published in Portuguese. This was a translation of a then recent text by the Frenchman Étienne Bézout (1730–1783).

Bézout, an examiner of the French navy schools, published between 1764 and 1769 a *Cours de Mathématiques* in six volumes (containing arithmetic, geometry and trigonometry, algebra, calculus, mechanics, and navigation) for the students of those schools. This course was hugely successful, and over the following decades either the full set or extracted parts were reprinted numerous times and translated into several languages.

For the new Faculty of Mathematics created in Coimbra in 1772, José Monteiro da Rocha (1734–1819), one of the main main people involved in the establishment of the Faculty, translated the first volume of Bézout's course, on arithmetic, and the section on plane trigonometry from the second volume — both to be used in the first year of mathematics. He also translated a textbook on mechanics by a different French author, to be used in the third year.

For the second year, which included algebra and the calculus, the parts from Bézout's course on these subjects were adopted. Their translations were published as volumes 1 and 2 of [Bézout, 1774]. It is not known who translated them into Portuguese, although by the 19th century it was said that the translator had been Fr. Joaquim de Santa Clara (1740–1818) — a Benedictine who graduated in theology but who also taught philosophy and mathematics in the early 1770s. Be as it may, there is a marked difference between this translation and those made by Monteiro da Rocha: while the latter adapted several passages and included additional material as he saw fit, [Bézout, 1774] is a very literal translation.

The volume on the calculus, [Bézout, 1774, II], presents a traditional Leibnizian version of the subject, with a strong geometrical tendency — particularly in the differential calculus.

Bézout casually accepts the existence of infinitely large and infinitely small quantities. His variable quantities increase (or decrease) by infinitely small degrees. Thus, the *differential* of a quantity is defined as the infinitely small difference between the values of that quantity in two consecutive moments. For instance, the differential of xy is $x\,dy + y\,dx$, because the difference between two consecutive states of xy is $(x+dx)(y+dy)-xy = x\,dy+y\,dx+dy\,dx$, and $dy\,dx$ must be omitted because it is infinitely small with regard to both $x\,dy$ and $y\,dx$. Accordingly, in order to calculate tangents, Bézout conceives a curve as a polygon with an infinite number of infinitely small sides. The tangent is the prolongation of one of these sides.

About two thirds of the differential calculus are taken up with geometrical applications — more precisely, applications to the study of curves: not only tangents (subtangents, subnormals) but also topics such as multiple points, points of inflexion, cusps, and radii of curvature. Yet another geometrical application is the determination of maxima and minima, which are treated as largest and smallest ordinates, so that the condition $dy/dx = 0$ comes from the tangent to a curve being parallel to the abscissas.

Another aspect of Bézout's calculus, consistent with this predominance of geometry, is the relative unimportance of the concept of function: the word "function" is first used 65 pages after "differential", in a section on multiple points of curves, as a mere abbreviation. Throughout the differential calculus, the object of study are *quantities*, fully geometrical or rep-

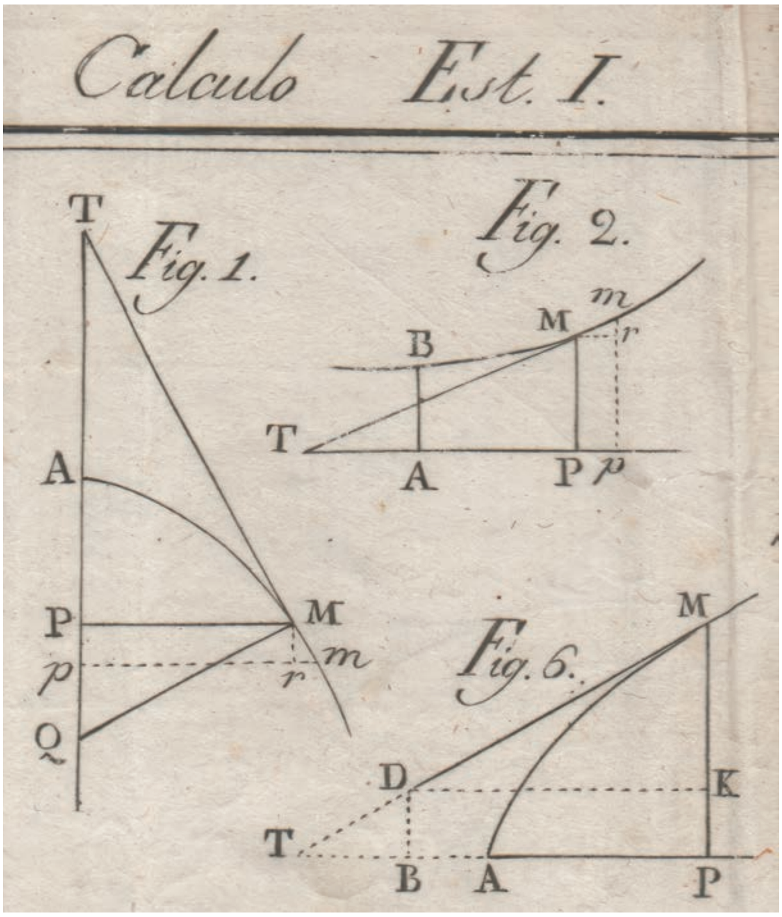

Figure 1.—Three figures from [Bézout, 1794] (similar to figures appearing in [Bézout, 1774]). In Fig. 1, line TM is tangent to the curve AM, and is obtained prolonging the infinitely small side Mm of this curve, regarded as a polygon; AP is the abscissa x, PM the ordinate y, $Pp = Mr = dx$, $mr = dy$; Mrm is an infinitely small triangle, similar to the finite triangle TPM, whence the subtangent PT is equal to $y\,dx/dy$.

resented geometrically.

This geometrical point of view was the norm at the time, in calculus textbooks all over Europe. The only major exception was Euler's treatises on the calculus (published between 1748 and 1770), where the primary object of study were *functions*. But these were treatises, not textbooks.

Still, it must be said that Bézout's integral calculus is much more analytical than his differential calculus: the integral is essentially defined as what we would call an antiderivative (which, again, was the most common approach at the time), so that the integral calculus deals naturally with expressions from the start; accordingly, the word "function" receives a definition at the beginning of the integral calculus[1]. There are some geometrical applications (areas, arc lengths and some volumes) but they occupy only about one fifth of the integral calculus.

2.2 The second translation (1794)

[Bézout, 1774] was too much of a literal translation, even including references to parts of Bézout's course that had not been translated nor adopted in Coimbra. In the 1790s, the same parts of Bézout's course were translated again, from scratch, by José Joaquim de Faria (1759–1828), who was at the time a substitute professor at the Faculty of Mathematics. The calculus volume appeared as [Bézout, 1794].

Unlike the previous one, this new translation is far from literal. There are minor adaptations to accommodate the text to the series of textbooks in use at the University, several calculations or arguments are shortened, and there are two small but relevant attempts to modernize the differential calculus: the word "function" is introduced somewhat earlier and is used more often; and a new (short) section is added, on Maclaurin and Taylor series.

At about the same time that this second translation came out, in France the calculus as a subject of teaching was changing dramatically. A new version of the calculus, inspired by Euler and Lagrange, and much more analytical than Bézout's, was being taught at the *École polytechnique*, founded in 1793. The changes introduced by José Joaquim de Faria were small steps in the same direction — certainly not caused by the new

[1] After having already been used, as was noticed above.

trends spreading from the *École polytechnique*; more likely, resulting from similar motivations, namely the growing gap between advanced, research-level works, which were typically very much analytical, and traditional introductory texts such as Bézout's. But Faria's changes were not enough to fundamentally change what was quickly becoming an outdated textbook.

3 José Anastácio da Cunha's fluxionary calculus

The earliest introduction to the calculus originally in Portuguese that is still extant[2] was written by the most original Portuguese mathematician of the 18th century, José Anastácio da Cunha (1744–1787), and it contains a remarkable definition of "fluxion" that has been described as the first rigorous analytic definition of the differential.

Cunha's introduction to the calculus was included in [Cunha, 1790], a short (little over 300 pages), very concise but very comprehensive, introduction to mathematics, from elementary geometry to some calculus of variations, organized in a logical way. This was a posthumous publication, and its text was never really finished, but as far as the calculus section is concerned, a manuscript text dated 1780 is known containing some of its main ideas.

Some of Cunha's personal opinions (such as his admiration for Newton, who preferred a geometrical style over algebra, his dislike of Euler, his distrust of conclusions drawn exclusively from analytical arguments) might lead us to expect from him a geometrically inclined version of the calculus. However, what we find is mostly analytical, albeit in an original way [Domingues, 2023]. It is certainly much more analytical than Bézout's calculus, and betrays more influence from Euler than Cunha would probably like to admit.

The book [Cunha, 1790] is divided into 21 "books" (so called following the Euclidean fashion; we would call them chapters). The calculus is introduced in "book" 15.

Book 15 opens with a crucial definition: "if an expression can assume more than one value, while another can assume only one, the latter will be called constant, and the former variable". This may seem trivial to a modern reader, but it was at the very least extremely unusual in the 18th century: a variable was almost always regarded as a quantity (not an expression) that *varied* (presumably over some sort of implicit time).

The second definition is built on the first one: "a variable always capable of assuming a value smaller than any proposed magnitude will be called infinitesimal". This means that, instead of infinitesimals being infinitely small quantities, as was then commonly the case, they are simply *expressions* that can assume arbitrarily small (but finite) values. In practice, Cunha's statements involving infinitesimals have the form "x infinitesimal makes $f(x)$ infinitesimal", which is equivalent to $\lim_{x\to 0} f(x) = 0$. Proposition 1 of book 15 states that if x is infinitesimal then $Ax + Bx^2 + Cx^3 + \&c.$ is also infinitesimal, and its proof is (as long as $Ax+Bx^2+Cx^3+\&c.$ is interpreted as a polynomial) an impeccable ε-δ argument.

The third definition is that of function: an expression A is a function of another expression B if the value of A depends on the value of B. This is not so remarkable, but it is worth noticing that "function" is defined at the outset (compare with what was said above about Bézout's text), which allows for the calculus to be about functions.

But the big highlight is the fourth definition, that of fluxion:

"Some magnitude having been chosen, homogeneous to an argument x, to be called fluxion of that argument, and denoted by dx; we will call fluxion of Γx, and will denote by $d\Gamma x$, the magnitude that would make $d\Gamma x/dx$ constant and $(\Gamma(x + dx) - \Gamma x)/dx - d\Gamma x/dx$ infinitesimal or zero, if dx were infinitesimal and all that does not depend on dx constant".

Notice that Cunha seems to combine the two main traditions in the calculus: the word "fluxion" is Newtonian, while the notation dx, $d\Gamma x$ is Leibnizian. However, this definition does not belong in either tradition. Youschkevitch [1973] said of it that "it was Cunha who, for the first time, formulated a rigorous analytical definition of the differential, taken up again and used later by the mathematicians of the nineteenth century". Mawhin [1990] was more specific, saying that it "corresponds to the modern definition of differential": $d\Gamma x$ is a linear function of dx (since $d\Gamma x/dx$ is constant) such that

$$\lim_{dx\to 0} \frac{\Gamma(x + dx) - \Gamma x - d\Gamma x}{dx} = 0.$$

[2] José Monteiro da Rocha is known to have written an introduction to the calculus in the 1760s, that was never published. The manuscript was at the Academy of Sciences of Lisbon in 1825, but its present whereabouts is unknown.

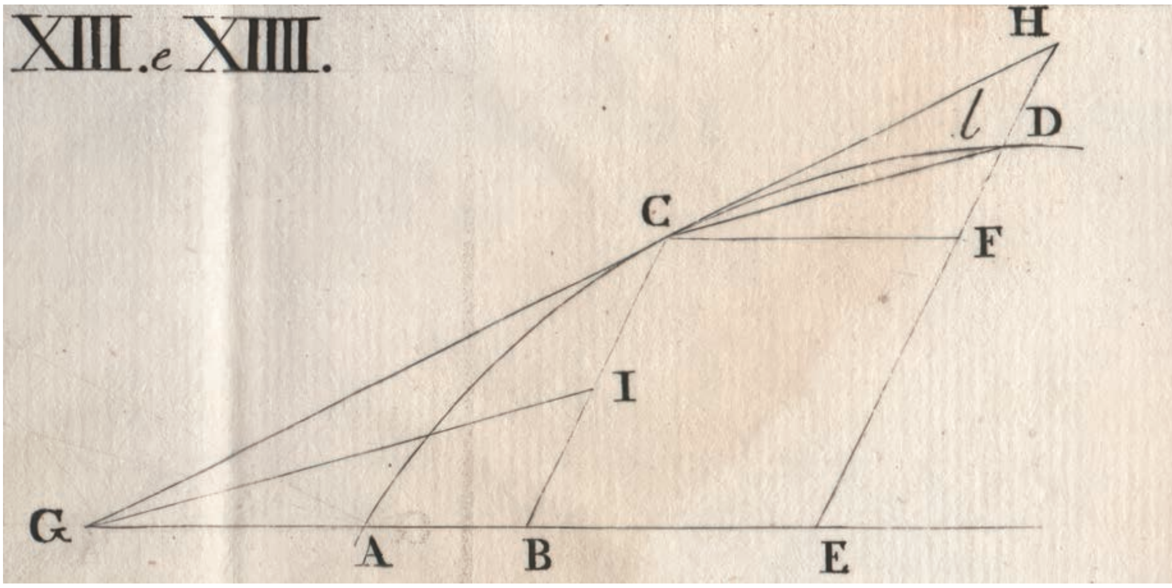

Figure 2.—A diagram for propositions 13 and 14 from book 15 of [Cunha, 1790]. *AB* is an abscissa and *BC* an ordinate (oblique coordinates!) of a curve *AD*. In prop. 13, Cunha proves that if *BE* is the fluxion of the abscissa *AB*, then parallelogram *BF* (that is, the parallelogram with diagonal *BF*) is the fluxion of the area *ACB*; for that, he assumes that the ordinate function is monotonic and trusts the diagram to convince the reader that area *CDF* is contained in the parallelogram with diagonal *CD* (not drawn): BF/BE is the height of parallelogram *BF*, and calling $(BF/BE)+\pi$ the distance from *D* to the axis of abscissas, it will be $CDF/BE<\pi$; this means that *AB* constant and *BE* infinitesimal would make BF/BE and $(ADE-ACB)/BE-BF/BE(=CDF/BE<\pi)$ infinitesimal, fulfilling the conditions in the definition of fluxion.

Such correspondence is not complete: for instance, $d\Gamma x$ is not explicitly stated to be a function of dx; and the existence of $d\Gamma x$ is not questioned — like all his contemporaries, Cunha assumed all functions to be differentiable. However, Cunha is definitely closer to a modern definition of differential than the usual definitions in the 18th century.

The propositions in book 15 can be roughly divided into two groups. Up to proposition 12, we find, in a very concise way, the fundamentals of what we call differential calculus: for instance, $d(x^n) = n\,x^{n-1}dx$ (prop. 2), $dx = x\,dlx$, l standing for hyperbolic (that is, natural) logarithm (prop. 8), the Taylor series of a function Γx (prop. 11), and the equality of mixed higher-order derivatives (prop. 12). All of this is proven (not always up to modern standards of proof, naturally) in an analytical way. For example, prop. 8 is obtained from the power series of the exponential:

$$dx = d\Big(1 + lx + \frac{1}{2}(lx)^2 + \frac{1}{6}(lx)^3 + $$
$$+\frac{1}{24}(lx)^4 + \frac{1}{120}(lx)^5 + \&c.\Big) =$$
$$= dlx + \frac{2}{2}(lx)dlx + \frac{3}{6}(lx)^2dlx + \frac{4}{24}(lx)^3dlx +$$
$$+\frac{5}{120}(lx)^4dlx + \&c. =$$
$$= \Big(1 + lx + \frac{1}{2}(lx^2) + \frac{1}{6}(lx)^3 + \frac{1}{24}(lx)^4 +$$
$$+\&c.\Big)dlx =$$
$$= x\,dlx.$$

In the rest of the book appear the simplest geometrical applications: the fluxions of the area under a curve, of the arc length of a curve, and of the volume of a simple solid.

Most of the following books are dedicated to applications or particular topics in the calculus. Book 16 is instead dedicated to trigonometry, but with a peculiar organization (for an introductory text) that makes the calculus central: one of the earliest propositions gives the fluxion of the sine, from there the power series for the sine and cosine are derived, and it is from these that comes the formula for the sine of the sum of two arcs.

In book 17, we find topics of elementary differential geometry of curves: multiple points, asymptotes, radius of curvature. In other words, geometrical applications similar to those that are so important in [Bézout, 1774].

Book 18 gives several techniques of integration (such as partial fraction decomposition), L'Hôpital's rule (proven using Taylor series expansions of the numerator and of the denominator), and the Bernoulli series of a function Γx.

Book 19 addresses very quickly (in only 6 pages) several aspects of differential equations: "exact fluxions", homogeneous equations, integrating factors, higher-order linear equations [Baroni, 2001].

Book 20 gives an introduction to the calculus of finite differences.

Book 21 is a miscellany, probably compiled from several short manuscripts left by Cunha on diverse topics, by whoever arranged for the final publication of [Cunha, 1790]. Some of these topics are not related to the calculus, while others are. The latter include a couple of improper integrals, the condition $d\Gamma x = 0$ for a maximum of Γx (which had not been given before), and a very short introduction to the calculus of variations.

Summing up, as an introduction to the calculus, the relevant sections in [Cunha, 1790] are very ambitious in scope, but often too brief; it was, generally, an up to date text at the time (more so than Bézout's); and, of course, its definition of fluxion (along with its handling of infinitesimals) was very innovative, even in an European context.

4 Early attempts at research

The Academy of Sciences of Lisbon was founded in the final days of 1779. This was the first institution in Portugal with the goal of promoting scientific research — including mathematical research.

In the 1790s two volumes of memoirs were published containing mathematics. In total, four of those memoirs can be classified under "calculus": three in the first volume, and one in the second.

In the first volume (published in 1797 but with articles written in the 1780s), two pieces concern an approximation method for integrals by Alexis Fontaine (1704–1771). The Academy had proposed for 1785 a prize for a proof of Fontaine's method and a study of its (rate of) convergence, which was won by Manuel Joaquim Coelho da Maia (1750–1817), one of the first batch of doctors in mathematics from Coimbra. The winning entry was the subject of harsh criticism by José Anastácio da Cunha, which prompted Monteiro da Rocha to write some additional comments, in defence of the Academy's honour. Coelho da Maia's solution [Maia, 1797] is indeed mediocre, but [Monteiro da Rocha, 1797] contains valuable additions about the rate of convergence of the method [Figueiredo, 2011, ch. 9].

Also in the first volume, there is a memoir by Francisco Garção Stockler (1759–1829) on the "true principles of the Method of Fluxions" — like Anastácio da Cunha, Stockler admired Newton and d'Alembert, and his purpose was to expand on ideas that those two mathematicians had supposedly only sketched. But he was neither very original nor very clear. Briefly, Stockler

1. defined "fluent" as a variable quantity, in the traditional 18th-century sense, explicitly admitting that a fluent increases or decreases in intervals of time — a modern reader might interpret Stockler's fluents as functions of a time variable;
2. then considered "hypothetical fluxions", which were ratios between increments or decrements of fluents and the corresponding time intervals, and "proper fluxions", which had an unclear definition (the increments or decrements that the fluents' "tendency" to increase or decrease could produce in a unit of time) but could be calculated as limits of hypothetical fluxions;
3. and finally used power series expansions to calculate those limits.

Stockler also published with the Academy a small booklet on limits [Stockler, 1794], far less ambitious but quite interesting. Inspired by the Swiss Simon l'Huilier (1750–1840), Stockler assumed that there are two cases when a variable has a limit: it can be an *increasing limit* or a *decreasing limit* (that is, they assumed that only monotonic variables could had limits). But, unlike l'Huilier, Stockler devoted a great deal of attention to *variables that decrease without limit*; in modern terms, these are variables with limit zero. The first section of [Stockler, 1794] uses elementary but careful ε-δ arguments to develop an extensive arithmetic of such variables. This is then used in the second section, on variables with (non-zero) limits, by means of a Fundamental Principle: if $Z = A \mp z$, where Z is a variable, A a constant, and z a variable that decreases without limit, then A is the limit of Z. This makes Stockler's proofs much less tiresome than those of l'Huilier, who had mostly written in algebraic language Greek-style exhaustion arguments. The third and fourth section deal with trigonometric, logarithmic, and exponential functions.[3] In spite of Stockler's careful treatment of more elementary limits, his handling of infinite series is not up to modern standards; for instance, he uses them to "prove" that the limit of any function of a variable equals the function of the limit of the variable. However, in a time when almost all limit arguments were vague at best, [Stockler, 1794] is worthy of note.

The second volume of memoirs from the Academy of Sciences (published in 1799) includes another article by Stockler on the calculus [Stockler, 1799]. This is a quite long (100 pages) attempt at simplifying and systematizing conditions for exact differentials (Stockler calls them "exact fluxions"). It is explicitly inspired by (early) works of Condorcet, who had tried to create a general theory of integration [Gilain, 1988], himself inspired by works of Fontaine and Euler.

5 Final remarks

The effective introduction of the calculus in Portugal occurred relatively late. However, in the final three decades of the 18th century a number of texts were published in Portuguese about the calculus, which was by then well established as part of mathematical curricula.

Also, the calculus had a prominent place in the first attempts at organized mathematical research in Portugal — partly reflecting the place it had in contemporary European mathematical research.

Most of the 19th century would be a period of stagnation in Portuguese mathematics, but that was not foreseeable around 1800.

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[3] For more details on the contents of [Stockler, 1794], see [Saraiva, 2001].

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